INTRODUCTION

HVE is a vehicle dynamics and collision simulation package that utilizes mechanical and geometric models of vehicles to conduct an analysis. There are various modules within the HVE simulation package that allow the user to conduct a simulation of either vehicle dynamics and/or a collision. HVE currently has a good vehicle database with approximately 200 vehicles. Time constraints limit not only the ability to create but also the number of vehicles that can be created by Engineering Dynamics Corporation (EDC). There are also a number of mechanical parameters that are not measured. These result in the simulation models either containing some “generic data” or not utilizing some data fields. The basis for the generic data has previously been published (Siddall, Day, 1996). The addition of DamageStudio into HVE has placed more emphasis on an accurate vehicle geometric model. Therefore, it is desirable that the number of vehicle models in the HVE program increase, including more modern vehicles.

Vehiclemetrics is compiling a new vehicle database for use in HVE to increase the number of vehicles for use in HVE drastically. The procedure currently used by EDC for building vehicle geometry, as well as various input mechanical parameters for the vehicles, has been published by Garvey (2000) and Day (1995) and is also presented annually at the HVE Forum. The goal of this paper is not to repeat a discussion of the various input parameters. Rather, it is to present not only an overview of the process that Vehiclemetrics uses to generate vehicle interior and exterior geometry but also a mechanical property dataset for use in HVE. We have highlighted areas where Vehiclemetrics’ method and EDC’s method differ. The intent of the Vehiclemetrics vehicle database is to supply modern vehicle mechanical datasets that are vehicle specific and, in turn, minimize the use of generic data. At this time, all vehicle parameters cannot be measured within budget constraints, and notes are made regarding planned areas of improvement and future research areas.

The most up to date measurement equipment and software are being utilized to acquire data for mechanical and geometric models of vehicles. The use of this equipment allows for efficient testing and subsequent model creation. This paper summarizes the procedure for the creation of geometric models and mechanical parameters needed for building the HVE vehicle database file.

GENERAL VEHICLE INFORMATION

The following general parameters are recorded through a vehicle inspection:

- Year
- Make
- Model
- Vehicle class
- Body style
- Similar vehicles (Sisters & Clones)
- Vehicle Identification Number
- Date of manufacture
- Odometer reading
- Driver position
- Number of axles
- Gross axle weight rating – front
- Gross axle weight rating – rear
- Gross vehicle weight rating
- Fuel level
Numerous photographs are taken of the vehicle. These photographs aid in the subsequent creation of the vehicle geometric models. Specific photographs are also taken of the vehicle’s rim for use as texture maps in the HVE vehicle model.

**VEHICLE GEOMETRY AND DIMENSIONAL DATA**

The three-dimensional vehicle geometry is acquired using a laser scanner. Multiple scans of the vehicle are taken to acquire the geometric data for both the interior and exterior of the vehicle. Trunk or cargo space, the engine compartment, suspension/brake assemblies, and the vehicle underbody are also scanned but are not yet utilized in current vehicle models. Based upon the scan data, the following dimensional specifications are measured:

i) Front overhang  
ii) Overall length  
iii) Overall width  
iv) Overall height  
v) Ground clearance  
vi) Track width  
vii) Wheelbase  
viii) Frontal area  
ix) Side area

Wheelbase and track width of the vehicle are also measured with a 4-wheel alignment system utilizing three imaging cameras to provide real-time 3D measurements. The data from the 4-wheel alignment system is used for establishing the four wheel positions in the vehicle model.

During the processing of the laser scan data, the axis co-ordinate system is set to the centre of gravity of the vehicle (positive X – forward, positive Y – right side, positive Z – downward). Once the laser scan data has been acquired and processed, the geometric surface topology of both the interior and exterior is created using commercially available modeling software. All surface topology is modeled using four-sided polygons for better and more predictable sub-division and/or tessellation. A semi-uniform grid spacing of the four-sided polygon method is also used for more uniform crush computations within DyMESH (Figure 1). The polygon count targeted for the current exterior modeling process is approximately 6000 to 7000 polygons. This is greater than the current count typically used by EDC (approximately 4000 polygons); however, based upon our tests, we have not experienced a significant increase in simulation times. As a quality check, a comparison of the modeled geometry versus our laser scan is completed (Figure 2).

![Figure 1: Typical vehicle in the HVE Environment](image-url)
The photographic documentation taken during our initial inspection of the vehicle also allows for texture mapping of various vehicle features and/or the entire vehicle exterior. Currently, HVE does not accept texture mapping for the vehicle body; however, this option could easily be implemented into vehicle geometric models if the program is modified in the future (Figure 3).

Figure 2: Comparison of HVE Model to Point Cloud Data

Figure 3: Texture Mapped Vehicle in the HVE Environment
VEHICLE MASS DATA

The vehicle mass data gathered during our vehicle inspection includes:

i) The total mass at each wheel position.
ii) The three-dimensional Centre of Gravity (CG) location of the vehicle.
iii) The unsprung mass at each wheel position.
iv) The mass of a tire and rim.

The total vehicle mass and CG location in the X and Y is determined by simultaneously measuring the force below each wheel using four wireless scales. The total vehicle mass is the sum of the mass at the four scales. The CG_{xy} location is calculated by using the measured total mass, wheelbase, and track width(s). The CG_x and CG_y locations are computed as indicated in Equations 1 and 2 (Milliken, 1995, p. 666-670) below:

\[
CG_{x, \text{total}} = WB - WB \times \frac{W_r}{W_t} \quad \text{[rearward of front axle]} \quad (1)
\]

\[
CG_{y, \text{total}} = \frac{W_{Rf}}{W_t} \left( t_f - \frac{(t_f - t_r)}{2} \right) - \frac{W_{Lf}}{W_t} \left( \frac{(t_f - t_r)}{2} \right) + \frac{W_{Rr}t_r}{W_t} - \frac{t_r}{2} \quad \text{[right of centre line]} \quad (2)
\]

The CG_z (height) is measured by raising the rear axle of the vehicle. The front wheels are located on slip plates to allow the vehicle to translate rearward and eliminate the introduction of horizontal forces. The vehicle is raised under the unsprung mass to prevent suspension sag.\(^1\) The inclination of the vehicle varies depending on the wheelbase of the vehicle. The vehicle typically undergoes a total change in inclination of 6 to 9 degrees in this test. Reaction loads at the loaded wheels (front), angle of the raised vehicle, wheelbase, and tire rolling radius, along with various other parameters, are measured monitored throughout the test.

\[CG_z, \text{total} = \frac{WB \times \Delta W_f}{W_t \times \tan \alpha} + r_{tire} \quad (3)\]

To assess the unsprung mass at the wheel, we utilize a quarter vehicle model (Figure 4), as summarized by Tsymberov (1996), and a commercially available suspension analyzer.

\[
f_{hop} = \frac{1}{2\pi} \sqrt{\frac{k_t + k_s}{m_U}} \quad (4)\]

\[
m_U = \frac{k_t + k_s}{(2 \times \pi \times f_{hop})^2}
\]

\(^1\)If excessive suspension sag occurs, the rear wheels are placed on fabricated aluminum boxes.

\(^2\)Wheel hop is the minimum contact force between wheel and ground during suspension test.
The above method of calculating unsprung mass is different from the method utilized by EDC. Currently, EDC utilizes the following assumptions for the calculation of unsprung mass:

- If the wheel has independent suspension, then the unsprung mass is assumed to be \( m_u = m_{\text{wheel}} = m_{\text{tire}} + m_{\text{rim}} \).
- If the wheel location has a solid axle, then the unsprung mass is assumed to be \( m_u = \frac{m_{\text{axle}}}{2} + m_{\text{wheel}} \).

HVE uses the CG height of the sprung mass and the unsprung mass from its vehicle models to perform calculations. To approximate location of the sprung mass and unsprung masses, we utilize the equations presented by Milliken (1995, p. 671-673). This methodology assumes the height of the unsprung mass is located at the wheel centre and the lateral location of the sprung mass is at the tire centre line. The equations we use to calculate the CG sprung mass are appended for reference. EDC uses these same equations.

**INERTIAL PARAMETERS**

Commercially available equipment to undertake whole vehicle inertial measurements are extremely costly and currently outside the scope of our budget. Therefore, we are utilizing the method being used by EDC to estimate inertial properties, as described by Garvey (2000), and through personal correspondence with personnel of EDC. The method currently used is based upon the National Highway Transportation Safety Association (NHTSA) inertia database that was previously published. A curve fit of this data is completed based upon the total vehicle mass. The unsprung mass inertias are also currently calculated based upon the same methods used by EDC.

Wheel inertia is a large component of the unsprung mass inertia. A test apparatus has recently been obtained to measure wheel (tire and rim) spin inertia, and an additional test device is being developed to measure wheel (tire and rim) steer inertia. We are currently conducting research to assess alternative methods of calculating or measuring vehicle inertia (total vehicle and sprung mass inertia).

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**SUSPENSION PARAMETERS**

The suspension parameters primarily consist of data relating to the springs (coil, leaf, or torsional springs); the anti-roll bar (auxiliary roll stiffness); and the shock absorbers at each wheel. We conduct tests to:

i) Measure the wheel centre rate,
ii) Measure the tire rate,
iii) Measure the auxiliary roll stiffness, and
iv) Estimate the damping rate at each wheel.

**Wheel Centre Rate and Tire Rate**

HVE does not use actual “spring stiffness” or geometry to transfer the load from the contact patch to the coil spring. Instead, it utilizes the wheel centre rate and the tire stiffness. The “wheel centre rate” is the vertical force per unit of vertical displacement of the wheel relative to the chassis (Milliken, 1995, p. 581). To obtain wheel centre rate measurements for the vehicle, an automotive alignment lift equipped with slip plates under the front and rear wheels is used. The vehicle is raised above the lift by lifting under the vehicle chassis until the suspension is fully extended and the wheels are airborne (Figure 5a). The vehicle is then lowered (through the rebound phase) until reaching its static ride height (Figure 5b) and then compressed downward (through the jounce phase) onto the lift (Figure 5c) by pulling down on the vehicle chassis.

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During the test, wheel movement relative to the chassis and tire deflection are measured in conjunction with the tire-ground contact force. The scales used to measure tire contact force are placed on the unlocked slip plates on the alignment lift. This allows the suspension to move freely through its designed range of motion. Wheel movement relative to the body is measured with string potentiometers, and tire deflection is measured using laser sensors. Measurements are recorded incrementally throughout the range of suspension travel. Wheel positions are also measured with the 4-wheel alignment system utilizing three imaging cameras to provide real-time 3D measurements.

The wheel centre rate in HVE is a linear approximation of wheel load versus displacement of the wheel relative to the chassis. Therefore, to assess the wheel centre rate for the vehicle, a linear curve fit is created using the collected data. The slope of the line represents the vehicle’s wheel centre rate at each individual wheel (Figure 6). The wheel centre rate utilized for the front and rear wheels is an average of the left and right sides.

During the test, wheel movement relative to the chassis and tire deflection are measured in conjunction with the tire-ground contact force. The scales used to measure tire contact force are placed on the unlocked slip plates on the alignment lift. This allows the suspension to move freely through its designed range of motion. Wheel movement relative to the body is measured with string potentiometers, and tire deflection is measured using laser sensors. Measurements are recorded incrementally throughout the range of suspension travel. Wheel positions are also measured with the 4-wheel alignment system utilizing three imaging cameras to provide real-time 3D measurements.

A similar analysis method is undertaken to approximate the static tire rate (Figure 7).

Full suspension travel distances are recorded. The maximum rebound is assessed by measuring the travel distance from ride height to full rebound with the vehicle lifted off of the lift. For some vehicles, compressing the suspension to full-jounce is not possible. In these cases, the travel to suspension stop is measured. If there is no stop, the coil spring dimensions are utilized to assess additional travel that would occur until the coil spring bottoms out.
**Alignment Data versus Jounce and Rebound**

During the wheel centre rate test, the wheels are equipped with targets for use with the 4-wheel alignment system. Changes in camber, toe, track width, wheelbase, and other parameters are measured through the travel of the suspension. The recorded parameters are then used to create the following tables for HVE:

1. Camber versus jounce/rebound,
2. Halftrack change versus jounce/rebound, and
3. Roll steer versus jounce/rebound.

Illustrations of an example set of data for a vehicle is in Figures 8 to 10.

HVE also accepts input data for anti-pitch versus jounce/rebound. Currently, there is no test method to assess anti-pitch versus jounce/rebound.

**Auxiliary Roll Stiffness**

The auxiliary roll stiffness is assessed by placing the vehicle on the alignment lift with a scale under each wheel. The longitudinal centre of gravity is calculated, and this location is marked on the vehicle. A hydraulic jack is placed under the vehicle at the longitudinal location of the centre of gravity on the passenger side rocker panel. The jack is raised, inducing a roll to the vehicle body. All of the wheels on the vehicle are free to translate laterally on the slip plates as the vehicle is lifted. Geometric measurements regarding body roll, suspension travel, and load transfer are recorded. Measurements are obtained at multiple angles of body roll. The vehicle is then restored to its original position, the front and rear (if equipped) anti-roll bar(s) are disconnected at both ends, and the test is repeated. The auxiliary roll stiffness is assessed by calculating the difference between both tests.

The front and rear measured roll stiffnesses for a vehicle are illustrated in Figure 11. The front auxiliary roll stiffness utilized in our mechanical model is the average value obtained from our front tests while the rear auxiliary roll stiffness is the average value obtained from our rear tests.
The method we utilize is different from the current method used by EDC. The auxiliary roll stiffness values obtained in the above-described technique are also compared to the values obtained by analyzing the geometric installation ratio of the anti-roll bar and the anti-roll bar physical measurements, as currently used by EDC. An alternative comparative method is also outlined by Milliken (1995, p. 592).

**Damping Rate**

To assess the damping rate at the wheel, we utilize a quarter vehicle model and the apparatus as summarized by Tsymberov (1996). For this testing procedure, each individual wheel is oscillated vertically from a frequency of 25 Hz to 0 Hz. The oscillation frequency and the load between the vehicle tire and suspension tester is measured. Body-to-wheel and wheel-to-ground displacements are recorded using string potentiometers and laser sensors previously used during the ride rate test. Accelerometers are also placed on the sprung and unsprung masses at each wheel position. The displacement data and accelerometer data are recorded at 1000 Hz.

The input necessary for HVE is the damping rate (C). The damping ratio ($\xi$) is defined as the amount of damping in a system (C) divided by the critical damping rate ($C_{cr}$). The critical damping rate is a function of the spring stiffness and tire stiffness at the wheel as well as the mass at that wheel.

If the damping ratio is less than 1, there is some “overshoot” in the system. If the damping ratio = 1, then the system is “critically” damped. If a system is critically damped, there is no oscillation in the vehicle body after being subjected to a force input. If a system has a damping ratio greater than 1, the system returns smoothly but slowly to its initial condition. Tsymberov (1996) reports that typical damping ratios of passenger cars are 0.2 to 0.4.

There are two methods currently available to assess the damping ratio or damping rate at each wheel.

I Use the phase angle and adhesion data from the suspension tester (damping ratio is calculated).

II Solve the system of differential equations for quarter vehicle model.

In each of these methods, we assume there is no damping in the tire.

We have not yet determined which of the above methods we will utilize for our database; however, the data required for both methods are recorded and the critical damping rate is also calculated.

The above methods of assessing damping rate are different from the current method utilized by EDC. EDC assumes $C = C_{cr}$. Currently, vehicles in the Vehiclemetrics database incorporate the same assumption as EDC; however, these values will be updated in a future release.

**STEERING PARAMETERS**

The vehicle is positioned on an automotive lift with rotating slip plates positioned underneath the front wheels. The steering assembly is rotated lock-to-lock while measuring steering wheel angle (rotation) and the independent front wheel (steer) angles. HVE allows for the input of a single steering gear ratio (steering wheel rotation/tire rotation) (deg/deg). The collected data is plotted and a linear curve fit is applied. A sample graph of the steering ratio test is illustrated in Figure 12.
The Ackermann angle and error are also measured, and the number of turns the steering wheel turns lock-to-lock is also recorded.

**BRAKE PARAMETERS**

The recommended method of applying braking to a vehicle when using SIMON is to calculate the brake torque at each wheel in response to a force being applied at the pedal. If other methods (i.e., Wheel Brake Force and Percent Available Friction) are used, the wheel spin degree of freedom and the simulation of ABS and ESS are not possible (EDC, 2005).

The equations utilized by HVE for the brake torque versus pedal force method are summarized below. The purpose of these various brake system parameters is to calculate the brake torque created at each wheel for a given pedal force input.

$$T_b = T_{ratio} \times (p - p_o)$$

Where:

- $T_b$ = Attempted brake torque at wheel [N.m]
- $T_{ratio}$ = Brake Torque Ratio (attempted brake torque per unit of line pressure) [N.m/kPa]
- $p$ = Current application pressure at wheel cylinder [kPa]
- $p_o$ = Pushout pressure [kPa]

To model the effects of brake proportioning, the proportioning pressure is identified and the pressure at the wheel cylinder(s) is calculated by the following:

$$p = p_{table}, \text{ if } p_{table} \leq p_{proportion}$$
$$p = p_{proportion} + \eta(p_{table} - p_{proportion}), \text{ for } p_{table} > p_{proportion}$$

$$p_{table} = F_{table} \times R \text{ [kPa]}$$

Where:

- $p$ = Brake system pressure at wheel [kPa]
- $F_{table}$ = User input from “At pedal” table [N]
- $R$ = Brake pedal ratio [kPa/N]
- $p_{proportion}$ = System pressure when proportion begins [kPa]
- $\eta$ = Proportioning ratio

The method we utilize to measure wheel brake force versus brake pedal force is a roller brake tester. The vehicle is driven onto a set of rollers where each axle is tested in sequence. Electric motors drive individual rollers for each wheel at 5 km/h. While the brake pedal is applied, pedal force versus brake force at each wheel is recorded until lock-up is achieved on a friction surface with $\mu = 0.9$. The brake force at each wheel is measured independently. This test procedure is repeated for each axle. Individual wheel drag values are also measured with the vehicle in Neutral. Drivetrain inertia can also be measured.

This method of measuring brake force allows us to bypass the calculation of brake line pressures. To apply our data to the brake torque calculations used in SIMON, we utilize our previously-measured tire rolling radius and assume the brake pedal ratio $(R) = 1$. Therefore, instead of utilizing brake line pressures, the pedal force is used for subsequent calculations. This results in the brake torque ratio $(T_{ratio})$ having the units of brake torque [N.m] produced per unit of pedal force [N]. This $T_{ratio}$ is the value utilized for the Vehiclemetrics database. As a result, the brake torque calculation remains the same:

$$T_b = T_{ratio} \times (F - F_o)$$

Where:

- $F$ = Pedal force
- $F_o$ = Pushout pedal force

A sample analysis of our brake tester data is illustrated in Figure 13. This Figure illustrates $T_{ratio}$ for front and rear wheel locations. It also illustrates the “$F_{proportion}$”

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4 Roller brake tester BDE2304 K, Snap-on Equipment Testing Division, GmbH.
and the “proportioning ratio (\(\eta\))” for the rear brake system for use in HVE.

![Brake Parameters](image)

**Figure 13:** Sample Brake Torque Ratio Analysis

As described, since pedal force versus wheel force is measured, the methodology used to calculate braking parameters is somewhat different from what is currently utilized for building vehicles by EDC.

**POWERTRAIN PARAMETERS**

The current method of deriving input parameters for wide-open throttle (WOT) and closed throttle horsepower and torque curves into the vehicle is analogous to the method utilized by HVE (Garvey, 2000). Future measurement of the WOT is planned through road testing; however, it has not yet been implemented.

Differential and transmission ratios are obtained through manufacturers’ specifications.

**STIFFNESS COEFFICIENTS**

The crush stiffness values are currently calculated using available published crash tests for frontal, side, and rear coefficients. When more than one vehicle crash test is available, an average value for the stiffness coefficients is presented. If a crash test for a specific vehicle cannot be located, then generic values reported by Siddall (1996) are assigned according to the proper vehicle class.

The bottom stiffness of the vehicles will be assigned the current stiffness values assigned to vehicles built by EDC. Methods to calculate top stiffness values are being researched.

**VEHICLE AERODYNAMIC DRAG**

The vehicle aerodynamic drag calculation employed in HVE is dependent upon the Aerodynamic drag constant \(C_a\), projected surface area \(A_p\), air density \(\rho\), and aerodynamic drag coefficient \(C_d\).

\[
C_a = \frac{1}{2} C_d \rho A_p \tag{5}
\]

For the Vehiclemetrics database, frontal, rear, and side areas are calculated using the laser scan data. If a published frontal \(C_d\) can be obtained, then it is incorporated into the vehicle model. If published data cannot be located, then the values reported by Garvey are used (passenger cars = .35, vans = .45, SUVs = .4, pickups = .45). An estimate of the frontal aerodynamic drag coefficient is planned using a road test conducted similar to SAE J1263.

**OTHER VEHICLE PARAMETERS**

Each of the Vehiclemetrics vehicles will come equipped with tires from the HVE Generic Tire Database. There are a number of vehicle parameters which are currently within the HVE vehicles with a “default” type parameter. These parameters, as well as their value, are listed below:

i) Vehicle torsional stiffness
ii) Drivetrain Inertia
iii) Suspension coulomb friction
iv) Suspension null band
v) Roll centre height
vi) Suspension linear, cubic stop rates, and energy ratio
vii) Steering stop stiffness and damping
viii) Steering column stiffness, friction, inertia
ix) Steering linkage play, mass, damping, and friction lag

Currently the Vehiclemetrics vehicle database will come equipped with the parameters as assumed by EDC. There are already plans to measure some of these parameters.
SUMMARY

A method has been developed and a new database is being compiled to provide HVE users an updated vehicle database. The methods utilized allow a vehicle to be tested in less than a day, and the final vehicle model can be completed in less than one week. The database is inclusive of both interior and exterior geometry as well as mechanical datasets. The model geometry is based upon laser scan data and is modeled using a variety of software applications. The increase in number of new geometry files complements the recent release of DamageStudio for use in EDSMAC4 and SIMON. The majority of the mechanical dataset will be vehicle specific. Research to obtain additional vehicle specific data is ongoing. The current number of vehicles in the database is approximately 75 and growing steadily. It is estimated that 100 to 150 new vehicles will be released on an annual basis. Ongoing support of and creation of this database is planned and the vehicle database will be available to HVE users in the near future (planned April 2012).

CONTACT INFORMATION

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REFERENCES


“SIMON Simulation Model,” Engineering Dynamics Corporation, 2005, p. 4-24 to 4-25.


Vehicle Year & Model Interchange List (Sisters & Clones List), provided by Gregory C. Anderson, of Scalia Safety Engineering, and distributed by Neptune Engineering Inc.
APPENDIX A - FORMULAS

Calculation of the $CG_{sprung}$ Location

$$CG_{x,sprung} = WB - \frac{W_T \times t_r - W_{fU} \times WB}{W_s} \text{ [rearward of front axle]}$$

$$CG_{y,sprung} = \frac{W_T}{W_s} \left[ \frac{W_{Rf}}{W_T} \left( t_f - \frac{(t_f - t_r)}{2} \right) - \frac{W_{lf.r}}{W_T} \left( \frac{(t_f - t_r)}{2} \right) + \frac{W_{Rr.t}}{W_T} \right] - \frac{W_{Rr.U.t}}{W_s}$$

$$\quad - \frac{W_{Rf.U}}{W_s} \left( t_f - \frac{(t_f - t_r)}{2} \right) - \frac{W_{lf.U}}{W_s} \left( \frac{(t_f - t_r)}{2} \right) - \frac{t_r}{2} \text{ [right of centre line]}$$

$$CG_{z,sprung} = \frac{W_T}{W_s} CG_z - \frac{W_{fU}}{W_s} R_{f} - \frac{W_{R,U}}{W_s} R_{r}$$
APPENDIX B - NOMENCLATURE

$CG_{x,\text{total}}$: Total vehicle centre of gravity in the x-axis

$CG_{y,\text{total}}$: Total vehicle centre of gravity in the y-axis

$CG_{z,\text{total}}$: Total vehicle centre of gravity in the z-axis

WB: Wheelbase

$W_f$: Total weight on the front axle

$W_r$: Total weight on the rear axle

$W_{Rf}$: Total weight on the right front wheel

$W_{Lf}$: Total weight on the left front wheel

$W_{Rr}$: Total weight on the right rear wheel

$\Delta W_f$: Change in total weight of the front axle

$t_f$: Front track width

$t_r$: Rear track width

$r_{\text{tire}}$: Tire rolling radius

$\alpha$: Inclination angle of the chassis during the centre of gravity test

$m_U$: Unsprung mass

$k_t$: Tire rate

$k_s$: Wheel centre rate

$f_{\text{hop}}$: Wheel hop natural frequency

$C$: Damping rate

$C_{cr}$: Critical damping rate