
Differences Between EDVDS and Phase 4

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ABSTRACT

Motor vehicle safety researchers have used the Phase 4 vehicle simulation model for several years. Because of its popularity and ability to simulate the 3-dimensional dynamics of commercial vehicles (large trucks and truck tractors towing up to three trailers), the Phase 4 model was ported to the HVE simulation platform. The resulting model is called EDVDS (Engineering Dynamics Vehicle Dynamics Simulator). This paper describes the procedures used in porting Phase 4 to the HVE platform. As a result of several assumptions made during the development of Phase 4, the port to EDVDS required substantial changes. The most significant modeling difference is the removal of the small angle assumption, allowing researchers to study complete vehicle rollover. Also significant is EDVDS's use of HVE's `GetSurfaceInfo()` function, allowing the vehicles' tires to travel over any 3-D terrain of arbitrary complexity. These and other changes in the model are described in the paper. The paper also includes a validation study, comparing Phase 4 and EDVDS simulation results. For most maneuvers, the results are quite similar. Differences become apparent as the maneuvers became more severe, owing to the removal of the small angle assumption and extensions to the tire model.

THE FIRST COMMERCIAL VEHICLE handling models were developed at the University of Michigan over 20 years ago. These models were developed with the objective of helping engineers and safety researchers evaluate new vehicle designs and meet tougher regulations imposed on the trucking

industry by the U.S. federal government. The first of these models, named Phase I, was a 15 degree-of-freedom pitch plane simulation of a vehicle towing a single trailer. Next came Phase II, a 3-dimensional simulation that extended the Phase I model by including roll and yaw degrees of freedom. Thus, the Phase II model was able to simulate the directional response from driver steering, as well as braking, inputs. The Phase III model extended the earlier work with better tire models, a path-follower model, models for double and triple trailers, and an improved braking model that included anti-lock. The project culminated in the Phase 4 model, which extended earlier work with improved suspension, brake torque and temperature, steering and frame compliance models. These projects are documented in references 1 through 5*.

The above models are traditional simulation models. To use them, the user supplies vehicle parameters (dimensions, inertias, suspensions, tires and brakes), driver controls (steering and braking inputs), payload data (inertias and location) and initial conditions (initial forward velocity). The model applies these data to a physical/mathematical model to determine the instantaneous external forces acting on the vehicle. The resulting forces and moments are used to calculate the current level of acceleration for each degree of freedom. These accelerations are then integrated twice (first to determine the velocity and second to determine the position) at the end of a small time increment. These calculations are repeated at updated time intervals to predict the vehicle's motion resulting from the driver inputs and other information. Interim calculation results, such as tire forces and suspension deflections, can also be analyzed. Vehicle designers and safety researchers use these results to help select brake system components, tires and suspension systems that maximize vehicle performance. Safety researchers also use the results to help identify accident causation and avoidability.

* Numbers in brackets designate references found at the end of the paper.

Simulation is thus accepted by vehicle design engineers and safety researchers as a useful tool. Although the Phase 4 model is quite powerful, its use has been limited by several factors. Most of these factors fall into one of two categories: First, owing to its complexity, program execution requires substantial time and effort to develop a valid input data set representing the subject vehicles. Second, the program was designed (in the 1970's) for use in batch mode, and had no user interface. The numeric output from Phase 4 was voluminous and, thus, was difficult and time-consuming to interpret. Potential for user error was high because there was no inherent means available for interpreting or visualizing the results. To address these issues, the Phase 4 model was ported to the HVE simulation environment [6,7]. The resulting program was named EDVDS (Engineering Dynamics Vehicle Dynamics Simulator [8]).

During the porting process, several issues arose that resulted in significant changes to the original Phase 4 model. These changes included removal of the small angle assumption

and initial equilibrium assumption, rewriting the equations of motion to allow full rollover, the addition of a drivetrain model, and the use of a more generalized terrain model.

This paper describes the EDVDS simulation model and compares it to its predecessor, Phase 4. An overview of the process of porting Phase 4 to the HVE simulation environment is presented, followed by a discussion of the basic vehicle, environment and execution models. Similarities and differences in the models are addressed. Finally, a validation study is presented comparing results from EDVDS and Phase 4.

MODEL OVERVIEW

EDVDS is a dynamic simulation analysis of a unit or combination vehicle (tractor towing up to three trailers). The first trailer is a semi-trailer, while the second and third trailers are full trailers with the front of the trailer supported by a dolly (fixed or converter type). Front and rear suspensions may have single or tandem axles. Tandem axles may be of the 4-spring or walking beam variety. The vehicle model includes a user-definable steering system, brake system and drivetrain.

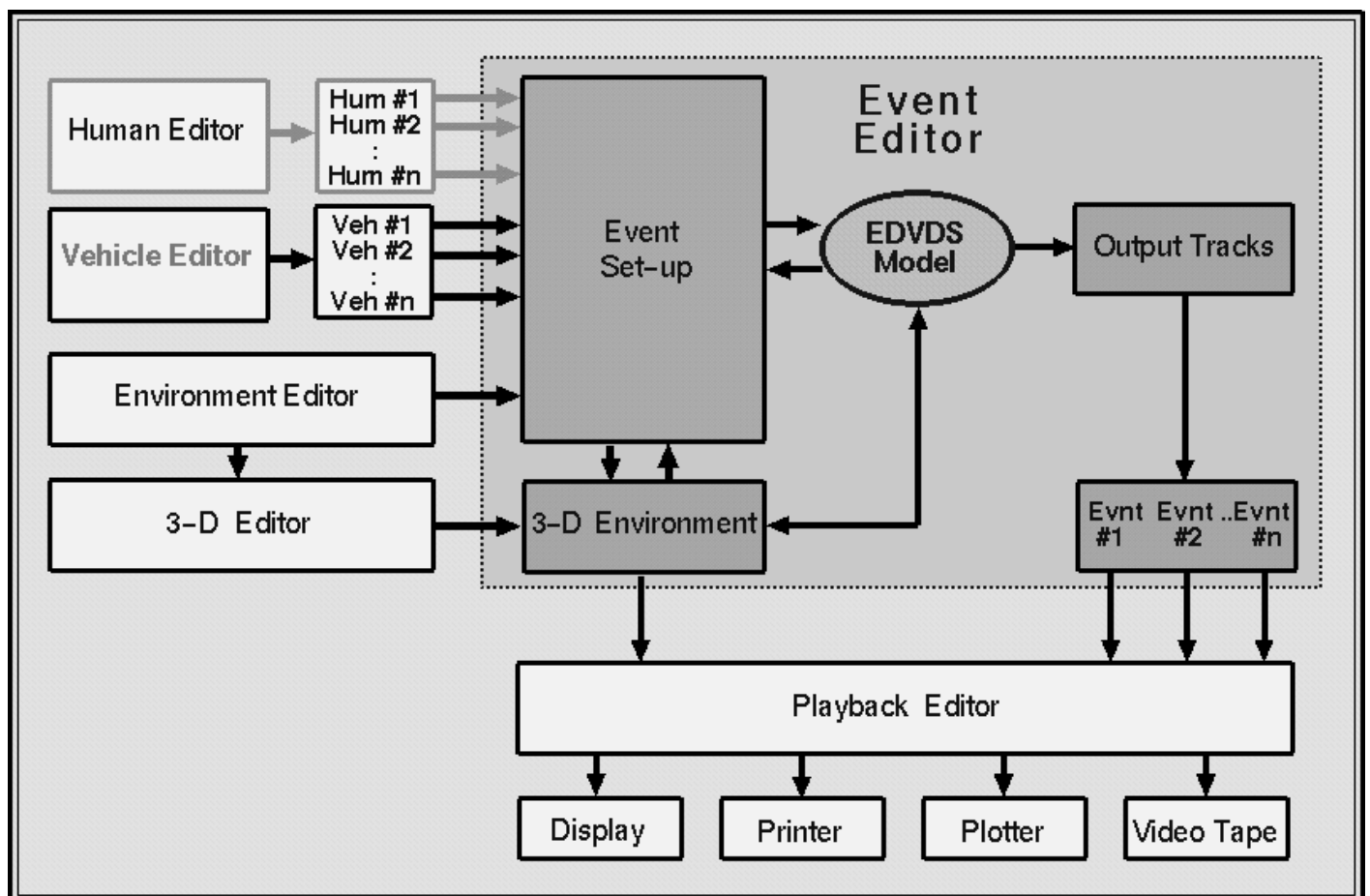


Figure 1 - HVE Simulation Environment, with EDVDS as the current simulation model in the Event Editor.

The mathematical model for EDVDS was derived from the Phase 4 program. Phase 4 was developed for the Motor Vehicle Manufacturer's Association and U.S. Federal Highway Administration by the University of Michigan Transportation Research Institute [1-5]. EDC ported the original Phase 4 code to execute in the HVE simulation environment (see Figures 1 and 2). The process of porting Phase 4 to the HVE environment involved the following steps:

- rewriting the model using the C programming language (Phase 4 is programmed in FORTRAN)
- replacing the Phase 4 input and output routines with HVE input and output interface functions

The following features of the Phase 4 model remain unchanged in EDVDS:

- Vehicle Configurations - The vehicle configurations supported by EDVDS are the same as those sup-

ported by Phase 4, that is a unit truck or tow vehicle pulling up to three trailers. The first trailer is a semi-trailer attached to the tow vehicle using a fifth wheel.

- Suspension Configurations - Like Phase 4, all EDVDS suspensions use solid axles; independent suspensions are not supported. The front axle of the tow vehicle is a single axle. All other suspensions may be of the single or tandem axle variety.
- Connection Forces - The basic procedure for calculating inter-vehicle connection forces and moments in EDVDS remains unchanged from Phase 4. Like Phase 4, fixed and converter dolly types are supported.
- Brake Model - Brake torques at each wheel are computed in the same manner for both programs (the Phase 4 anti-lock model is not implemented in EDVDS).

The basic EDVDS vehicle model is shown in Figure 3.



Figure 2 - HVE simulation environment with an EDVDS simulation of an alternate ramp traversal, used for studying the effect of torsional frame compliance on the handling and control of a set of highway doubles.

An interim model, internally named EDPH4, resulted from the straight port to HVE. EDPH4 was essentially identical to Phase 4, except that it ran in the HVE environment. Preliminary validation was performed to ensure that the Phase 4 model was intact after the port. The EDPH4 code was then modified as follows:

- The equations of motion were rewritten and extended to allow complete 3-dimensional motion. The small angle assumption was eliminated.
- The equations of motion were generalized to allow the vehicle positions to be at any location at the start of the simulation (Phase 4 assumes the tow vehicle begins the simulation in equilibrium at the earth-fixed origin).
- The Phase 4 user-written ROAD subroutine was replaced by HVE's `GetSurfaceInfo()` function, allowing the vehicles to drive on an arbitrary 3-D surface.
- The in-line solution of the equations of motion was replaced by a simultaneous solutions procedure.
- The suspension and tire models were extended and made more robust.
- The HVE Drivetrain model was added in order to model tractive effort.
- The Phase 4 driver model was replaced with by the HVE Path Follower Model.

Equations of Motion

Three coordinate systems are required for the equations of motion:

- The vehicle-fixed x,y,z coordinate system is fixed to the vehicle's CG and defines motion with respect to the vehicle sprung mass, with x defined along the longitudinal vehicle axis, y defined to the right, and z directed down. Vehicle rotations are defined about these axes as roll (about the x axis), pitch (about the y axis) and yaw (about the z axis).
- The earth-fixed X,Y,Z coordinate system is fixed to the earth (as its name implies). The current vehicle sprung mass position is defined by this coordinate system.
- The tire axis system is fixed to each wheel position, and defines the direction of the tire axis, x' , relative to the vehicle-fixed coordinate system due to its current steer angle.

As shown in Figure 3, the vehicle sprung mass includes six degrees of freedom (X,Y,Z , roll, pitch, yaw). Each axle includes two degrees of freedom (axle z and roll). Each wheel also includes a spin degree of freedom for dynamic braking calculations.

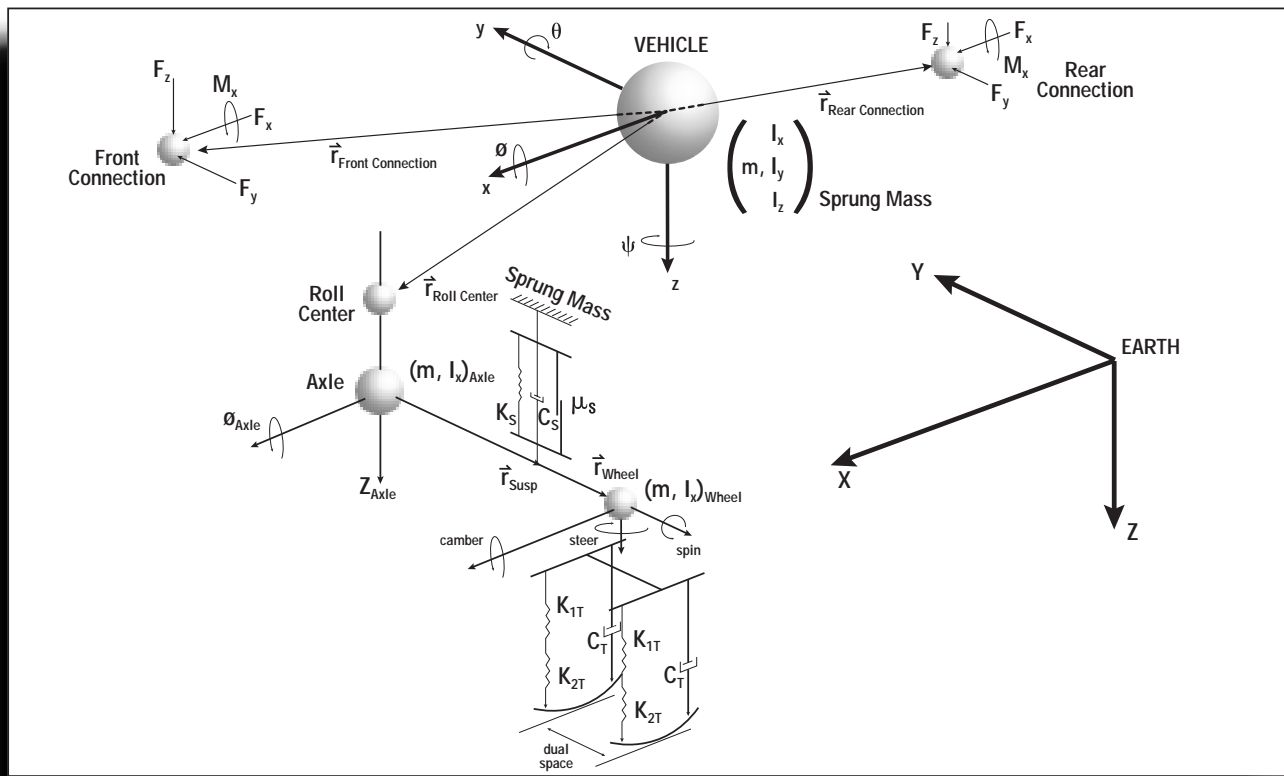


Figure 3 - EDVDS physical/mathematical vehicle model

The required equations of motion for the each vehicle sprung mass are:

$$\Sigma F_x = m(\dot{u} + wq - vr)$$

$$\Sigma F_y = m(\dot{v} + ur - wp)$$

$$\Sigma F_z = m(\dot{w} + uq - vp)$$

$$\Sigma M_x = I_x \dot{p} + qr(I_z - I_y)$$

$$\Sigma M_y = I_y \dot{q} + pr(I_x - I_z)$$

$$\Sigma M_z = I_z \dot{r} + pq(I_y - I_x)$$

where

m = vehicle sprung mass

I_x, I_y, I_z = roll, pitch and yaw rotational moments of inertia

u, v, w = forward, lateral and vertical velocities (vehicle-fixed components)

p, q, r = roll, pitch and yaw angular velocities (about the vehicle-fixed x,y and z axes, respectively)

$\Sigma F_x, \Sigma F_y, \Sigma F_z$ = summation of external forces in the vehicle-fixed x, y and z directions, respectively

$\Sigma M_x, \Sigma M_y, \Sigma M_z$ = summation of external moments about the vehicle-fixed x, y and z axes, respectively

The equations of motion for each axle are:

$$\Sigma F_{z,u} = m_u \ddot{z}_{axle}$$

$$\Sigma M_{x,u} = I_{x,u} \ddot{\phi}_{axle}$$

where

m_u = axle unsprung mass (including wheels)

$I_{x,u}$ = axle roll moment of inertia

z_{axle} = axle roll center vertical displacement (vehicle-fixed)

ϕ_{axle} = axle roll angle (vehicle-fixed)

Finally, the equation of motion for wheel spin for each wheel is:

$$\Sigma M_w = I_w \ddot{\Omega}$$

where

ΣM_w = summation of moments (brake and drive torque) acting about the spin axis

I_w = wheel total spin inertia (including tire, rim and rotating brake components)

Ω = wheel rotational displacement about spin axis

EDVDS solves these equations of motion at discrete, user-defined time intervals. The current accelerations are numerically integrated to predict position and velocity at the start of the next timestep. EDVDS uses *Hamming's Modified Predictor-Corrector* integration method [9]. Use of a predictor-corrector method helps to ensure stable results.

External Forces

External forces are applied at the tire-road interface and at the inter-vehicle connections. Because these forces completely define the motion of the vehicle(s), it is important that the user fully understand how they are calculated.

Tire Forces

Pneumatic tires do not behave in an easy-to-calculate manner. Rather, tire properties are functions of several variables and are extremely non-linear.

Vertical tire force, F_z , is calculated directly from tire radial stiffness and current tire deflection (that is, the difference between undeflected tire radius and the current distance from the wheel center to the road surface). EDVDS extended the original Phase 4 model by the use of a 2-step stiffness model, as shown in Figure 4. Mathematically, this is expressed as:

$$F_z = k_1 \delta_1 + k_2 (\delta_2 - \delta')$$

where

k_1 = tire initial radial stiffness

k_2 = tire secondary radial stiffness

δ_1 = tire radial deflection up to δ'

δ_2 = tire radial deflection greater than δ'

δ' = radial tire deflection secondary deflection point (see Figure 4)

The simulation terminates with an error message if the current tire deflection exceeds the maximum allowable value, δ_{max} , as shown in Figure 4.

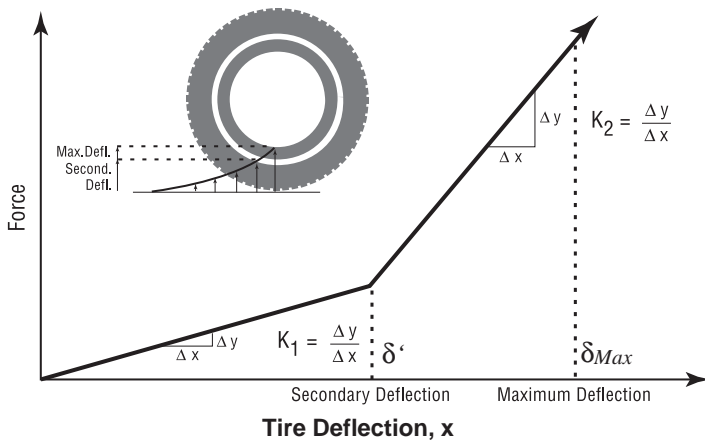


Figure 4 - 2-Step radial tire stiffness is dependent on current tire deflection.

Longitudinal tire force, F_x' , and lateral tire force, F_y' , are calculated using one of two user-specified tire models: the *linear model* or *semi-empirical model*. These tire modeling options are described below.

Linear Tire Model - The linear tire model is the simplest of models and should be used only for non-limit maneuvers. In fact, the simulation will terminate if the longitudinal tire slip exceeds 10 percent or the lateral tire slip exceeds 0.10 radians (these limits are editable).

The linear tire model uses the tire's longitudinal and cornering stiffnesses to calculate F_x' and F_y' , respectively. The linear model has no load- or speed-dependence. Rather, a simple linear relationship is assumed:

$$F_x' = C_s S$$

and

$$F_y' = C_\alpha \alpha$$

where

- F_x' = Longitudinal Tire Force (tire axis system)
- C_s = Tire Longitudinal Stiffness
- S = Longitudinal Slip
- F_y' = Lateral Tire Force (tire axis system)
- C_α = Tire Cornering Stiffness
- α = Tire Slip Angle = $\text{ATAN2}(v_{Wheel}, u_{Wheel})$

These relationships are shown in Figures 5 and 6. Note that tire forces in the linear tire model are not assumed to be limited by friction because, as mentioned above, the simulation terminates before a friction-limiting condition exists.

Semi-empirical Tire Model - The basis for the EDC semi-empirical tire model is the HSRI tire model, developed at the University of Michigan Transportation Research Institute [5]. The model was extended to allow large tire slip angles, drive torque (i.e., tire forces that accelerate the vehicle) and drive and/or brake torque when the vehicle is rolling backwards. Reference 5 includes a description of the HSRI model. An overview of the extended model is provided below for purposes of comparison.

The semi-empirical tire model describes empirically what is occurring at the tire-road shear interface, according to the current tire-road conditions. It employs a simplified theory assuming an adhesion region and a sliding region. The major assumptions made by the tire model are:

- The contact patch can be divided into two regions: an adhesion region and a sliding region.
- The shear force generated in the adhesion region depends on the elastic properties of the tire and the shear force generated in the sliding region depends on the frictional properties at the tire-road interface.

The inputs required by the tire model are:

- F_o = Vertical load for middle of up to three test loads.
- V_o = Longitudinal velocity for middle of up to three test speeds.
- μ_p, μ_s, S_p = Peak and slide tire-road friction and longitudinal slip at peak friction, at F_o and V_o (these data generate a μ -slip curve at F_o, V_o).
- C_α = Cornering stiffness at F_o, V_o .

The equations for the semi-empirical tire model require the above data at one load and speed. If the HVE tire data for the selected tire include data for more than one load or speed, the rates of change for those parameters with respect to load and/or speed are also used by the model. For example, if C_α data for more than one load are supplied, then $\frac{\partial C_\alpha}{\partial F_z}$ is calculated from the difference in C_α for the maximum and minimum tire test loads. Similar partial derivatives are also calculated for $\mu_p, \mu_s,$ and S_p . During execution, the current vertical tire load, F_z' , and forward velocity component, V_x' , are calculated. Based on the test tire parameters and the current

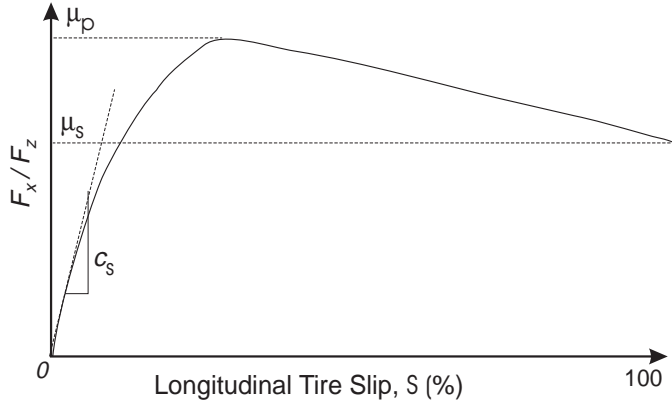


Figure 5 - F_x/F_z vs longitudinal slip, S

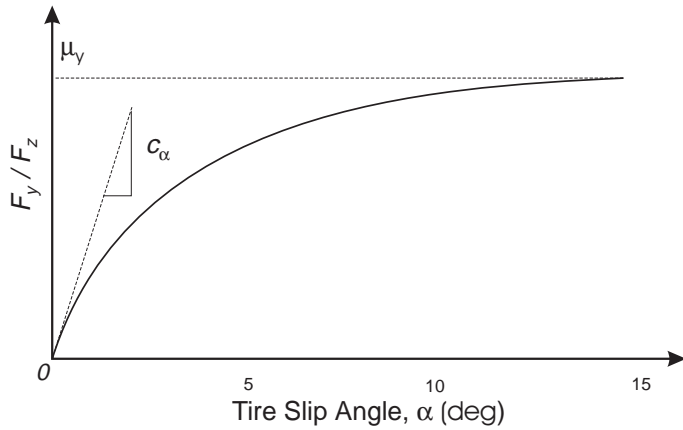


Figure 6 - F_y/F_z vs slip angle, α

tire load and velocity, the following effective values are calculated:

$$C'_\alpha = C_\alpha + (F_{z'} - F_o) \frac{\partial C_\alpha}{\partial F_z} + (V_{x'} - V_o) \frac{\partial C_\alpha}{\partial V}$$

$$\mu'_p = \mu_p + (F_{z'} - F_o) \frac{\partial \mu_p}{\partial F_z} + (V_{x'} - V_o) \frac{\partial \mu_p}{\partial V}$$

$$\mu'_s = \mu_s + (F_{z'} - F_o) \frac{\partial \mu_s}{\partial F_z} + (V_{x'} - V_o) \frac{\partial \mu_s}{\partial V}$$

$$S'_p = S_p + (F_{z'} - F_o) \frac{\partial S_p}{\partial F_z} + (V_{x'} - V_o) \frac{\partial S_p}{\partial V}$$

From these data, the following intermediate parameters are computed:

$$a = (1.0 - S'_p)^2 (1.0 + S'_p)$$

$$b = (1.0 - S'_p)(\mu'_s(S'_p + 2.0) - \mu'_p(2.0S'_p + 1.0))$$

$$c = (\mu'_s - \mu'_p)\mu'_s$$

$$B = -\frac{b + \sqrt{b^2 - 4ac}}{2a}$$

$$A = \mu'_s + B$$

$$C = \mu'_s + B(1.0 - S'_p)$$

From the above values, the effective friction, μ , and longitudinal slip, C_s , are then computed as follows:

$$C_s = \frac{C^2 F_{z_0} (1.0 - S'_p)}{4.0 S'_p (C - \mu'_p)}$$

$$\mu = A - BS$$

Based on the above parameters, the fraction of the adhesion region of the total contact patch, X_s/L (where L is the total length of the contact patch, and X_s is the distance from the front of the contact patch to the point where sliding starts), is calculated as follows:

$$D_t = \sqrt{(C_s S)^2 + (C'_\alpha \sin \alpha)^2}$$

$$X_s/L = \frac{\mu F_z (1.0 - |S|)}{2.0 D_t}$$

X_s/L is limited to 1.0 (note that $X_s/L = 1.0$ means there is no sliding region). If $X_s/L = 1.0$, the longitudinal and lateral tire forces, F_x and F_y , respectively, are:

$$F_x = \frac{C_s S}{1.0 - |S|}$$

$$F_y = \frac{-C'_\alpha \sin \alpha}{1.0 - |S|}$$

If $X_s/L < 1.0$, there is some sliding at the tire-road interface. For this condition, the tire forces in the adhesion region and sliding region are computed separately. $F_{x'}$ and $F_{y'}$ tire forces in the adhesion region are:

$$F_{x_{Adhesion}} = C_s S \left(\frac{\mu F_z}{2.0 D_t} \right)^2 (1.0 - |S|)$$

and

$$F_{y_{Adhesion}} = -C'_a \sin \alpha \left(\frac{\mu F_z}{2.0 D_t} \right)^2 (1.0 - |S|)$$

The tire force components in the sliding region are:

$$F_{x_{Sliding}} = \mu F_z (1.0 - X_s/L) \left(\frac{S}{\sqrt{S^2 + \sin^2 \alpha}} \right)$$

and

$$F_{y_{Sliding}} = -\mu F_z (1.0 - X_s/L) \left(\frac{\sin \alpha}{\sqrt{S^2 + \sin^2 \alpha}} \right)$$

The total tire force is the sum of the force in the adhesion and sliding regions,

$$F_{x'} = F_{x_{Adhesion}} + F_{x_{Sliding}}$$

and

$$F_{y'} = F_{y_{Adhesion}} + F_{y_{Sliding}}$$

The aligning torque, A_t , is approximated as follows:

$$A_t \cong -F_y X'_p \left(\frac{X_s}{L} \right) + F_{x'} F_{y'} C'_y$$

In the above development, the original HSRI model has been modified in two ways: First, the tangent function used by the HSRI model has been replaced by the sine function. This change allows the EDC tire model to properly handle slip angles throughout the continuous range $-\pi \leq \alpha \leq \pi$. (Note that $\tan(\alpha) \rightarrow \infty$ as $\alpha \rightarrow 90$ degrees, resulting in an infinite lateral tire force and resulting integration failure in the HSRI model.) Second, longitudinal slip, S , has been replaced by $|S|$ in the EDC model to allow for drive torque at the tire-road interface.

Connection Forces

The equations of motion include constraint forces at the inter-vehicle connection locations. The constraint equations are solved for each integration timestep by calculating the earth-fixed X,Y,Z coordinates of the connections on each vehicle, and assuming a constraint force exists between the connections, acting along a line between the connection points. The constraint force, F_c , at the connection is:

$$\vec{F}_c = (\vec{R}_1 - \vec{R}_2) \cdot K_c + (\vec{V}_1 - \vec{V}_2) \cdot C_c$$

where

$\vec{R}_1 - \vec{R}_2 =$ the earth-fixed distance between the connection points

$K_C =$ constraint spring rate for the connection

$\vec{V}_1 - \vec{V}_2 =$ the relative earth-fixed linear velocity between the connection points

$C_C =$ constraint damping rate for the connection

For 5th wheel connections between vehicles, the relative roll angular displacements and velocities are used, in conjunction with torsional stiffness and damping factors, to calculate the roll moment, M_ϕ , transferred between tow vehicle and trailer:

$$M_\phi = (\phi_1 - \phi_2) K_\phi + (\dot{\phi}_1 - \dot{\phi}_2) C_\phi$$

where

$\phi_1 - \phi_2 =$ relative roll angle between vehicles

$K_\phi =$ roll stiffness between vehicles

$\dot{\phi}_1 - \dot{\phi}_2 =$ relative roll velocity between vehicles

$C_\phi =$ roll damping between vehicles

This method allows for the ability to model the roll compliance and moment transfer between vehicles within the vehicle train. An important example of the need for this moment transfer is the rollover of a tractor-trailer: Without this moment transfer during a rollover, one of the vehicles might remain upright (obviously, this is not possible for two vehicles connected by a 5th wheel). The roll stiffness parameter, K_ϕ , is actually a frame torsional stiffness parameter that defines vehicle twist about an axis parallel to the vehicle-fixed x axis. The elevation of this axis relative to the CG is user-assignable, and is normally located at the frame height.

Suspension Forces

The suspension is modeled by linear springs, damping and coulomb friction on each axle. The springs are separated by a user-specified lateral spring spacing. The suspension is free to move only in the vehicle-fixed z direction, and to rotate about the user-entered suspension roll center, so longitudinal and lateral forces from the tires are applied directly at the spring locations, resulting in forces and moments applied about the sprung mass center of gravity (CG). The total suspension force, F_S , at each spring location is:

$$F_S = K\delta + C\dot{\delta} + F_\mu + \frac{K_{rs}}{\bar{r}_s}\phi_{Axle}$$

where

- K = linear spring rate
- δ = spring deflection
- C = damping rate
- $\dot{\delta}$ = suspension deflection rate
- F_μ = suspension coulomb friction
- K_{rs} = auxiliary roll stiffness of sway bar (see next section)
- \bar{r}_s = spring y-coordinate
- ϕ_{Axle} = vehicle-fixed axle roll angle

A friction null band is applied to the friction value to prevent friction force in the absence of suspension velocity.

Axle roll is calculated using the current spring force and lateral spring spacing. Roll stiffness may be increased with the addition of an anti-sway bar (see Auxiliary Roll Stiffness, below).

As a result of the changes made to the original Phase 4 model, the static suspension force no longer drops out of the equations. In EDVDS, for a vehicle in equilibrium, the force reported for each suspension is that force required to support the sprung mass in static equilibrium.

Tandem Axles - For tandem axles, braking causes a redistribution of the suspension force between the leading and trailing axles as follows:

$$SF_{LeadingAxle} = SF_{LeadingAxle} + \left(\frac{F_{Shift} T_{Brake}}{\Delta x} \right)$$

and

$$SF_{TrailingAxle} = SF_{TrailingAxle} - \left(\frac{F_{Shift} T_{Brake}}{\Delta x} \right)$$

where

- $SF_{Leading Axle}$ = Total suspension force on lead axle
- $SF_{Trailing Axle}$ = Total suspension force on trailing axle
- F_{Shift} = User-entered Inter-tandem Load Transfer Coefficient (a negative coefficient reduces the suspension force on the lead axle.)
- T_{Brake} = Total brake torque for all axles on the tandem axle set
- Δx = Tandem axle spacing

Auxiliary Roll Stiffness - The EDVDS suspension model allows the user to study the effects of an anti-sway bar. This device increases the roll stiffness by an amount

$$\Delta K_\phi = K_{rs} T_w$$

where K_{rs} is the torsional stiffness of the anti-sway bar and T_w is the track width. The effect of additional roll stiffness is to decrease the roll angle of the sprung mass. The additional roll moment is resisted in the springs and, thus, shows up as an increase/decrease in the vertical spring forces (this latter effect on suspension force was apparently omitted in the Phase 4 code).

Brake System

EDVDS uses HVE's *At Pedal* driver controls option. Using this option, the user enters a table of *Brake Pedal Force vs Time*. The concept of brake pedal application force does not apply to air brake systems that use a treadle valve. Therefore, all HVE heavy truck models have the brake *Pedal Ratio* set equal to 1.0; thus, an entry into the brake force table is really *System Pressure vs Time*.

EDVDS also uses the following wheel brake options:

T_{Lag} - Time lag between the time of pedal application and the onset of pressure rise at the wheel.

T_{Rise} - Time required to fill the air chamber and begin producing brake torque

Brake Torque Ratio - The resulting brake torque per unit of air pressure in the brake chamber.

T_{Lag} and T_{Rise} are often ignored for passenger cars. However, for truck air brake systems, T_{Lag} and T_{Rise} may be significant and should be considered.

The current brake torque, TQ_{Brake} , at each wheel is calculated as follows:

$$TQ_{Brake} = BTQ * P_{Brake}$$

where

- BTQ = Brake torque ratio for the wheel
- P_{Brake} = Current brake pressure at the wheel, including time lag and time rise

The Phase 4 anti-lock model has not yet been implemented in EDVDS.

Steering System

The steer angle at each steerable wheel is determined from the current user-entered steer angle. The current angle is determined from the HVE Steer Table. The user has two options for entering steer angles:

At Axle - The angle at each wheel is derived directly from the steer table for each wheel. Left and right wheels need not have the same steer angle.

At Steering Wheel - The angle at each wheel is the product of the user-entered steering wheel angle and the vehicle's steering gear ratio. The steer angles at both wheels are initially equal, subject to additional roll steer.

Dollies

Two types of dollies are supported by EDVDS:

- **Converter Dolly** - The fifth wheel articulates about the pitch axis, and the drawbar is rigidly attached to the dolly. The result is that brake torque is resisted at the pintle hook of the tow vehicle. Thus, trailer braking results in a vertical load transfer to the rear of the tow vehicle.
- **Fixed Dolly** - The fifth wheel is fixed to the trailer and is not free to articulate about its pitch axis. The drawbar is hinged. Thus, there is no load transfer to the tow vehicle.

These two dolly types are shown in Figure 7.

Drivetrain

EDVDS employs the HVE drivetrain model. This model includes a user-defined engine, transmission and differential. These elements are described below.

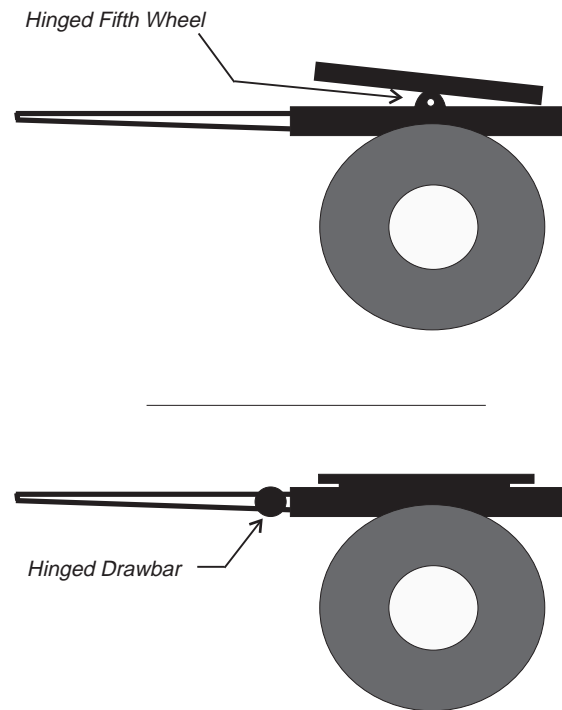


Figure 7 - Converter (above) and Fixed (below) Dollies. Note that a converter dolly transfers its vertical drawbar load due to braking to the tow vehicle.

Engine - The engine is modeled using torque vs RPM tables for wide-open-throttle (WOT) and closed throttle (CT) conditions. Data for the WOT and CT tables are included in the HVE vehicle model. WOT data are normally obtained from the engine manufacturer's data sheet. Data for the CT table are normally calculated from pumping losses, according to the engine displacement and compression ratio [12].

Transmission and Differential - The transmission and differential are modeled according to the manufacturers' ratios for the specified component. Up to 12 forward transmission ratios and three differential ratios are possible. Again, these data are included in the vehicle model according to the component manufacturer (Eaton, Rockwell, Timken, etc.).

Driver Controls - During event set-up, the user specifies tables for throttle position and gear shifts as a function of time. At run-time, the simulation calculates the current engine speed from the current drive wheel RPM, transmission and differential ratios. The current tractive effort is then calculated according to the throttle position and available engine torque for the current engine RPM.

Path Follower Driver Model

EDVDS uses the HVE Path Follower Driver Model [7]. This is an extended version of the model described in reference 13. The model uses up to eight user-defined path positions and orientations, a description of the driver (sample interval, preview distance, maximum path error and lateral acceleration), and a description of the steering system (initial steer angle, maximum steer velocity, steer correction and damping factors). An optional neuro-muscular filter is available. The model then simulates the steering required to cause the vehicle to maintain the desired path.

Not Implemented

The following portions of the Phase 4 model have not been implemented in EDVDS:

- Semi-empirical Brake Model
- Anti-lock Brake Model
- Table Look-up Brake Model
- Table Look-up Tire Model
- Table Look-up Suspension Model

The HVE Vehicle/Tire model includes the parameters required for the Semi-empirical Brake Model and Table Look-up Tire Model; thus it is likely that these (or similar) models will soon be implemented for EDVDS.

VALIDATION

To provide a direct comparison of the results produced for each model, a copy of the Phase 4 source code, dated 1-18-91, was obtained from the University of Michigan Transportation Research Institute. The FORTRAN source code was compiled on an IBM-compatible PC.

The process of porting Phase 4 to the HVE environment involved two steps: First, the Phase 4 code was rewritten in C and modified to run in HVE (see references 10 and 11 for an explanation of the procedures). This intermediate version was called EDPH4 because it essentially was the same as Phase 4, except it ran in the HVE simulation environment. EDPH4 underwent a preliminary validation that involved running essentially duplicate data sets through EDPH4 and Phase 4 to confirm the same results were obtained from each model.

The second step involved modification of the EDPH4 model, as described earlier. This model became EDVDS, Version 1.00. Again, preliminary validations were performed comparing EDVDS results with those from EDPH4 and Phase 4.

After preliminary validation, a detailed validation study was performed involving several different vehicle configurations and maneuvers. Input data sets were produced for Phase 4, EDPH4 and EDVDS, and runs were executed using each model for purposes of comparison. In addition, an actual experimental handling maneuver was included to compare simulation results with actual experimental data.

The following experimental validation studies were performed:

- Straight Truck - Step Steer
- Triples - Combined Steering and Braking
- Tractor-trailer - Heavy Steering and Braking
- 2-Axled Vehicle - Loss of Control
- Tractor-trailer - Combined Braking and Steering

The results from each of these validation experiments are discussed below.

Straight Truck - Step Steer

This simulation of a straight tandem axle truck was included in reference 5. The event involves a single 8 degree (at the axle) step steer input; the initial velocity was 30 ft/sec.

The vehicle has a single solid axle front suspension and tandem axle rear suspension. Its total weight was 44,500 lb (with payload). Other vehicle parameters are included in reference 15. The linear tire model was used.

Comparison of Results

The simulation results for Phase 4, EDPH4 and EDVDS are shown in Figure 8.

Figure 8a shows the driver steering input, the only driver control used in this event.

Path X,Y results (Figure 8b) reveal substantially similar results for all three models. Orientation results (Figure 8c) reveal that the roll angle predicted by EDVDS is less than the Phase 4 and EDPH4 models. This difference is related to the way the auxiliary roll stiffness is handled by EDVDS (as explained earlier, the EDVDS suspension model includes the effect of the sway bar force at the suspension; Phase 4 does not). The sway bar on this vehicle is also much stiffer than is typically seen, thus the effect is quite large. The pitch angle predicted by EDVDS illustrates a common result related to the fact that Phase 4 assumes the vehicle is initially in equilibrium, while EDVDS does not. The initial transient pitch response occurs because the vehicle is empty and over-sprung at the rear. As the vehicle settles to its equilibrium position, the softer front suspension settles faster, resulting in a pitch oscillation. Yaw results compare favorably, although the result diverges as the vehicle continues along its path. The reason for the divergence is the extremely stiff rear axle sway bar used in the original study [5] and, thus, included for comparison in the EDVDS simulation. In EDVDS, this sway bar directly affects the suspension and tire forces; in Phase 4 it does not. Changing the front/rear distribution of tire forces reduces the vehicle's overall understeer gradient, and thus, slightly increases its lateral acceleration gain and resulting path curvature.

Lateral acceleration results are shown in Figure 8e. The results are quite similar, although the overshoot is greater in the Phase 4 results. This is possibly due to the fact that

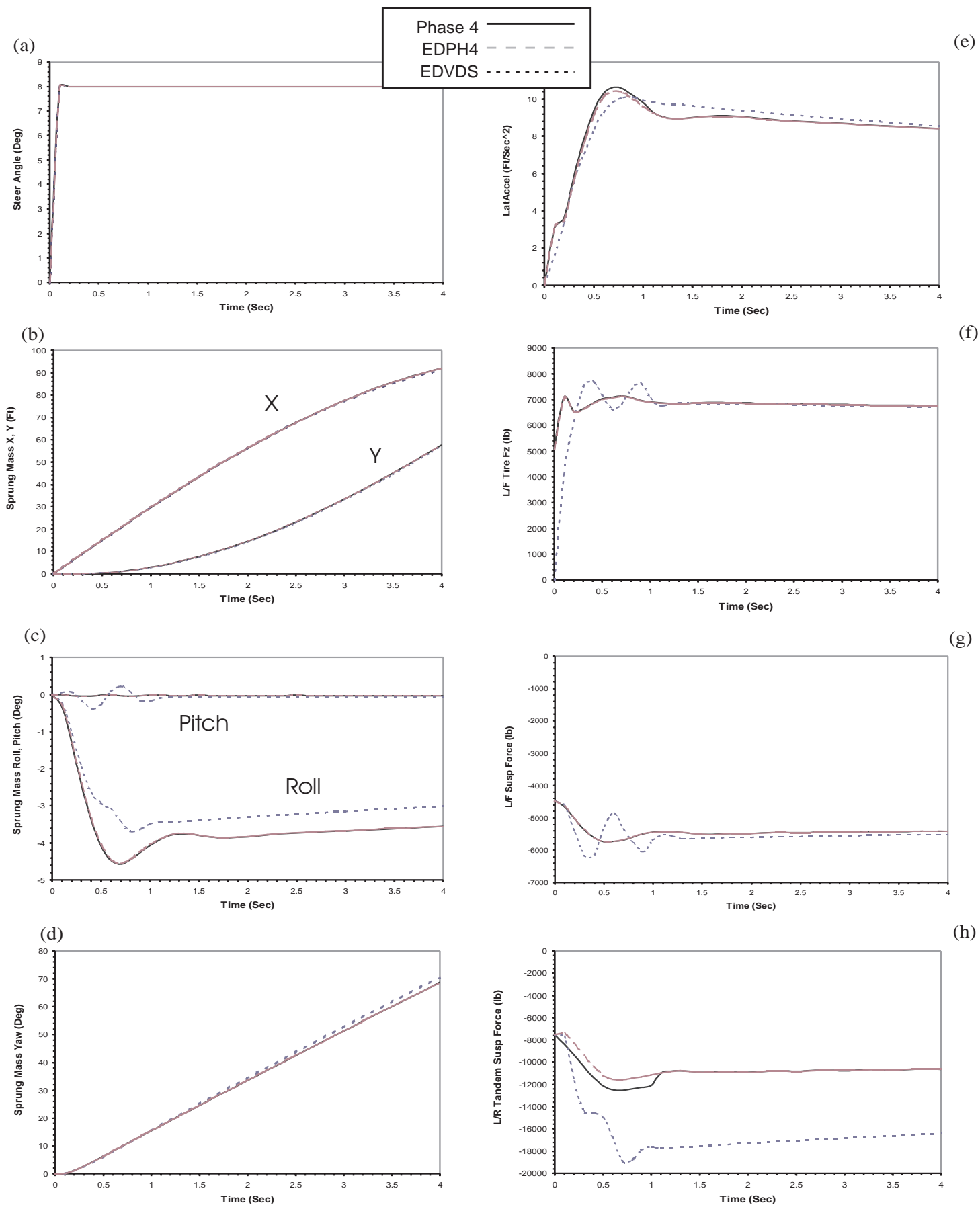


Figure 8 - Validation Results, Test 1, Straight Truck - Step Steer

steering begins early in the event, before the EDVDS vehicle has stabilized. There is also a discontinuity of unknown origin in the Phase 4 results.

Tire and suspension loads are shown in Figures 8f through 8h. These figures illustrate the effect of auxiliary roll stiffness on vertical tire and suspension forces. Recall the vehicle is steering to the right, so left-side vertical tire and suspension forces increase due to lateral load transfer. Figure 8f shows substantially similar results between models for vertical force at the left front tire. You can see the tire bouncing a little as the vehicle settles. Figure 8g shows the suspension force for the same wheel location, again substantially similar. Figure 8h shows the suspension force for the left rear tandem suspension. This axle includes the sway bar described earlier. The effect of this (unusually stiff) sway bar is clearly seen in the EDVDS model.

In summary, general vehicle behavior is quite similar, although specific differences are observable in the detailed results, most probably related to how the sway bar is incorporated into the suspension model.

Triples - Combined Steering and Braking

This is a slightly modified version of another run included in reference 5. It involves a tandem axle tractor towing three trailers. The first trailer is a semi-trailer; the second and third trailers are full trailers. The second trailer uses a converter dolly with a rigid drawbar and the third trailer uses a fixed dolly with a hinged drawbar (an illustration of these drawbars is shown in Figure 7). The driver inputs include a single 10 degree (at the axle) step-steer, followed by a 5 psi brake application. The initial velocity is 44 ft/sec.

The tow vehicle has a single solid axle front suspension and tandem axle rear suspension. Its total weight was 17,390 lb. The weight of the first trailer was 58,265 lb (with payload); the weight of the second and third trailers was 17,300 lb each (including dollies), without payloads. The remaining vehicle parameters are included in reference 15. The semi-empirical tire model was used.

Comparison of Results

The simulation results for Phase 4, EDPH4 and EDVDS are shown in Figure 9.

Figures 9a and 9b show the driver steering and braking inputs, respectively. It should be noted that, although these are table inputs, the current level of steering and braking is modeled using interpolation according to the current simulation time.

Figure 9c shows the X,Y path coordinates of the tractor. After traveling a distance, the paths begin to diverge due to differences between the EDC and Phase 4 semi-empirical tire models, described earlier.

Figures 9d and 9e show the tractor sprung mass orientation for each model. The results are substantially similar. The initial settling of the EDVDS model is seen again, the reasons for which were the same as in the first run (see above). The yaw angle diverges slightly after several seconds, again due to differences in the tire models.

Figures 9f and 9g show substantially similar results for tractor forward velocity and acceleration, respectively.

Figure 8h shows the yaw angle for trailer 3. Again, the results are substantially similar, but begin to diverge slightly due to differences in the semi-empirical tire models.

Figures 9c through 9h all exhibit an interesting behavior in the Phase 4 and EDPH4 results at t=7 seconds: The results either terminate or become unstable. The reason for this behavior was traced to the calculation of the earth-fixed yaw angle for trailer 2's dolly. Dolly yaw angles are calculated in Phase 4 as:

$$\Psi_{Dolly} = \arcsin\left(\frac{DD_Y}{TOL}\right)$$

where DD_Y is the difference in the dolly earth-fixed connection Y-coordinates at each end, and TOL is the total distance between the dolly endpoints. One can see that the above expression would not allow the dolly yaw angle to increase beyond 90 degrees, regardless of the actual heading angle of the second trailer. The resulting dolly connection forces are incorrectly calculated and quickly become enormous. The resulting dynamic instability causes an integration error and the run terminates. On inspection, calculation of the dolly yaw angle is not dependent on the TOL , but rather on the X-difference in dolly connection points. Thus, the corrected expression is:

$$\Psi_{Dolly} = ATAN2(DD_Y, DD_X)$$

where DD_X is the difference in earth-fixed connection X-points. The EDVDS results reveal correct vehicle behavior.

In summary, the Phase 4 and EDVDS results for this simulation are quite similar until the second trailer's dolly reaches a heading angle of 90 degrees, causing the Phase 4 run to terminate abnormally.

Tractor-trailer - Steering and Heavy Braking

This is a run obtained from an early case study using Phase 4. It involves a severe maneuver resulting in loss of control of a tandem axle tractor towing a semi-trailer. The

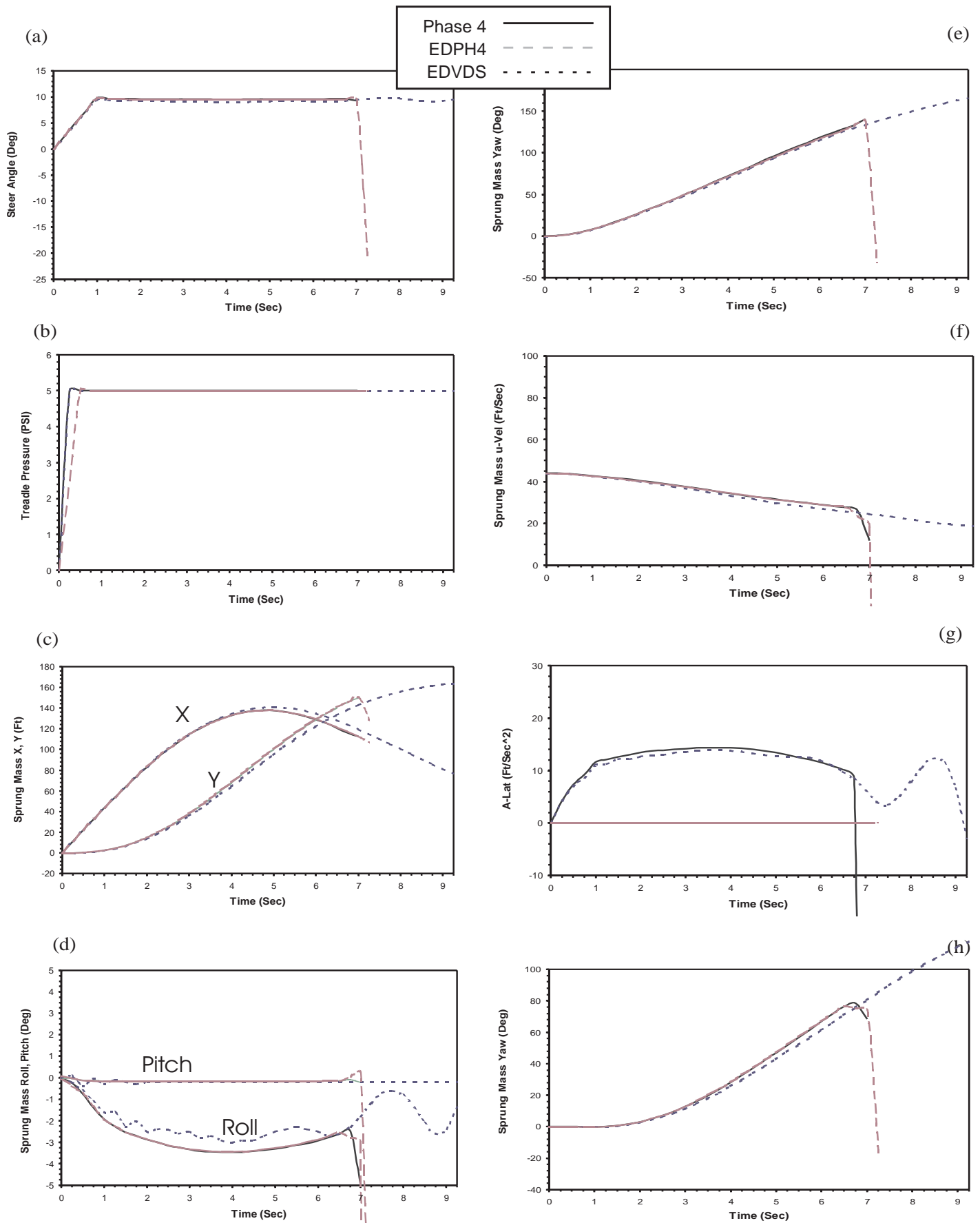


Figure 9 - Validation Results, Test 2, Triples - Combined Braking and Steering

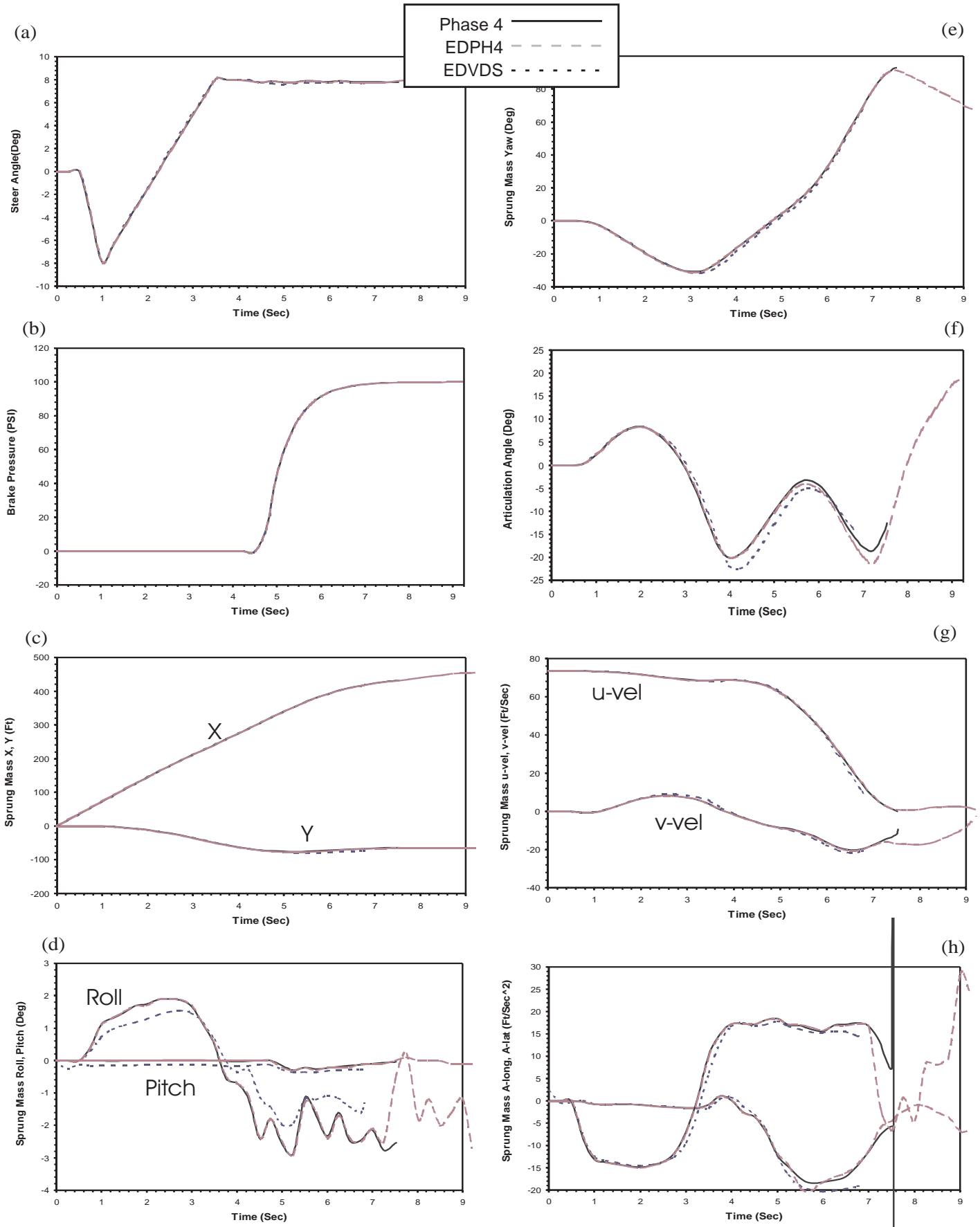


Figure 10 - Validation Results, Test 3, Tractor-trailer Steering and Heavy Braking

driver inputs include a single -8 degree (at the axle) step-steer, followed by a heavy (100 psi) brake application. The initial velocity is 73.5 ft/sec. The tow vehicle weight 14,535 lb. The weight of the semi-trailer was 22,265 lb (with payload). The remaining vehicle parameters are included in reference 15. The semi-empirical tire model was used.

Comparison of Results

The simulation results for Phase 4, EDPH4 and EDVDS are shown in Figure 10.

Figures 10a and 10b show the driver steering and braking inputs, respectively.

Figure 10c shows the results for tractor X,Y path coordinates. The results are substantially similar. Because the brakes are locked on both units, the difference between Phase 4 and EDVDS tire models has a much smaller effect and the path divergence is less than in the previous validation.

Figures 10d through 10f show the tractor roll, pitch and yaw angles, and the trailer articulation angle. The simulated roll angle is less for EDVDS owing to the updated method for handling the auxiliary roll stiffness (the tractor has sway bars at the front and rear suspensions). The simulated pitch angles are quite small, with the EDVDS results showing the vehicle settling in. The tractor yaw angle and trailer articulation angle are substantially similar.

Figure 10g shows the tow vehicle forward and lateral velocities, and Figure 10h shows the longitudinal and lateral accelerations. All results are substantially similar.

2-Axled Vehicle - Loss of Control

This is a sample run illustrating the models' use for studying the dynamics of a passenger vehicle. It involves an S-turn with moderate braking. In addition to Phase 4, EDPH4 and EDVDS, the EDVSM simulation model was also included in the study as a recognized reference. The initial velocity is 95.33 ft/sec. The driver inputs include a +3.5 degree (at the axle) followed by a -3.5 degree steer. A 10 psi brake application is applied during the run. Ultimately, the vehicle spins out. The vehicle weight is 2601 lb and has solid front and rear axles. The remaining vehicle parameters are included in reference 15. The semi-empirical tire model was used.

Comparison of Results

The simulation results for Phase 4, EDPH4, EDVDS and EDVSM are shown in Figure 11.

Figures 11a and 11b show the driver steering and braking inputs, respectively.

Figure 11c shows the X and Y path results for all four models. The differences represent the effects of the tire models: While the Phase 4 and EDPH4 results are nearly identical (as expected), the EDVDS and EDVSM results

diverge. The EDVDS results diverge for reasons explained earlier (differences between EDC and Phase 4 semi-empirical tire model). The EDVSM results diverge simply because its tire model is totally different.

Figure 11d shows an important difference in results for vehicles in high-G turns: In the Phase 4/EDPH4 model, the tires do not saturate, and the vehicle turns back to the left as a result of the left steer. However, in both EDVSM and EDVDS (with modified tire model), the simulation shows the tires saturating. As a result, the vehicle does not steer back to the left, rather it spins out.

Figures 11e and 11f show the simulated forward and lateral velocities, respectively. The simulated forward velocity shows the same trend, but EDVSM and EDVDS are different from Phase 4/EDPH4, as expected. The lateral velocities are significantly different, as would be expected from the results shown in Figure 11d.

Figure 11g shows the longitudinal acceleration. The Phase 4/EDPH4 results show an unstable oscillation related to tire saturation. The EDVSM/EDVDS results are substantially similar and consistent with the observed path.

Figure 11h shows the lateral acceleration. The results are consistent with the observed path for each model: The lateral accelerations for Phase 4 and EDPH4 show the reversal associated with the driver steering input, while EDVSM and EDVDS show a leveling off of the lateral acceleration until the vehicle slows.

Tractor-trailer Combined Steering and Braking

This example from Reference 2 includes an instrumented tractor-trailer performing a combined braking and steering maneuver at an initial speed of 27 mph. The tow vehicle is a White COE 4x6 142 in. wheelbase tractor with a 12,000 lb front axle and 34,000 lb rear tandem axle. All suspensions use elliptical leaf springs. The trailer is a 40 ft semi-trailer with a 34,000 lb rear tandem axle leaf spring suspension. The tires on all units were 10.00-20. The drive tires used a lug tread; all others were rib style. The remaining data were interpreted from reference 2.

Steering angles were measured using potentiometers. Brake pressures were measured using strain gauge pressure transducers. Tractor orientation and acceleration were measured using triaxial accelerometers. Yaw rate was measured using a rate gyro. Articulation was measured using a Beckman Helipot. All test data were logged on two Honeywell Visicorder (light beam) recorders.

The experimental results were not available in tabular form, thus, they were scaled from graphs in reference 2. In this validation study, the EDVDS simulation used vehicle dimensions and inertias from reference 2, and Generic Class 3

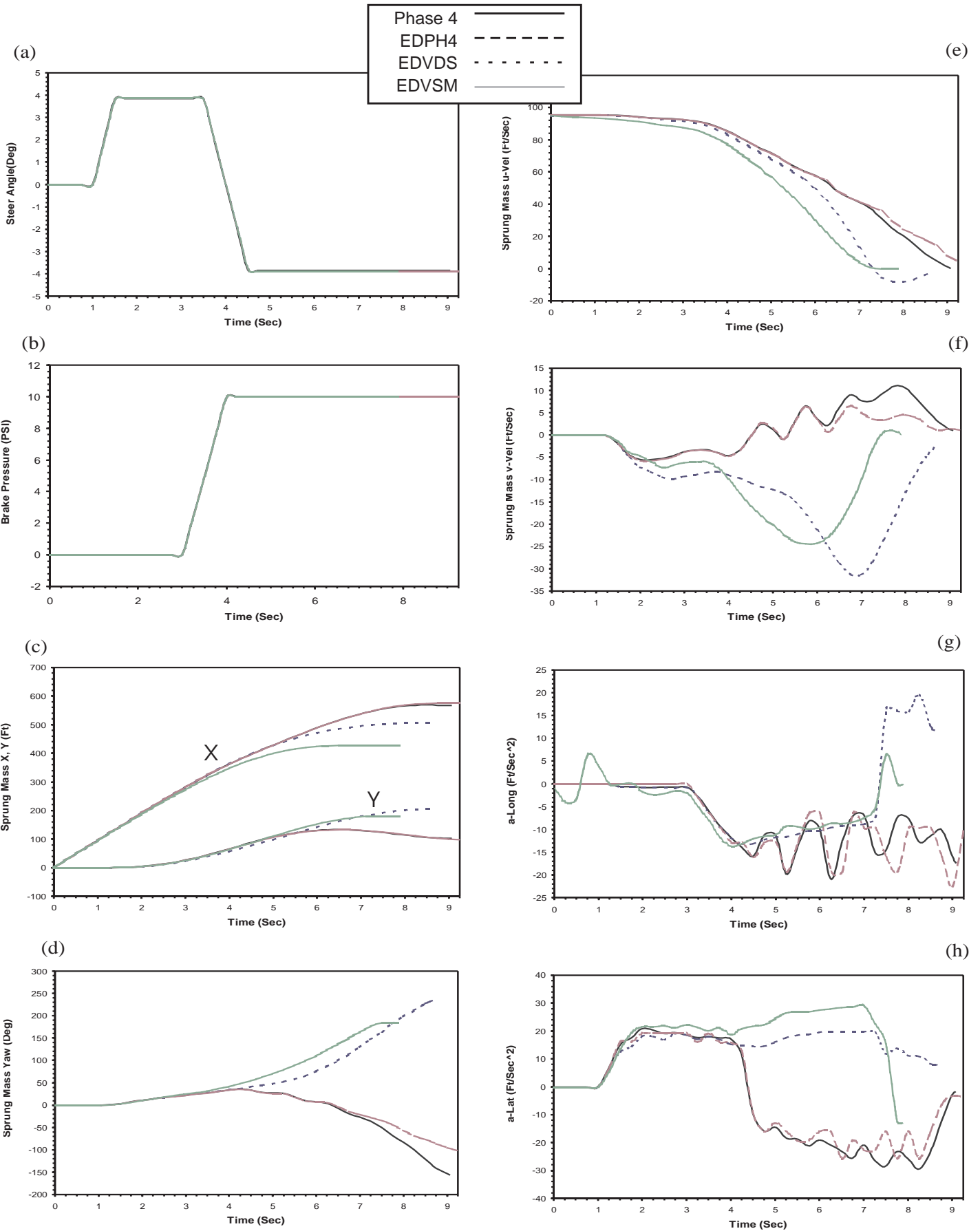


Figure 11 - Validation Results, Test 4, 2-Axled Vehicle Loss of Control

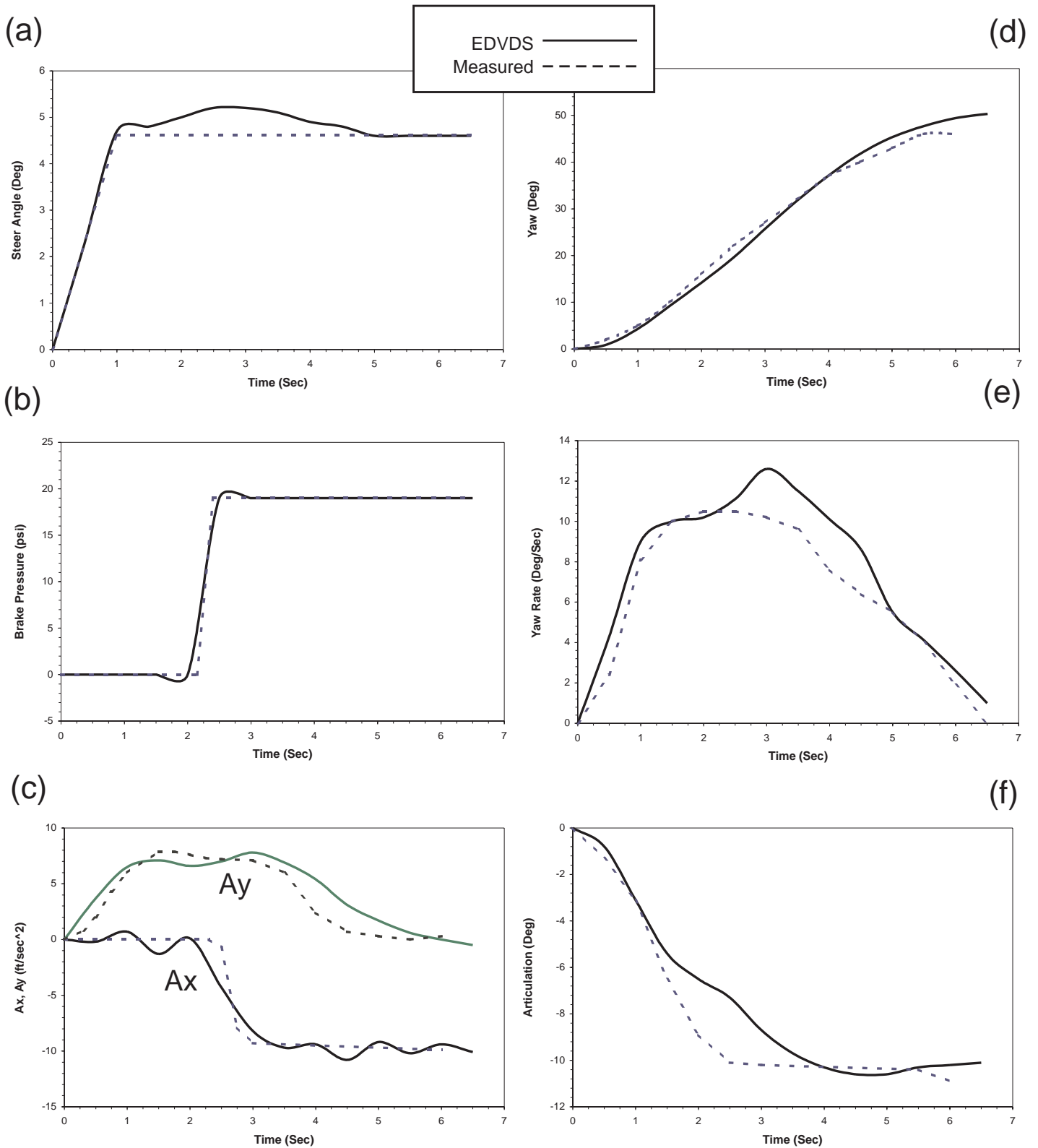


Figure 12 - Validation Results, Test 5, Combined Steering and Braking

suspensions and tires from the HVE Vehicle Database [17]. The vehicle parameters are available in reference 15. The simulation used the semi-empirical tire model.

Comparison of Results

The experimental results and EDVDS simulation results are shown in Figure 12. The experimental results appear to be significantly filtered; the instrumentation used for measurement and recording would not create such smooth plots. Thus, some interpretation is necessary while making comparisons.

Figures 12a and 12b show the driver steering and braking inputs. The measured value was reported as the average of the right and left wheels. The simulated steering input is for the right front wheel only, and shows the effect of the front suspension roll steer.

Measured and simulated trailer articulation angles are shown in Figure 12c. The agreement is excellent, although there is more noise in the simulation results, when compared to the filtered measurements.

Figure 12d shows measured and simulated tractor yaw angle. The agreement is excellent, especially considering the use of generic tire and suspension parameters in the EDVDS simulation.

Simulated yaw rate, Figure 12e, also agrees with measured results, except for the period between 2.2 and 5 seconds. A heavy brake application begins at 2.15 seconds, and the simulation results seem to reflect this as a momentary increase in yaw rate; however, the increase does not appear in the measured results. It is not clear if the difference is due to the simulation or the measurement.

Trailer articulation angle is shown in Figure 12f. The match is good, except for a divergence between 1 and 4 seconds. The final articulation angle match is excellent; it is not clear why the intermediate results differ during the transient phase.

In summary, this comparison between simulated and experimental values for this combined steering and braking maneuver agrees very well for all available test parameters.

DISCUSSION

The inherent assumption of initial equilibrium in the Phase 4 vehicle model is sometimes an advantage. For example, no oscillation related to settling occurs at the start of the simulation. However, there are also disadvantages. In particular, it is not possible to begin a simulation on a sloped surface. Also, the simulation must begin with the tow vehicle's CG at the earth-fixed origin at zero heading angle (although removing this restriction would have been a small programming effort when compared to rewriting the entire code). After considerable evaluation of the pros and cons, the

decision was made in favor of robustness (i.e., the elimination of assumptions in the EDVDS code).

The HVE Event Editor has an *AutoPosition* feature that allows the user to position an entire combination vehicle while dragging it over the surface of a complex 3-dimensional terrain. To increase interactivity while positioning the vehicle(s) using this method, tire deflections are ignored. Thus, if *AutoPosition* is used, the vehicle(s) are not in perfect equilibrium after positioning. If initial equilibrium is important, the user can over-ride the default position by manually entering a different CG elevation (i.e., by dropping the initial CG elevation by about an inch).

The development of EDPH4 was an unintentional byproduct of the EDVDS development project. Because of the decision to modify significant portions of the Phase 4 model, it was viewed as crucial that the existing Phase 4 vehicle dynamics model be ported intact before its modification. Thus, any differences found between Phase 4 and EDPH4 are attributable to the port, while differences between EDPH4 and EDVDS are attributable to the intentional changes to the model.

A complete set of validation results from Phase 4, EDPH4 and EDVDS is available for each of the runs described in this report. The printed document is approximately 600 pages in length. In addition, input files for the Phase 4 runs and the HVE Case files for EDPH4 and EDVDS (*EdvdsValidationSuite*) are also available.

Finally, the author wishes to express his sincere appreciation and gratitude to the MVMA (now called the American Automobile Manufacturers Association, or AAMA), the U.S. Federal Highway Administration, and, most of all, the University of Michigan, for their landmark original work which provided the basis for the EDVDS model. Without their initial contributions, EDVDS could not have been developed.

CONCLUSIONS

1. Although the EDVDS model was derived from the Phase 4 model, the resulting EDVDS model was significantly different from Phase 4. The basic force producers (i.e., suspension, tire and inter-vehicle connection models) remained mostly intact (this report has addressed changes to the tire and suspension models), however, the vehicle dynamics model using these force producers was completely rewritten.

2. The resulting vehicle dynamics model did not incorporate the small angle assumption for sprung mass roll and pitch, thus EDVDS may be used for rollovers.

3. The resulting model did not include a force producer for the exterior body of each unit, either with the ground or between units. Therefore EDVDS was not valid for rollovers during which the vehicle body contacted the pavement or jackknife wherein the vehicle bodies contacted each other.

4. The addition of a drivetrain model in EDVDS permits its use in tractive effort and gradability studies.

5. A validation study comparing results from EDVDS and Phase 4 revealed differences. These differences ranged from negligible to significant, depending on the severity of the maneuver.

6. The extended EDC semi-empirical tire model permitted the study of severe handling maneuvers wherein tire slip angles approach and exceeded 90 degrees. Results compared favorably with other well-validated models.

7. A problem in the Phase 4 mathematical model was identified and eliminated, thus permitting earth-fixed dolly yaw angles to exceed ± 90 degrees.

8. The ability to drive on a 3-D surface of arbitrary complexity greatly extended the functionality of the EDVDS model for applications, such as alternate ramp traversals and pavement edge dropoff studies.

9. User interaction was substantially improved by replacing Phase 4's batch processing interface with the HVE simulation environment.

10. The availability of complete HVE vehicle data sets improved the ability to use the EDVDS model. Simulation using generic vehicle data resulted in a very good match between EDVDS simulation results and experimental data.

RECOMMENDATIONS

1. The semi-empirical brake model, brake anti-lock model, and table look-up tire model should be implemented, as these capabilities would be useful to the design engineer.

2. The model should be extended to allow interaction between the body and the roadway (i.e., complete rollover and slide to rest), as this would be useful for the heavy vehicle crash reconstruction engineer.

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