EXTENSIONS AND REFINEMENTS
OF THE CRASH COMPUTER PROGRAM PART II
USER'S MANUAL FOR THE CRASH COMPUTER

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This report contains detailed instructions for users of the CRASH computer program. It also outlines the analytical basis of calculations within the program.
FOREWORD

This user's manual for the CRASH program has been prepared in partial completion of the Work Statement of Contract No. DOT-HS-5-01124 with the National Highway Traffic Safety Administration (NHTSA), U. S. Department of Transportation.

The opinions, findings and conclusions expressed in this manual are those of the author and are not necessarily those of NHTSA.

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ACKNOWLEDGEMENTS

The author wishes to acknowledge contributions of Mr. James P. Lynch, of Calspan, to development of the format and coding of the CRASH computer program.
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<td>----------</td>
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</tbody>
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1. INTRODUCTION

The CRASH* computer program (References 1 and 2) is an accident investigation aid aimed at achieving uniformity in the interpretation of physical evidence from automobile accidents. It is designed to accommodate a range of accident evidence, from VDIs only at one extreme to complete definitions of rest and impact positions as well as damage dimensions at the other. Multiple outputs of speed change and/or impact speed are provided with identification of the basis for each approximation. In this manner, it is possible for the user to select the approximation result based on the most reliable items of evidence while assuring that the various different items of evidence are at least grossly compatible. A schematic flow chart of the CRASH program is depicted in Figure 1.

The computer costs for time-sharing operation of the CRASH program in its present form ranges from approximately $1.00 to $5.00 per case, depending on the extent of the input information (i.e., the upper end of the cost range corresponds to a combination of trajectory and damage analyses).

The structural development of the computer program (i.e., solution procedures, program logic, input and output formats) is considered to be complete. However, stored tables of vehicle parameter data and of empirical coefficients that are applied in the programmed calculations must be recognized as being "first approximations" that can be substantially refined. Therefore, the overall accuracy range of approximately 12% that has been indicated in initial trial applications to staged collisions (References 1 and 2) is not considered to be representative of the potential accuracy of the reconstruction technique.

In the following, the questions that must be answered by the program user are presented and discussed. The output format is then defined. Details of the calculation procedures are given in the Appendices.

*Calspan Reconstruction of Accident Speeds on the Highway.
Figure 1 SCHEMATIC FLOW CHART OF THE CRASH COMPUTER PROGRAM
2. REQUIRED RESPONSES TO QUESTIONS

The following general comments apply to all responses.

(1) All individual items of entry must be separated by either one or more blank spaces or a single comma.

(2) Subsequent to completion of the response to a given question, the carriage return must be entered.

(3) Errors may be corrected by use of special deletion keys that are system dependent (see your reference time-sharing manual). Alternatively, the deliberate entry of an illegal or out-of-range response will produce a rejection of the response and a repeat of the given question.

(4) In yes or no responses, the first letters are sufficient.

Discussions of individual questions are presented in the following paragraphs.

2.1 ENTER A DESCRIPTIVE TITLE (72 CHARACTERS MAX.)

This question is considered to be self-explanatory. The title should include the date of the computer run so that the program version can be identified in the event of future revisions.
2.2 ENTER SIZE CATEGORIES FOR VEHICLE #1 AND VEHICLE #2

LEGAL CATEGORIES: SUBCOMPACT
COMPACT
INTERMEDIATE
FULL SIZE
BARRIER

SAMPLE: COMPACT FULL       NOTE: 1ST LETTERS ARE O.K.

The specified vehicle categories serve to define dimensions, radii of
gyration, weights and structural crush properties, through a table look-up,
for the CRASH calculations. The actual weights, if known, can be entered in
response to a subsequent question. The other items of vehicle data that are
stored in the present version of the program are presented in Table 1. Note
that the final data item, T, for the vehicles and the final four items, X_F,
X_R, Y_S and T, for the barrier in Table 1 are stored for use in the optional
generation of input data for the SMAC program. They are not used in the
CRASH calculations. The symbols used in Table 1 are defined as follows:

\[
\begin{align*}
    a, b & = \text{Distances along vehicle X axis from the total vehicle center of gravity to the center lines of the front and rear wheels, respectively, inches (both entered as positive quantities).} \\
    M & = \text{Total vehicle mass, lb}-\text{sec}^2/\text{in.}
\end{align*}
\]
Table 1

VEHICLE PARAMETER DATA

<table>
<thead>
<tr>
<th>Sub</th>
<th>Compact</th>
<th>Compact</th>
<th>Intermediate</th>
<th>Full Size</th>
<th>Barrier</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>44.7</td>
<td>52.7</td>
<td>57.3</td>
<td>60.5</td>
<td>50.0</td>
<td>inches</td>
</tr>
<tr>
<td>b</td>
<td>46.6</td>
<td>54.8</td>
<td>59.7</td>
<td>63.0</td>
<td>50.0</td>
<td>inches</td>
</tr>
<tr>
<td>M</td>
<td>5.71</td>
<td>8.51</td>
<td>9.86</td>
<td>12.42</td>
<td>10⁶</td>
<td>lb sec²/in</td>
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<tr>
<td>RSQ</td>
<td>1963.</td>
<td>2635.</td>
<td>2998.</td>
<td>3588.</td>
<td>10⁶</td>
<td>inches²</td>
</tr>
<tr>
<td>Col. 3 (=F)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1b/in²</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>130.5</td>
<td>154.6</td>
<td>281.8</td>
<td>307.5</td>
<td>0.0</td>
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<tr>
<td>B</td>
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<td>69.57</td>
<td>33.82</td>
<td>36.89</td>
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<tr>
<td>G</td>
<td>144.94</td>
<td>171.78</td>
<td>1174.3</td>
<td>1281.1</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Col. 3 (=R,L)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1b/in²</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>82.21</td>
<td>111.8</td>
<td>43.72</td>
<td>49.19</td>
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<tr>
<td>B</td>
<td>42.76</td>
<td>58.16</td>
<td>47.23</td>
<td>53.13</td>
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<tr>
<td>G</td>
<td>79.04</td>
<td>107.5</td>
<td>20.24</td>
<td>22.77</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Col. 3 (=B)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1b/in²</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>65.98</td>
<td>78.18</td>
<td>85.51</td>
<td>93.28</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>13.20</td>
<td>15.64</td>
<td>17.11</td>
<td>18.66</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>164.97</td>
<td>195.45</td>
<td>213.78</td>
<td>233.21</td>
<td>0.0</td>
<td></td>
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<tr>
<td>(X_F)</td>
<td>74.7</td>
<td>85.7</td>
<td>94.8</td>
<td>100.5</td>
<td>50.0</td>
<td>inches</td>
</tr>
<tr>
<td>(X_R)</td>
<td>-83.5</td>
<td>-100.0</td>
<td>-110.8</td>
<td>-119.6</td>
<td>-50.0</td>
<td>inches</td>
</tr>
<tr>
<td>(Y_S)</td>
<td>31.1</td>
<td>35.7</td>
<td>38.4</td>
<td>39.6</td>
<td>50.0</td>
<td>inches</td>
</tr>
<tr>
<td>T</td>
<td>51.2</td>
<td>57.7</td>
<td>60.0</td>
<td>63.1</td>
<td>50.0</td>
<td>inches</td>
</tr>
</tbody>
</table>
RSQ = Radius of gyration squared for complete vehicle in yaw, inches squared.

A, B, G = Empirical coefficients of unit-width structural crush properties obtained from crash test data.

$X_F, X_R$ = Distances along vehicle X axis from the total vehicle center of gravity to the boundaries of the vehicle at the front and rear, respectively, inches ($X_R$ is entered as a negative quantity).

$Y_S$ = Distance along vehicle fixed Y axis from the total vehicle center of gravity to the boundary of the vehicle at the side (i.e., one-half of the total vehicle width), inches.

$T$ = Track at front and rear wheels (average), inches.

The values for A, B and G listed in Table 1 must be recognized as gross first approximations based on very limited test data including a range of model years. Obviously, the accuracy of the damage-based CRASH approximations of speed changes can be substantially improved, within the existing program framework, by the availability and proper utilization of more extensive test data. The categorization of vehicles in relation to crush properties should be extended to reflect model years and structural configurations as well as sizes, when adequate test data are available. Also, the effects of underride/override on the interpretation of vehicle crush dimensions should be incorporated when applicable test data become available.
The initial four and the final four items of vehicle data listed in Table 1 are based on a limited survey of 1971-72 model automobiles. Vehicles that were included in the existing categories are listed in Table 2. In view of recent changes in the vehicle population, extensions and refinements of the included vehicle categories should precede any extensive applications of the computer program. It should be noted that the CRASH-related research to date has been exploratory and developmental in nature. Therefore, the data tables contain "reasonable approximations" rather than rigorously established data with which minimum errors would be expected. Despite this fact, the results have been very encouraging (References 1 and 2).

2.3 ENTER A 7 CHARACTER VDI FOR VEHICLE #1
FORM: 12LYEW2

The user is referred to SAE Technical Report No. J224a (Reference 3). The damage analysis portion of the CRASH program does not accept a 00 entry in columns 1 and 2 (i.e., rollover). Also, because of limitations on available crush data, it does not distinguish between classification codes entered in column 5.

2.4 ENTER A 7 CHARACTER VDI FOR VEHICLE #2
FORM: 12LYEW2

See 2.3.

2.5 ARE ANY ACTUAL WEIGHTS KNOWN?
(ANSWER YES OR NO)

A "no" response to this question will produce an automatic weight entry corresponding to the mass, M, listed in Table 1 for the given vehicle category. Since the accuracy of program results is obviously degraded by the use of such "representative" values, an effort should be made to obtain accurate weights, including cargo and occupant weights.
# Table 2

**VEHICLE MODELS ('71, '72) INCLUDED IN SIZE CATEGORIES**

<table>
<thead>
<tr>
<th>Sub Compact</th>
<th>Compact</th>
<th>Intermediate</th>
<th>Full Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volkswagen Beetle</td>
<td>Maverick</td>
<td>Chevelle</td>
<td>Chevrolet</td>
</tr>
<tr>
<td>Toyota 1200</td>
<td>Camero</td>
<td>Torino</td>
<td>Galaxie</td>
</tr>
<tr>
<td>Datsun 1200</td>
<td>Dart</td>
<td>Coronet</td>
<td>Polara</td>
</tr>
<tr>
<td>Vega</td>
<td>Hornet</td>
<td>Matador</td>
<td>Ambassador</td>
</tr>
<tr>
<td>Pinto</td>
<td></td>
<td>Skylark</td>
<td>Monterey</td>
</tr>
<tr>
<td>Fiat 850</td>
<td></td>
<td></td>
<td>LeSabre</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>New Yorker</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Fleetwood</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Continental</td>
</tr>
</tbody>
</table>
The "default" values for mass, M, in Table 1 are based on averages of a number of vehicles in each size category (see Table 2), including allowances for liquid weight and two occupants.

2.6 ENTER THE ACTUAL WEIGHT OF VEHICLE #1
FORM: 3625. (LBS)

If weights are looked up in references, the weight of occupants and cargo should be added to the "curb" weight (i.e., including liquids). The "shipping" or "dry" weight does not include coolant, fuel or oil.

2.7 ENTER THE ACTUAL WEIGHT OF VEHICLE #2
FORM: 3625. (LBS)

See 2.6.

2.8 ARE BOTH REST AND IMPACT POSITIONS KNOWN?
(ANSWER YES OR NO)

A "no" response to this question will limit the program results to ΔV approximations based on damage evidence only.

If a "yes" response is entered, a series of detailed questions related to the spinout trajectories will be presented. In preparation for those questions, the trajectory evidence should be organized on a reference coordinate system as defined in the following paragraphs.

Reference Coordinate System

The space-fixed rectangular coordinate system that is used to define measured spinout trajectories for analysis via the CRASH program is shown in Figure 2. The relationship of the X' and Y' axes reflects the aeronautical convention for three-dimensional coordinates, in which the Z' axis points downward. In the selected planar coordinate system, the Y' axis is directed
to the right of the X' axis and the angle $\psi$ is measured in the clockwise direction with respect to the X' axis.

While the position and orientation of the space-fixed reference coordinate system for a given case are arbitrary, it is generally desirable to relate them to permanent reference points at the accident scene (e.g., curb lines, utility poles, etc.).

2.9 ENTER REST POSITIONS AND HEADINGS FOR VEHICLE 1 AND VEHICLE 2
FORM: XCR1(FT) YCR1(FT) PS1R1(DEG)
      XCR2(FT) YCR2(FT) PS1R2(DEG)

On a space-fixed coordinate system (Figure 2) defined for the scene of a given case, enter the positions and orientations, at rest, of the two vehicles. Note that the heading angles may be entered as either positive or negative quantities.
2.10 ENTER IMPACT POSITIONS AND HEADING ANGLES
FOR VEHICLE 1 AND VEHICLE 2
FORM: XC10(FT) YC10(FT) PS110(DEG) XC20(FT)
YC20(FT) PS120(DEG)

In the established space-fixed coordinate system, enter the positions
and orientations of the two vehicles at initial contact. A dimensional check,
using the undeformed vehicle dimensions, should be made to insure an accurate
definition of initial contact. Note that the optional generation of inputs
for the SMAC program includes application of a routine which adjusts the
initial vehicle positions to eliminate any initial interference. Heading angles
may be entered as either positive or negative quantities.

2.11 DID ROTATIONAL AND/OR LATERAL SKIDDING
OF VEHICLE #1 OCCUR?

The terms "rotational and/or lateral skidding" are used herein to
refer to the vehicle motions in a portion of the trajectory during which the
front and rear wheels do not run in the same tracks.

A "no" response to this question will result in the separation velocity
of vehicle #1 being approximated entirely on the basis of the specified rolling
resistances of the wheels (or the longitudinal deceleration), as limited by the
specified tire-terrain friction coefficient, and the total distance traveled
between separation and rest (i.e., the traditional $V^2 = 2as$ relationship).

2.12 DID ROTATIONAL (YAW) AND/OR LATERAL SKIDDING
OF VEHICLE #1 STOP BEFORE REST POSITION WAS
REACHED? (ANSWER YES OR NO)

A relatively common occurrence in the vehicle trajectories subsequent
to a collision, in which rotational and/or lateral skidding occurs, is an
abrupt change in the direction of vehicle motion as the skidding stops and the
longitudinal motion continues (either forward or backward) in a direction
determined by the heading direction at the stop of the skidding. Note
that damage-locked wheels and/or steered angles can produce a curved path in the final, nonskidding portion of the trajectory.

The response to this question must be based on a detailed review of tire mark and other trajectory evidence.

2.13 ENTER POSITION AND HEADING OF VEHICLE #1 AT END OF ROTATIONAL (YAW) AND/OR LATERAL SKIDDING
FORM:  XC11(FT) YC11(FT) PS111(DEG)

Entries in response to this question must define the position and heading of the vehicle at the point in the trajectory at which the front and rear wheels start to run in the same tracks.

2.14 WAS THE SPINOUT PATH OF VEHICLE #1 BETWEEN SEPARATION AND [REST, STOP OF SKIDDING] CURVED? (ANSWER YES OR NO)

If a "no" response is entered, the direction of the resultant linear velocity at separation is determined by the rest (or stop of skidding) position. The resultant linear velocity at separation is used, in turn, to establish the velocities of the two vehicles at impact, by means of conservation of momentum relationships. Thus, a curved trajectory can introduce significant error in the velocity calculations by indicating an erroneous direction of motion at separation (Figure 3). The present question is aimed at introducing a correction in the direction of motion subsequent to separation in those cases where clear evidence of a curved trajectory exists. In most cases, a "no" response is appropriate.
Figure 3  EFFECTS OF CURVED TRAJECTORY ON INDICATED DIRECTION OF MOTION SEPARATION
2.15 ENTER AN INTERMEDIATE POSITION OF VEHICLE #1 ON THE CURVED PATH BETWEEN SEPARATION AND [REST, STOP OF SKIDDING]
FORM: XC21(FT) YC21(FT)

An approximate correction in the direction of motion at separation is introduced at this point. It is assumed that the direction of motion at separation is tangential to a circular arc fitted through the positions at separation and rest (or end of skidding) and the intermediate position entered in response to this question.

The response to this question must be based on a detailed review of tire mark and other trajectory evidence.

2.16-2.20 Same questions as 2.11 through 2.15 in relation to vehicle #2. The same discussions and comments apply.

2.21 WHICH DIRECTION DID VEHICLE #1 ROTATE?
ANSWER: NONE CW CCW

For the case of purely lateral skidding, the answer "none" is appropriate. The direction of rotation is required to permit calculation of the total amount of rotation.

2.22 DID VEHICLE #1 ROTATE MORE THAN 360 DEGREES BETWEEN SEPARATION AND REST? (ANSWER YES OR NO)

Rotations of more than 360 degrees are relatively rare. The response to this question clearly must be based on evidence available in the specific case.

2.23, 2.24 Same questions as 2.21 and 2.22 in relation to vehicle #2. The same discussions and comments apply.
2.25 ENTER THE NOMINAL TIRE-GROUND FRICTION COEFFICIENT
FORM: MU

The user is referred to texts on accident reconstruction which contain
tables of representative ranges of tire-ground friction coefficients on
different surfaces (e.g., Baker, J. S., "Traffic Accident Investigator's
Manual for Police," Reference 4). Where possible, an attempt should be made
to obtain directly measured friction data (e.g., stopping tests, pendulum
tests, etc.) in view of the critical importance of this data item to the
accuracy of the trajectory analysis.

The trajectory analysis portion of the CRASH program is based on a
calculation of the energy dissipation, through work done against friction
forces, during the spinout. Therefore, it is possible, in a straightforward
manner, to handle a case in which surfaces with different friction coefficients
are traversed. An "equivalent" single friction coefficient that will dissipate
the same amount of energy can be determined in the following manner:

$$\mu_{eq} = \frac{\mu_1 S_1 + \mu_2 S_2 + \ldots + \mu_n S_n}{(S_1 + S_2 + \ldots + S_n)}$$

where $\mu_{eq} =$ "equivalent" single friction coefficient
$S_i =$ distance traveled through region with
friction coefficient = $\mu_i$
ROLLING RESISTANCE MAY BE ENTERED EITHER AS (1) THE DECIMAL PORTION OF FULL ROTATIONAL LOCK-UP AT EACH WHEEL OR (2) THE LEVEL OF LONGITUDINAL DECELERATION, IN G UNITS, PRODUCED BY ROTATIONAL RESISTANCE AT THE WHEELS. SELECT OPTION (1) OR OPTION (2).

The related CRASH calculations do not distinguish which of the individual wheels are the sources of longitudinal deceleration. Therefore, an equivalent use of options 1 and 2 will yield identical results from CRASH. Option 1 is designed for use in the generation of SMAC inputs (an optional output of CRASH) or for establishing a detailed record of the basis for the CRASH reconstruction (e.g., LF wheel locked by damage, engine braking at rear wheels, etc.).

(OPTION 1) ENTER ROLLING RESISTANCES OF WHEELS OF VEHICLE 1 (DAMAGE, BRAKES, ENGINE BRAKING, TIRES, 0.0 TO 1.0, WHERE 1.0 = LOCKED)

To achieve a given longitudinal deceleration level via this form of rolling resistance input, consideration must be given to the entered value for the tire-ground friction coefficient. The decimal quantity, between 0.0 and 1.00, that is entered for each wheel defines the portion of the full available friction force acting at that individual wheel in the longitudinal direction. Since an equal distribution of weight is assumed in this approximation, the sum of the individual decimal quantities entered for the four wheels may be divided by four and multiplied by the tire terrain friction coefficient to obtain the longitudinal deceleration level in G units.

As previously noted, this option serves to provide a record of the rolling resistances at individual wheels used in the CRASH reconstruction (i.e., it defines how the total longitudinal deceleration was produced). It can also serve to generate corresponding inputs for the SMAC program. For these purposes, the entry sequence of RF, LF, RR, LR is important.
2.28 (OPTION 2) ENTER LONGITUDINAL DECELERATION, IN G UNITS, PRODUCED BY ROTATIONAL RESISTANCE AT THE WHEELS

For this option, a single entry is required. It should be noted that the entry must be less than or equal to the tire-ground friction coefficient entered in response to 2.25.

2.29, 2.30 Same questions as 2.27 and 2.28 in relation to vehicle #2. The same discussions and comments apply.

2.31 ARE ANY ACTUAL DAMAGE DIMENSIONS KNOWN? (ANSWER YES OR NO)

It is highly desirable, for the purpose of achieving accurate results, to enter actual measured damage dimensions whenever possible. The required dimensions are (1) the width of the crushed region, (2) the extent of the damage (i.e., the depth of indentation) at several positions within the damaged region and (3) the location of the midpoint of the damaged region with respect to the center of mass of the vehicle. The required dimensions are depicted in Figure 4.

The dimension D to the midpoint of the damaged region must be entered with an algebraic sign corresponding to that of the y (end impact) or x (side impact) coordinate of the damage midpoint. This dimension, in combination with the direction of the resultant force, is used to determine the effective mass of the subject vehicle at the point that achieves a common velocity with the corresponding point on the other vehicle in the collision (see Figure 5).

In the absence of actual damage dimensions, the CRASH program generates approximations of the required dimensions on the basis of the entered vehicle damage indices. The procedure for generating approximate dimensions is outlined in the following paragraphs.
Figure 4  DAMAGE DIMENSIONS
Figure 5  MOMENT ARMS OF RESULTANT FORCE ON VEHICLES 1 AND 2
APPROXIMATION OF DAMAGE DIMENSIONS
FROM VDI INPUTS

1) IF COL 3 OF VDI1 = F OR B, GO TO 4

2) COL 7

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<th>DEFLI</th>
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<tr>
<td>2</td>
<td>.2090Ys</td>
</tr>
<tr>
<td>3</td>
<td>.3775Ys</td>
</tr>
<tr>
<td>4</td>
<td>.6265Ys</td>
</tr>
<tr>
<td>5</td>
<td>.8755Ys</td>
</tr>
<tr>
<td>6</td>
<td>1.1245Ys</td>
</tr>
<tr>
<td>7</td>
<td>1.3735Ys</td>
</tr>
<tr>
<td>8</td>
<td>1.6225Ys</td>
</tr>
<tr>
<td>9</td>
<td>1.7470Ys</td>
</tr>
</tbody>
</table>

(Ys from Table Lookup. See Table 1)

3) COL 4

<table>
<thead>
<tr>
<th></th>
<th>DI</th>
<th>LI</th>
<th>GO TO</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0</td>
<td>0.69XF - 0.79XR</td>
<td>7</td>
</tr>
<tr>
<td>P</td>
<td>0</td>
<td>0.37XF - 0.57XR</td>
<td>7</td>
</tr>
<tr>
<td>Z</td>
<td>+0.5XR</td>
<td>-XR</td>
<td>8</td>
</tr>
<tr>
<td>Y</td>
<td>0.5XF</td>
<td>XF</td>
<td>8</td>
</tr>
<tr>
<td>F</td>
<td>0.69XF</td>
<td>0.63XF</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>+0.79XR</td>
<td>-0.43XR</td>
<td>8</td>
</tr>
</tbody>
</table>

(XF, XR (Neg) from Table 1)
4) IF COL 3 = B, GO TO 5

<table>
<thead>
<tr>
<th>COL 7</th>
<th>DEFLI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.0625X_F)</td>
</tr>
<tr>
<td>2</td>
<td>(0.1875X_F)</td>
</tr>
<tr>
<td>3</td>
<td>(0.3125X_F)</td>
</tr>
<tr>
<td>4</td>
<td>(0.4375X_F)</td>
</tr>
<tr>
<td>5</td>
<td>(0.5625X_F)</td>
</tr>
<tr>
<td>6</td>
<td>(0.7355X_F)</td>
</tr>
<tr>
<td>7</td>
<td>(0.8960X_F)</td>
</tr>
<tr>
<td>8</td>
<td>(0.9960X_F)</td>
</tr>
<tr>
<td>9</td>
<td>(1.046X_F)</td>
</tr>
</tbody>
</table>

GO TO 6.

5) | COL 7 | DEFLI          |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-0.042X_R)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(-0.126X_R)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(-0.210X_R)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(-0.294X_R)</td>
<td>((X_R) (Neg) from Table 1)</td>
</tr>
<tr>
<td>5</td>
<td>(-0.3785X_R)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(-0.5045X_R)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(-0.6785X_R)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(-0.8595X_R)</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>(-0.950X_R)</td>
<td></td>
</tr>
</tbody>
</table>
6) | COL 4 | DI | LI | GO TO |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0</td>
<td>2Y_s</td>
<td>9</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0.666Y_s</td>
<td>7</td>
</tr>
<tr>
<td>Z</td>
<td>0.333Y_s</td>
<td>1.332Y_s</td>
<td>8</td>
</tr>
<tr>
<td>Y</td>
<td>-0.333Y_s</td>
<td>1.332Y_s</td>
<td>8</td>
</tr>
<tr>
<td>R</td>
<td>0.666Y_s</td>
<td>0.666Y_s</td>
<td>8</td>
</tr>
<tr>
<td>L</td>
<td>-0.666Y_s</td>
<td>0.666Y_s</td>
<td>8</td>
</tr>
</tbody>
</table>

(Y_s from Table 1 Lookup)

7) \[\begin{align*}
\text{CI1} &= 0.0 \\
\text{CI2} &= \text{DEFLI} \\
\text{CI3} &= \text{DEFLI} \\
\text{CI4} &= 0.0 \\
\text{GO TO} &= 10
\end{align*}\]

8) \[\begin{align*}
\text{CI1} &= 0.0 \\
\text{CI2} &= 0.5*\text{DEFLI} \\
\text{CI3} &= 0.5*\text{DEFLI} \\
\text{CI4} &= \text{DEFLI} \\
\text{GO TO} &= 10
\end{align*}\]

9) \[\begin{align*}
\text{CI1} &= \text{DEFLI} \\
\text{CI2} &= (0.666+0.333*\text{RHOI})*\text{DEFLI} \\
\text{CI3} &= (0.333+0.666*\text{RHOI})*\text{DEFLI} \\
\text{CI4} &= \text{RHOI*DEFLI} \\
\text{GO TO} &= 10
\end{align*}\]

\[C = C_1[1-\frac{\rho}{L}(1-\rho)]\]

10) IF COL 6 = N, LI = 16.0 INCHES
REPLACE DI, LI, CII, CII, .... CIN WITH ANY DIRECT ENTRIES

The approximation procedure that has been outlined above again reflects the exploratory nature of this research in which the emphasis has been on program structure as opposed to rigorously established approximation techniques. Obviously, the approximations of DI, LI and of the damage profiles, CII, could be refined by means of detailed correlations of those items with investigator rated VDI s for a large number of vehicles. Such a procedure has been beyond the limited scope of this research effort to date. Corresponding refinements of the above approximation technique, within the existing program framework, could be achieved by means of appropriate adjustments of the coefficients used in the programmed equations.

In relation to the above approximations and the crush properties presented in Table 1, one complicating factor is worthy of mention. In the reporting of damage in staged collisions, the distinction between induced and direct-contact damage is generally obscured. On the other hand, the VDI, by definition, is limited to direct-contact damage (Reference 3). Thus, the damage basis for the fitted empirical crush properties may differ from that of the VDI s. In any future refinements of the computer program, this aspect of the damage analysis procedure should receive priority attention.

2.32 ENTER DAMAGE DIMENSION MEASURED ALONG INVOLVED [END, SIDE] OF VEHICLE #1
FORM: L(I) (INCHES)

See Figure 4. For compatibility with the VDI, the entered dimension L(I) should be based on direct-contact damage, as opposed to induced damage (see discussion of preceding question).
2.33 ENTER EXTENT OF DAMAGE (CHANGE FROM ORIGINAL [END, SIDE] DIMENSION OF VEHICLE #1 (INCHES) AT EDGES OF DAMAGED REGION AND AT EQUALLY SPACED INTERMEDIATE POSITIONS. NOTE: ENTRY OF TWO, FOUR, OR SIX DIMENSIONS IN SEQUENCE FROM EITHER SIDE.

The optional entry of two points is aimed at convenience in angled, full-frontal contacts where the damage surface is relatively flat. The four and six point options permit greater detail in the definitions of damage when adequate measured data are available. See Figure 4 for definition of the extent dimensions. The entry sequence may proceed from either side of the damaged region.

2.34 ENTER DISTANCE ALONG VEHICLE #1 AXIS BETWEEN C.G. AND MIDDLE OF DAMAGED REGION
FORM: D (INCHES)

The dimension D to the midpoint of the damaged region (see Figure 4) must be entered with an algebraic sign corresponding to that of the y (end impact) or x (side impact) coordinate of the damage midpoint. See Figure 5 and further discussion in 2.31.

2.35-2.37 Same questions as 2.32 through 2.34 in relation to vehicle #2. The same discussions and comments apply.

3.38 ARE THE DIRECTIONS OF THE PRINCIPAL FORCES KNOWN MORE ACCURATELY THAN THE GIVEN CLOCK DIRECTIONS (ANSWER YES OR NO)

The clock directions in the VDI's are rounded off to the nearest 30 degrees. In cases where a more accurate definition of the principal force direction is available (e.g., a 45° impact which would be rounded off to 01 or 02) the more accurate angle should be entered.

A "no" response will result in the use of the clock directions.
2.39 ENTER DIRECTIONS OF PRINCIPAL FORCE FOR VEHICLE 1 AND VEHICLE 2 (DEGREES, MEASURED FROM STRAIGHT-AHEAD DIRECTION, ± 180°)
FORM: ANG1 ANG2 (DEG)

The compatibility of the entered directions of the principal force on the two vehicles in the impact configuration should be carefully checked (i.e., the forces acting on the two vehicles should be equal and opposite in direction).
3. OUTPUT FORMAT

On the following pages, the computer printout for a sample run is presented.
CRASH
CALSPAN RECONSTRUCTION OF ACCIDENT SPEEDS ON THE HIGHWAY

NOTE: ANSWER ALL QUESTIONS AS DIRECTED
SEPARATE ALL NUMERIC ENTRIES BY ONE OR MORE BLANKS
LITERAL RESPONSES ARE ALSO FREE FORM

ENTER A DESCRIPTIVE TITLE (72 CHARACTERS MAX.)
90 DEG REAR-SIDE AT 40 MPH, UCLA-ITL SIDE IMPACT SERIES

ENTER SIZE CATEGORIES FOR VEHICLE # 1 AND VEHICLE # 2
LEGAL CATEGORIES: SUBCOMPACT
COMPACT
INTERMEDIATE
FULLSIZE
BARRIER
SAMPLE: COMPACT FULL
NOTE: 1ST LETTERS ARE O.K.
1 1

ENTER A 7 CHARACTER VDI FOR VEHICLE # 1
FORM: 12LYEn2
01DENm2

ENTER A 7 CHARACTER VDI FOR VEHICLE # 2
FORM: 12LYEn2
1OL2En3

ARE ANY ACTUAL WEIGHTS KNOWN? (ANSWER YES OR NO) N

ARE BOTH REST AND IMPACT POSITIONS KNOWN? (ANSWER YES OR NO) Y

ENTER REST POSITIONS AND HEADING FOR VEHICLE 1 AND VEHICLE 2
FORM: XCH1(FT) YCH1(FT) PSI1(DEG) XCH2(FT) YCR2(FT) PSI2(DEG)
50.0 -21.0 -141.0 19.8 -60.9 -336.0

ENTER IMPACT POSITIONS AND HEADING ANGLES FOR VEHICLE 1 AND VEHICLE 2
FORM: XC10(FT) YC10(FT) PSI10(DEG) XC20(FT) YC20(FT) PSI20(DEG)
-13.3 0.0 0.0 0.0 5.25 -90.0

DID ROTATIONAL AND/OR LATERAL SKIDDING OF VEHICLE # 1 OCCUR?
(ANSWER YES OR NO) Y

27

ZQ-5708-V-3
DID ROTATIONAL (YAW) AND/OR LATERAL SKIDDING OF VEHICLE 1 STOP BEFORE REST POSITION WAS REACHED? (ANSWER YES OR NO)

Y

ENTER POSITION AND HEADING OF VEHICLE 1 AT END OF ROTATIONAL (YAW) AND/OR LATERAL SKIDDING

FORM: XCI1(FT.) YCI1(FT.) PSI11(DEG.)

39.0 -20.0 -173.0

WAS THE SPINOUT PATH OF VEHICLE 1 BETWEEN SEPARATION AND STOP OR ROTATION CURVED? (ANSWER YES OR NO)

N

DID ROTATIONAL AND/OR LATERAL SKIDDING OF VEHICLE # 2 OCCUR? (ANSWER YES OR NO)

Y

DID ROTATIONAL (YAW) AND/OR LATERAL SKIDDING OF VEHICLE 2 STOP BEFORE REST POSITION WAS REACHED? (ANSWER YES OR NO)

N

WAS THE SPINOUT PATH OF VEHICLE 2 BETWEEN SEPARATION AND REST CURVED? (ANSWER YES OR NO)

N

WHICH DIRECTION DID VEHICLE 1 ROTATE?

ANSWER: NONE CW CCW

CCW

DID VEHICLE 1 ROTATE MORE THAN 360 DEGREES BETWEEN SEPARATION AND REST? (ANSWER YES OR NO)

N

WHICH DIRECTION DID VEHICLE 2 ROTATE?

ANSWER: NONE CW CCW

CCW

DID VEHICLE 2 ROTATE MORE THAN 360 DEGREES BETWEEN SEPARATION AND REST? (ANSWER YES OR NO)

N

ENTER THE NOMINAL TIREE-GROUND FRICTION COEFFICIENT

FORM: MU

0.8

ROLLING RESISTANCE MAY BE ENTERED EITHER AS:
1 --- THE DECIMAL PORTION OF FULL ROTATIONAL LOCKUP AT EACH WHEEL.
2 --- THE LEVEL OF LONGITUDINAL DECELERATION, IN G UNITS, PRODUCED BY ROTATIONAL RESISTANCE AT THE WHEELS.

ENTER 1 OR 2

1
ENTER ROLLING RESISTANCES OF WHEELS OF VEHICLE 1
(DAMAGE, BRAKES, ENGINE BRAKING, TIRES, 0.0 TO 1.0, WHERE 1.0=LOCKED)
FORM: RF LF RR LR
1.0 0.0 0.0 0.0

ENTER ROLLING RESISTANCES OF WHEELS OF VEHICLE 2
(DAMAGE, BRAKES, ENGINE BRAKING, TIRES, 0.0 TO 1.0, WHERE 1.0=LOCKED)
FORM: RF LF RR LR
0.0 0.0 1.0 1.0

ARE ANY ACTUAL DAMAGE DIMENSIONS KNOWN? (ANSWER YES OR NO)

ENTER DAMAGE DIMENSION Measured ALONG INVOLVED END OF VEHICLE # 1
FORM: L(1) (INCHES)
10.0

ENTER EXTENT OF DAMAGE (CHANGE FROM ORIGINAL END DIMENSIONS OF
VEHICLE 1 (INCHES) AT EDGES OF DAMAGED REGION AND AT EQUALLY SPACED
INTERMEDIATE POSITIONS.
NOTE: TWO, FOUR, OR SIX POINTS ALLOWED - ENTRY POINT FROM EITHER SIDE.
FORM: C1 C2 C3 C4 C5 C6 (INCHES)
12.0 14.0 14.0 24.0

ENTER DISTANCE ALONG VEHICLE # 1 AXIS BETWEEN C.G. AND MIDDLE OF
DAMAGED REGION
FORM: D (INCHES)
0.0

ENTER DAMAGE DIMENSION MEASURED ALONG INVOLVED SIDE OF VEHICLE # 2
FORM: L(2) (INCHES)
90.0

ENTER EXTENT OF DAMAGE (CHANGE FROM ORIGINAL SIDE DIMENSIONS OF
VEHICLE 2 (INCHES) AT EDGES OF DAMAGED REGION AND AT EQUALLY SPACED
INTERMEDIATE POSITIONS.
NOTE: TWO, FOUR, OR SIX POINTS ALLOWED - ENTRY POINT FROM EITHER SIDE.
FORM: C1 C2 C3 C4 C5 C6 (INCHES)
0.0 12.7 14.3 14.2

ENTER DISTANCE ALONG VEHICLE # 2 AXIS BETWEEN C.G. AND MIDDLE OF
DAMAGED REGION
FORM: D (INCHES)
-11.2

29
ARE THE DIRECTIONS OF THE PRINCIPAL FORCES KNOWN MORE ACCURATELY THAN THE GIVEN CLOCK DIRECTIONS? (ANSWER YES OR NO)

THANK YOU VERY MUCH

**** SUMMARY OF CRASH RESULTS ****

90 DEG REAR-SIDE AT 40 MPH, UCLA-IT SE SIDE IMPACT SERIES

VEHICLE # 1

<table>
<thead>
<tr>
<th>IMPACT SPEED</th>
<th>SPEED CHANGE</th>
<th>BASIS OF RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPH</td>
<td>MPH</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>LONG.</td>
<td>LATERAL</td>
</tr>
<tr>
<td>31.3</td>
<td>14.7</td>
<td>-10.4</td>
</tr>
</tbody>
</table>

SPINOUT TRAJECTORIES AND CONSERVATION OF LINEAR MOMENTUM

| 14.9         | -12.9        | -7.4             |

SPINOUT TRAJECTORIES AND DAMAGE

DAMAGE DATA ONLY
### Speed Change

<table>
<thead>
<tr>
<th>IMPACT SPEED MPH</th>
<th>SPEED CHANGE MPH</th>
<th>BASIS OF RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TOTAL</td>
<td>LONG.</td>
</tr>
<tr>
<td>39.6</td>
<td>14.7</td>
<td>-10.3</td>
</tr>
<tr>
<td></td>
<td>14.9</td>
<td>-1.4</td>
</tr>
</tbody>
</table>

COLLISION CONDITIONS

<table>
<thead>
<tr>
<th>VEHICLE # 1</th>
<th>VEHICLE # 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>XC10' = -13.3 FT.</td>
<td>XC20' = 0.0 FT.</td>
</tr>
<tr>
<td>YC10' = 0.0 FT.</td>
<td>YC20' = -5.3 FT.</td>
</tr>
<tr>
<td>PSI10 = 0.0 DEGREES</td>
<td>PSI20 = -90.0 DEGREES</td>
</tr>
<tr>
<td>PSI1D0 = 0.0 DEG/SEC</td>
<td>PSI2D0 = 0.0 DEG/SEC</td>
</tr>
<tr>
<td>U10 = 37.3 MPH</td>
<td>U20 = 39.6 MPH</td>
</tr>
<tr>
<td>V10 = 0.0 MPH</td>
<td>V20 = 0.0 MPH</td>
</tr>
</tbody>
</table>

SEPARATION CONDITIONS

<table>
<thead>
<tr>
<th>VEHICLE # 1</th>
<th>VEHICLE # 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>XCS1 = -13.3 FT.</td>
<td>XCS2 = 0.0 FT.</td>
</tr>
<tr>
<td>YCS1 = 0.0 FT.</td>
<td>YCS2 = -5.3 FT.</td>
</tr>
<tr>
<td>PSIS1 = 0.0 DEG</td>
<td>PSIS2 = -90.0 DEG</td>
</tr>
<tr>
<td>US1 = 29.4 MPH</td>
<td>US2 = 29.4 MPH</td>
</tr>
<tr>
<td>VS1 = 10.4 MPH</td>
<td>VS2 = 10.4 MPH</td>
</tr>
<tr>
<td>PSISD1 = -136.4 DEG/SEC</td>
<td>PSISD2 = -156.8 DEG/SEC</td>
</tr>
</tbody>
</table>
### SUMMARY OF DAMAGE DATA

<table>
<thead>
<tr>
<th>VEHICLE # 1</th>
<th>VEHICLE # 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TYPE</strong> ------ INTERMEDIATE</td>
<td><strong>TYPE</strong> ------ INTERMEDIATE</td>
</tr>
<tr>
<td><strong>WEIGHT</strong> ------ 3809.9 LBS. *</td>
<td><strong>WEIGHT</strong> ------ 3809.9 LBS. *</td>
</tr>
<tr>
<td><strong>VDI</strong> ------ 01FDEM2</td>
<td><strong>VDI</strong> ------ 10LZEM3</td>
</tr>
<tr>
<td><strong>L</strong> ------ 76.8 IN.</td>
<td><strong>L</strong> ------ 96.7 IN.</td>
</tr>
<tr>
<td><strong>C1</strong> ------ 12.0 IN.</td>
<td><strong>C1</strong> ------ 0.0 IN.</td>
</tr>
<tr>
<td><strong>C2</strong> ------ 14.0 IN.</td>
<td><strong>C2</strong> ------ 12.7 IN.</td>
</tr>
<tr>
<td><strong>C3</strong> ------ 14.0 IN.</td>
<td><strong>C3</strong> ------ 14.3 IN.</td>
</tr>
<tr>
<td><strong>C4</strong> ------ 24.0 IN.</td>
<td><strong>C4</strong> ------ 14.2 IN.</td>
</tr>
<tr>
<td><strong>C5</strong> ------ 0.0 IN.</td>
<td><strong>C5</strong> ------ 0.0 IN.</td>
</tr>
<tr>
<td><strong>C6</strong> ------ 0.0 IN.</td>
<td><strong>C6</strong> ------ 0.0 IN.</td>
</tr>
<tr>
<td><strong>D</strong> ------ 0.0 IN.</td>
<td><strong>D</strong> ------ -71.2 IN.</td>
</tr>
<tr>
<td><strong>RHO</strong> ------ 1.00 *</td>
<td><strong>RHO</strong> ------ 1.00 *</td>
</tr>
<tr>
<td><strong>ANG</strong> ------ 30.0 DEG. *</td>
<td><strong>ANG</strong> ------ 300.0 DEG. *</td>
</tr>
</tbody>
</table>

### DIMENSIONS AND INERTIAL PROPERTIES

<table>
<thead>
<tr>
<th>A1</th>
<th>57.3 INCHES</th>
<th>A2</th>
<th>57.3 INCHES</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>59.7 INCHES</td>
<td>B2</td>
<td>59.7 INCHES</td>
</tr>
<tr>
<td>TH1</td>
<td>60.0 INCHES</td>
<td>TH2</td>
<td>60.0 INCHES</td>
</tr>
<tr>
<td>II</td>
<td>29560.0 LB-SEC**2-IN</td>
<td>I2</td>
<td>29560.0 LB-SEC**2-IN</td>
</tr>
<tr>
<td>XR1</td>
<td>94.8 INCHES</td>
<td>XR2</td>
<td>94.8 INCHES</td>
</tr>
<tr>
<td>XR1</td>
<td>-110.8 INCHES</td>
<td>XR2</td>
<td>-110.8 INCHES</td>
</tr>
<tr>
<td>YS1</td>
<td>38.4 INCHES</td>
<td>YS2</td>
<td>38.4 INCHES</td>
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</tbody>
</table>

### ROLLING RESISTANCE

<table>
<thead>
<tr>
<th>VEHICLE # 1</th>
<th>VEHICLE # 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RF</strong> ------ 1.00</td>
<td><strong>RF</strong> ------ 0.0</td>
</tr>
<tr>
<td><strong>LF</strong> ------ 0.0</td>
<td><strong>LF</strong> ------ 0.0</td>
</tr>
<tr>
<td><strong>RR</strong> ------ 0.0</td>
<td><strong>RR</strong> ------ 1.00</td>
</tr>
<tr>
<td><strong>LR</strong> ------ 0.0</td>
<td><strong>LR</strong> ------ 1.00</td>
</tr>
<tr>
<td><strong>MU</strong> ------ 0.80</td>
<td></td>
</tr>
</tbody>
</table>
DO YOU WISH TO GENERATE A SMAC INPUT FILE?  (ANSWER YES OR NO)

Y

DO YOU WISH SMAC INPUT FILE PRINTED ON YOUR TERMINAL?  (ANSWER YES OR NO)

Y

SIMULATION MODEL OF AUTOMOBILE COLLISIONS (USING CRASH RESULTS)
90 DEG REAR-SIDE AT 40 MPH, UCLA-IFFE SIDE IMPACT SERIES

<table>
<thead>
<tr>
<th></th>
<th>0.0</th>
<th>4.0</th>
<th>0.025</th>
<th>0.001</th>
<th>0.01</th>
<th>0.001</th>
<th>30.0</th>
<th>5.0</th>
<th>2.0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-159.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>50.39</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>0.0</td>
<td>-63.00</td>
<td>-90.0</td>
<td>0.0</td>
<td>69.17</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51.3</td>
<td>59.7</td>
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<td>29300.0</td>
<td>0.0</td>
<td>9.000</td>
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4. REFERENCES


APPENDIX

5.1 Spinout Analysis

In Reference 5, Marquard defines relationships for approximating the initial linear and angular (yaw) velocities of a vehicle in a spinout trajectory (i.e., at the point of separation subsequent to a collision) on the basis of the energy dissipated during its changes in position and orientation between separation and rest. He includes the cases of freely rotating wheels and of locked wheels, each with the front wheels limited to the straight-ahead position of steering.

In the case of freely rotating wheels, the linear and angular velocities of the vehicle are decelerated alternately as the heading direction changes with respect to the direction of the linear velocity. When the vehicle slides laterally, the side forces at the front and rear tires tend to have the same direction despite the existence of a yaw velocity (Figure 6a). Therefore, during this phase of the motion, the angular velocity tends to remain constant while the linear velocity is decelerated. When the longitudinal axis is aligned with the direction of the linear velocity, the side forces at the front and rear tires act in opposite directions and the angular velocity is decelerated while the linear velocity tends to remain constant (Figure 6b). A SMAC-generated example of the time-histories of angular and linear velocity, for the case of no braking, is shown in Figure 7. Marquard defines a different form of solution for the case of locked wheels, whereby the ratio of angular to linear displacement during the spinout is used to determine empirical coefficients.

The derivation of equations in Reference 5 is not completely presented. Therefore, some details of the assumptions must be deduced from the final form of the equations.
Figure 6  SCHEMATIC OF ROLL OUT AFTER COLLISION
Figure 7  LINEAR AND ANGULAR VELOCITY VS TIME
In the following outline of the derivation, the time derivative of a variable is indicated by a dot over the symbol for the variable and the subscript S is used to indicate the value of a variable at the point of separation.

Spinout Without Braking

In Figure 8, an idealized plot of the time history of linear and angular velocities is depicted. The symbols $T_1$ and $T_2$ are used to represent the times, subsequent to separation, at which the linear and angular velocities, respectively, reach zero values. The areas under the velocity plots are, of course, equal to the corresponding linear and angular displacements. If it is assumed that reasonable approximations of the areas under the two velocity plots can be obtained from the triangles shown in Figure 8 as broken lines, the following relationships can be applied.

\[ \Delta \psi = \left( \frac{\psi_S}{2} \right) T_1, \quad \text{and} \]

\[ S = \left( \frac{S_S}{2} \right) T_2 \]  

(1)

(2)

During the periods of angular deceleration, the magnitude of that deceleration can be approximated as

\[ \dot{\psi} = \frac{\mu g}{k^2} \left( \frac{a+b}{2} \right) \]

(3)

where

$\mu$ = nominal tire-ground friction coefficient.

$g$ = acceleration of gravity, in/sec$^2$.

$k^2$ = radius of gyration squared for complete vehicle in yaw, in$^2$.

$(a+b)$ = wheelbase, inches.
Figure 8  IDEALIZED PLOT OF VELOCITIES VS TIME
From equation (3) the actual deceleration time of the angular velocity can be approximated as

\[
\dot{\psi} = \frac{\psi}{\dot{\psi}} = \frac{2\psi_k^2}{\mu g(a+b)}
\]  

(4)

During the linear deceleration portion of the motion (i.e., for orientations near that of broadside sliding), the tire side forces, which are perpendicular to the wheel planes, act at a changing angle with respect to the direction of the linear velocity. If the average value of the cosine of the angle during that portion of the motion is taken to be 0.85, the average magnitude of the linear deceleration during periods of linear deceleration can be approximated as

\[
\ddot{S} = 0.85 \mu g
\]  

(5)

The corresponding actual deceleration time of the linear velocity can be approximated by

\[
\dot{t}_2 = \frac{\dot{S}}{S} \approx \frac{\dot{S}}{0.85 \mu g}
\]  

(6)

The total time required to stop both the linear and the angular motions can be expressed, from equations (4) and (6) as

\[
T = \dot{t}_1 + \dot{t}_2 = \frac{2\psi_k^2}{\mu g(a+b)} + \frac{\dot{S}}{0.85 \mu g}
\]  

(7)
If it is assumed that both phases of the motion end at approximately the same time,

\[ T \approx T_1 \approx T_2, \text{ then from (1) and (2),} \]

\[ \frac{2(\Delta \psi)}{\dot{\psi}_s} \approx \frac{2S}{S_s} \approx T \quad (8) \]

From (8),

\[ \frac{\dot{S}_s}{\dot{\psi}_s} \approx \frac{S}{\Delta \psi} \quad (9) \]

Substitution of (8) and (9) into (7) yields

\[ \frac{2(\Delta \psi)}{\psi_s} = \frac{2k^2}{(a+b)u_g} + \frac{S}{0.85 u_g \Delta \psi} \quad (10) \]

Solution of (10) for \( \dot{\psi}_s \) yields

\[ \dot{\psi}_s = \sqrt{\frac{u_g (\Delta \psi)^2}{\left(\frac{k^2}{(a+b)}\right) \Delta \psi} + \frac{S}{1.70} \text{ sgn}(\Delta \psi)} \quad (11) \]

From (7) and (8),

\[ \dot{S}_s = 1.70 \left[ \frac{u_g (\Delta \psi)}{\psi_s} - \frac{k^2 |\dot{\psi}_s|}{(a+b)} \right] \quad (12) \]
Equations (11) and (12) correspond to the relationships defined by Marquard in Reference 5. The relationships defined by (11) and (12) were extended to include the case of partial braking, in the following manner.

If $\theta$ is used to define the decimal portion of full deceleration produced by braking or wheel damage, where $0 \leq \theta \leq 1.00$, a linear deceleration of $0.85 \mu g$ occurs during $t_1$, the deceleration time of the angular velocity. Therefore, the linear velocity to be decelerated in the corresponding phase of motion is reduced to

$$S_1 = S_s - 0.85\theta \mu gt_1$$  \hspace{1cm} (13)

The total time required for linear deceleration is reduced to

$$t_2 = \frac{S_1}{0.85 \mu g} = \frac{S_s}{0.85 \mu g} - \theta t_1$$  \hspace{1cm} (14)

Therefore, the total time required to stop both the linear and the angular motions becomes

$$T = t_1 + t_2 = \frac{S_s}{0.85 \mu g} + (1-\theta) \frac{2_{\psi_s}k^2}{(a+b) \mu g}$$ \hspace{1cm} (15)

With the introduction of $\theta$, equations (11) and (12) become

$$\psi_s = \sqrt{\frac{\mu g (\Delta \psi)^2}{(k^2 |\Delta \psi| (1-\theta) + S \frac{1.70}{a+b}}) \cdot \text{sgn} \Delta \psi}$$  \hspace{1cm} (16)

$$S_s = 1.70 \left[ \frac{\mu g (\Delta \psi)}{\psi_s} - \frac{k^2 |\psi_s| (1-\theta)}{(a+b)} \right]$$  \hspace{1cm} (17)
Application of equations (16) and (17) to a number of SMAC-generated (Reference 8) spinout trajectories revealed several shortcomings. First, it was found that a residual linear velocity frequently exists at the end of the rotational motion. Thus, equations (8) and (9) can introduce large errors. Next it was found that the shapes of the plots of linear and angular velocity vs. time change substantially as functions of the initial ratio of linear to angular velocity, affecting the accuracy of simple linear approximations of the areas under the curves. Finally, the transitions between the different deceleration rates in the linear and angular motions do not occur abruptly. Rather, slope changes in the plots of velocities against time occur gradually, producing rounded "corners" in the curves (e.g., see Figure 7). As a result of the transitions, the effective deceleration rates in the two modes of motion are somewhat smaller than those corresponding to the full value of tire-ground friction.

To improve the accuracy of the approximations, provision was made for introduction of a residual linear velocity at the end of the rotational motion and empirical coefficients, in the form of polynomial functions of the initial ratio of linear to angular velocity. Since the velocity ratio is initially unknown, a solution procedure was developed whereby several trial values of the ratio, based on an approximate equation, are used to obtain multiple solutions. The solution for which the velocity ratio most closely matches the corresponding trial value is retained. The residual linear velocity is approximated on the basis of the distance traveled subsequent to the end of the rotational motion. The corresponding derivation of equations is outlined in the following.

The total time required to stop the angular motion is approximated by

$$T_1 = \alpha_1 \frac{\Delta \psi}{\psi_s} = t_1 + t_2$$

**TOTAL ANGULAR** (18)
The actual time of angular deceleration,

\[ t_1 = \frac{2\psi s k^2}{(a+b)\mu g a_2} \]  \hspace{1cm} (19)

The actual time during which linear acceleration occurs,

\[ t_2 = \frac{(S_s - S_1)}{a_4 \mu g} - \frac{\alpha_3 \delta t_1}{a_4} \] \hspace{1cm} (20)

The change in linear velocity during time \(T_1\), can be approximated as

\[ S_1 = \left(\frac{S_s + S_1}{a_s}\right) T_1 \] \hspace{1cm} (21)

From (18) and (21)

\[ a_1 \frac{\Delta\psi}{\psi_s} = \frac{\alpha_5}{\alpha_1} \frac{S_1}{(S_s + S_1)} \] \hspace{1cm} (22)

From (18), (19) and (20),

\[ a_1 \frac{\Delta\psi}{\psi_s} \frac{2\psi s k^2}{(a+b)\mu g a_2} \left(1 - \frac{\alpha_3 \delta}{\alpha_4}\right) + \frac{S_s - S_1}{a_4 \mu g} \] \hspace{1cm} (23)

From (22),

\[ (S_s - S_1) = \frac{\alpha_5}{\alpha_1} \frac{S_1 \Delta\psi}{\alpha_1} - 2S_1 \] \hspace{1cm} (24)
Substituting (24) in (23),

\[ \ddot{\psi}_s^2 + B \dot{\psi}_s + C = 0 \]  

(25)

where

\[ B = \frac{S_1 | \Delta \psi |}{D} \]  

(26)

\[ C = \frac{a_4 \mu g (\Delta \psi)^2}{2D} \]  

(27)

\[ D = \frac{a_4 k^2 | \Delta \psi | (1 - \frac{a_3}{a_4})}{\alpha_2 (a+b)} + \frac{a_5 S_1}{2a_1} \]  

(28)

From (23)

\[ \dot{S}_s = \dot{S}_1 + 2a_4 \left\{ \frac{\alpha_1 \mu g \Delta \psi}{2 \psi_s} - \frac{\dot{\psi}_s k^2 \left( \frac{a_3}{a_4} \right)}{(a+b) \alpha_2} \right\} \]  

(29)

The detailed solution procedure for equations (25) through (29) is outlined in the following section. It should be noted that the developed equations reduce to the form of (16) and (17) when the residual linear velocity is set to zero and the coefficients, \( a_1 \), are set to constant values. Also, the developed relationships apply to the case of fully locked wheels as well as rotating wheels, eliminating the need for a separate "locked wheel" procedure such as that defined in Reference 5.
SOLUTION PROCEDURE

Subroutine SPIN II

Inputs:

\[ X'_{c1}, Y'_{c1}, \psi'_1 = \text{Position and orientation at end of rotation (feet and degrees)} \]

\[ X'_{cs}, Y'_{cs}, \psi'_s = \text{Position and orientation at separation (feet and degrees)} \]

\[ S_1 = \text{Residual linear velocity at end of rotation (ft/sec)} \]

\[ a+b = \text{Wheelbase, inches} \]

\[ k^2 = \text{Radius of gyration squared for complete vehicle in yaw, } \text{in}^2 \]

\[ \mu = \text{Nominal tire-ground friction coefficient} \]

\[ \theta = \text{Decimal portion of full deceleration, } 0 \leq \theta \leq 1.000 \]

\[ g = \text{Acceleration of gravity} \]
\[ = 386.4 \text{ inches/sec}^2 \]

1. \[ S_1 = 12 \sqrt{(X'_{c1} - X'_{cs})^2 + (Y'_{c1} - Y'_{cs})^2} \text{ inches} \]

2. \[ \Delta \psi = \frac{(\psi'_1 - \psi'_s)}{57.3} \text{ radians} \]
3. \[ \gamma_s = \arctan \left( \frac{Y'_{cl} - Y'_{cs}}{X'_{cl} - X'_{cs}} \right) \]

4. For \( \theta = 1.0 \),

\[ \rho' = 1.408 \left( \frac{S_1}{|\Delta \psi|} \right) - 32 \]

For \( \theta < 1.0 \),

\[ \rho' = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \]

where \( a = (1-\theta) \times 8.52 \times 10^{-4} \)

\( b = 0.94 - 0.23\theta \)

\( c = 40.64 - 8.64\theta - \frac{S_1}{|\Delta \psi|} \)

5. \[ \rho_1 = 0.70\rho' \]

\[ \rho_2 = 0.85\rho' \]

\[ \rho_3 = \rho' \]

\[ \rho_4 = 1.15\rho' \]

\[ \rho_5 = 1.30\rho' \]

6. For each \( \rho_j \), calculate \( \alpha_{ij} \) where

\( i = 1, 2, 3, 4, 5 \)

\( j = 1, 2, 3, 4, 5 \)

For \( 0 \leq \rho_j \leq 250 \),

\[ \alpha_{ij} = a_{10} + a_{11}\rho_j + a_{12}\rho_j^2 + a_{13}\rho_j^3 \]
For $250 < \alpha_j$,

$$\alpha_{ij} = K_1$$

where

$$z_{ij} = \alpha_{i0} + \alpha_{i1} \rho_j + \alpha_{i2} \rho_j^2 + \alpha_{i3} \rho_j^3$$

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7. \[ D_j = \left\{ \frac{\alpha_{4j} k^2 |\Delta \psi|}{a_{2j}(a+b)} \left( 1 - \frac{\alpha_{3j}^2}{\alpha_{4j}} \right) \right\} + \frac{\alpha_{5j} S_j}{2a_{1j}} \]

8. \[ B_j = \frac{12S_j |\Delta \psi|}{D_j} \]

9. \[ C_j = \frac{\alpha_{1j} \alpha_{4j} \mu g(\Delta \psi)^2}{2D_j} \]

10. \[ \psi_{sj} = \left\{ \frac{B_j}{2} + \frac{1}{2} \sqrt{B_j^2 + 4C_j} \right\} \text{ sgn} (\Delta \psi) \text{ rad/sec} \]

11. \[ S_{sj} = 12S_j + 2\alpha_{4j} \left\{ \frac{\alpha_{1j} \mu g(\Delta \psi)}{2\psi_{sj}} - \left| \psi_{sj} \right| k^2 \left( 1 - \frac{\alpha_{3j}^2}{\alpha_{4j}} \right) \right\} \text{ inches/sec} \]
12. \[ \beta_j = \frac{\rho_j |\psi_{sj}|}{S_{sj}} \] 

Let \( n \) be the value of \( j \) for which \( |\beta_j| \) is smallest.

13. \[ \psi_s = 57.3 \psi_{sn} \text{ degrees/sec} \]

14. \[ S_s = S_{sn} \text{ inches/sec} \]

15. \[ U_s = S_s \cos (\gamma_s - \psi_s) \text{ inches/sec} \]

16. \[ V_s = S_s \sin (\gamma_s - \psi_s) \text{ inches/sec} \]

17. Return with starting values:

\[ U_s \text{ inches/sec} \]
\[ V_s \text{ degrees/sec} \]
\[ \psi_s \text{ degrees/sec} \]

5.2 **Damage Analysis**

Hand calculation techniques for damage analysis that yield reasonable estimates of the impact velocity in frontal collisions (i.e., the relative velocity of approach) have been developed by Emori (full width contact only, Reference 6) and by Campbell (partial width contact, Reference 7), using linear approximations of the relationship between residual crush and impact velocity. The SMAC program (Reference 8) applies a similar analytical approach to the entire peripheral structure, and it has been demonstrated to yield good approximations of both impact velocity and speed change, \( \Delta V \), in general collision configurations including oblique, non-central impacts. The objective of the present research has been to develop a simple, closed-form
damage analysis technique that is applicable to general collision configurations.

Central Collisions

In the case of central collisions (i.e., where the line-of-action of the collision force passes through the centers of masses of the two vehicles, Figure 9) the extents and areas of residual crush on the two vehicles provide a basis for estimating the relative velocity at impact of the vehicles. The following simplified linear analysis provides relationships for such estimates.

In Figure 9, the symbols are defined as follows.

\[ M_1, M_2 = \text{Masses of Vehicles 1 and 2, lb sec}^2/\text{in.} \]
\[ K_1, K_2 = \text{Linear approximations of peripheral crush stiffness of contact areas of Vehicles 1 and 2, for increasing load, lb/in.} \]
\[ X_1, X_2 = \text{Displacements of centers of masses, inches.} \]
\[ X = \text{Displacements of peripheral interface, inches.} \]

In the following derivation, the time derivatives of a variable is indicated by a dot over the symbol for the variable, and the subscript zero is used to indicate the initial value of a variable at zero time.

Application of Newton's Second Law to the system depicted in Figure 9 yields

\[ M_1 \ddot{X}_1 = - \left( \frac{K_1 K_2}{K_1 + K_2} \right) (X_1 - X_2) \]  \hspace{1cm} (1)

\[ M_2 \ddot{X}_2 = \left( \frac{K_1 K_2}{K_1 + K_2} \right) (X_1 - X_2) \]  \hspace{1cm} (2)
Figure 9  SCHEMATIC REPRESENTATION OF CENTRAL COLLISION
To facilitate solution of equations (1) and (2), let \( \delta = x_1 - x_2 \), \( \dot{\delta}_0 = x_{10} - x_{20} \). Then equations (1) and (2) can be restated as

\[
\ddot{\delta} + \left( \frac{K_1K_2}{K_1+K_2} \right) \left( \frac{M_1+M_2}{M_1M_2} \right) \delta = 0
\]

(3)

Solving (3) for the maximum relative displacement,

\[
(\delta)_{\text{max}} = (x_{10} - x_{20}) \sqrt{\frac{(K_1+K_2)M_1M_2}{K_1K_2 (M_1+M_2)}} \text{ inches}
\]

(4)

Let \( \delta_1 = x_1 - x \), \( \delta_2 = x - x_2 \). For force equilibrium,

\[
K_1\delta_1 = K_2\delta_2 \text{ lbs}
\]

(5)

and, by definition

\[
\delta_1 + \delta_2 = \delta \text{ inches}
\]

(6)

Solution of (5) and (6) for \( \delta_1 \) yields

\[
\delta_1 = \left( \frac{K_2}{K_1+K_2} \right) \delta \text{ inches}
\]

(7)

Equation (4) can be restated in the following form,

\[
\dot{x}_{10} - \dot{x}_{20} = \sqrt{\frac{(M_1+M_2)K_1K_2(\delta)^2_{\text{max}}}{M_1M_2 (K_1+K_2)}} \text{ in/sec}
\]

(8)
From (7), (6) and (5),

\[ \dot{x}_{10} - \dot{x}_{20} = \sqrt{\frac{(M_1 + M_2)(K_1 \delta_1^2 + K_2 \delta_2^2)}{M_1 M_2}} \text{ in/sec} \]  
(9)

The energy absorbed in peripheral crush of Vehicles 1 and 2 can be expressed as

\[ E_1 = \frac{1}{2} K_1 \delta_1^2 \text{ lb in} \]  
(10)

\[ E_2 = \frac{1}{2} K_2 \delta_2^2 \text{ lb in} \]  
(11)

Substitution of (10) and (11) into (9) yields

\[ \dot{x}_{10} - \dot{x}_{20} = \sqrt{\frac{(M_1 + M_2)^2 (E_1 + E_2)}{M_1 M_2}} \text{ in/sec} \]  

From Conservation of Momentum, the common velocity, \( V_c \), may be obtained,

\[ V_c = \frac{M_1 \dot{x}_{10} + M_2 \dot{x}_{20}}{M_1 + M_2} \text{ in/sec} \]  
(13)

The velocity changes experienced by Vehicles 1 and 2 during the approach period of the collision are

\[ \Delta V_1 = \dot{x}_{10} - V_c = \left( \frac{M_2}{M_1 + M_2} \right) (\dot{x}_{10} - \dot{x}_{20}) \text{ in/sec} \]  
(14)

\[ \Delta V_2 = V_c - \dot{x}_{20} = \left( \frac{M_1}{M_1 + M_2} \right) (\dot{x}_{10} - \dot{x}_{20}) \text{ in/sec} \]  
(15)
From (14), (15) and (12), these velocity changes (approach period) can be expressed as

\[ \Delta V_1 = \sqrt{\frac{2(E_1 + E_2)}{M_1 \left(1 + \frac{M_1}{M_2}\right)}} \text{ in/sec} \]  (16)

\[ \Delta V_2 = \sqrt{\frac{2(E_1 + E_2)}{M_2 \left(1 + \frac{M_2}{M_1}\right)}} \text{ in/sec} \]  (17)

**Non-Central Collisions**

In the more general case of non-central collisions, a common velocity is achieved at the regions of collision contact rather than at the centers of gravity. For example, in the offset frontal collision depicted in Figure 10, a common velocity is reached at point P.

In Figure 10, the collision force acting on Vehicle 1,

\[ F_x = -M_1 \ddot{x}_1 = -M_1 (\ddot{x}_p - h \dot{\psi}_1) \]  (18)

The corresponding moment acting on Vehicle 1,

\[ F_x h_1 = -I_1 \ddot{\psi}_1 = -M_1 k_1^2 \ddot{\psi}_1 \]  (19)

where \( k^2 = \text{radius of gyration squared of Vehicle 1 in yaw, in}^2 \).

From (19), the angular acceleration of Vehicle 1,

\[ \ddot{\psi}_1 = \frac{F_x h_1}{M_1 k_1^2} \]  (20)
Figure 10 OFFSET FRONTAL COLLISION
Substitution of (20) in (18) yields

\[ \ddot{x}_p = -\frac{F}{M_1} \left( \frac{k_1^2 + h_1^2}{k_1^2} \right) \quad (21) \]

\[ \ddot{x}_1 = -\frac{F}{M_1} = \left( \frac{k_1^2}{k_1^2 + h_1^2} \right) \ddot{x}_p \quad (22) \]

Let \( \gamma_1 = \frac{k_1^2}{k_1^2 + h_1^2} \), then from (22),

\[ \ddot{x}_1 = \gamma_1 \ddot{x}_p \quad (23) \]

Integration of equation (23) over the time interval during which a common velocity is reached at point P yields

\[ \dot{x}_1 = \gamma_1 \dot{x}_p, \text{ or} \quad (24) \]

\[ \Delta V_1 = \gamma_1 \Delta V_1 \quad (25) \]

where \( \Delta V_1 \) is the velocity change during the approach period of the collision at point P.

From (21), the effective mass of Vehicle 1 acting at point P may be expressed as \( \gamma_1 M_1 \). Similarly, the effective mass of Vehicle 2 acting at point P may be expressed as \( \gamma_2 M_2 \). Substitution of the effective masses into equations (16) and (17) yields expressions for the velocity change (approach period) at point P.
\[ \Delta V'_{1} = \sqrt{\frac{2(E_{1} + E_{2})}{\gamma_{1}M_{1}(1 + \gamma_{1}M_{1}/\gamma_{2}M_{2})}} \text{ in/sec} \]  

(26)

\[ \Delta V'_{2} = \sqrt{\frac{2(E_{1} + E_{2})}{\gamma_{2}M_{2}(1 + \gamma_{2}M_{2}/\gamma_{1}M_{1})}} \text{ in/sec} \]  

(27)

From equation (25) and the corresponding expression for Vehicle 2, the velocity changes (approach period) at the center of gravity of the two vehicles are obtained.

\[ \Delta V_{1} = \sqrt{\frac{2\gamma_{1}(E_{1} + E_{2})}{M_{1}(1 + \gamma_{1}M_{1}/\gamma_{2}M_{2})}} \]  

(28)

\[ \Delta V_{2} = \sqrt{\frac{2\gamma_{2}(E_{1} + E_{2})}{M_{2}(1 + \gamma_{2}M_{2}/\gamma_{1}M_{1})}} \]  

(29)

It should be noted that when \( \gamma_{1} = \gamma_{2} = 1.00 \), equations (28) and (29) reduce to the central-impact relationships of equations (16) and (17).

In Figure 11 and Figure 5, further relationships required to approximate the effects of intervehicle friction are depicted. The dimensions \( h_{1} \) and \( h_{2} \) are approximated on the basis of (1) the midpoint of the collision contact region and (2) the existence of a tangential velocity (columns 1 and 2 of the VDI).
Figure 11  INTERSECTION COLLISION

\[ m_1 v_{1o} \quad m_1 v_{1o} + m_2 v_{2o} = m_1 v_{1f} + m_2 v_{2f} \]
In the cited study by Emori (Reference 6) and in the SMAC program (Reference 2) a simple friction coefficient has been shown to yield reasonable approximations of collision responses. Inherent in the present analytical treatment is the assumption that the residual crush provides a direct measure of the energy absorbed by compressive forces between the two vehicles and that the additional work done by tangential shear forces does not provide directly measurable damage evidence. It should be noted that the front end of the impacting vehicle in an intersection collision is generally distorted laterally, but that corresponding measurement techniques have not been established.

Absorbed Energy

The calculation of absorbed energy is based on residual crush and is patterned after that developed by Campbell (Reference 7). The only significant difference is in the treatment herein of the energy absorbed without residual crush as being proportional to the contact width rather than a constant. The following relationship is applied.

\[ E_i = \int_0^w \left( A_i C + B_i \frac{C^2}{2} + G_i \right) \, dw \text{ in lbs} \]  

(30)

where \( E_i \) = Energy absorbed by vehicle i, inch lbs.

\( C = f(w) \) = Residual crush of vehicle i, inches.

\( w \) = Width dimension of damaged region, inches.

\( A_i, B_i, G_i \) = Empirical coefficients of unit width properties obtained from crash test data.
Values for $A_i$, $B_i$, and $G_i$, corresponding to the energy absorbed in barrier crashes with "standard" test weights, are stored in a table that is categorized for four vehicle sizes and for the front, side and rear of each vehicle size (Table 1). It should be noted that the frontal values in Table 1 are based on Campbell's data but that the side and rear data are gross approximations only, that are based on fragmentary crash test data. Actual vehicle weights are used in the solution of equations (28) and (29).

The developed calculation procedure permits a two, four or six point definition of the damage profile. By default, four-point definitions are generated on the basis of column 7 of the VDI and on three "representative" types of damage profiles. The integration of equation (30) is based on trapezoidal approximations of the damage region, yielding the following equation.

\[
E_i = \frac{L_i}{3} \left[ \frac{A_i}{2} (C_{i1}^2 + 2C_{i2}^2 + 2C_{i3}^2 + C_{i4}^2) + \frac{B_i}{6} (C_{i1}^2 + 2C_{i2}^2 + 2C_{i3}^2 + 2C_{i4}^2 + C_{i1}C_{i2} + C_{i2}C_{i3} + C_{i3}C_{i4}) + 3 G_i \right] \text{ inch lbs (31)}
\]

The "equivalent barrier speed" as defined by Campbell (Reference 7) is not equal to the speed change, $\Delta V$, in low speed collisions, since the rebound velocity produced by the coefficient of restitution is not included. At the velocity intercept of the linearized fit of impact velocity plotted against residual crush, elastic behavior is indicated for the vehicle crush. The total speed change, $\Delta V$, should, therefore, be equal to twice the impact velocity in that velocity range, in the absence of an energy absorbing bumper device.
The impact speeds without residual crush that are indicated by Campbell's linear fits (no actual data points at impact speeds below 15 MPH) suggest substantially higher coefficients of restitution than, for example, the values presented by Emori in Reference 6. Without more definitive information on the actual magnitude and variation of the coefficient of restitution as a function of both deflection extent and position on the vehicle periphery, the complexity of introducing a corresponding refinement in the damage analysis technique cannot be justified. Therefore, the damage analysis procedure defined herein tends to underestimate $\Delta V$ in low speed collisions.