PROGRESS IN THE THEORETICAL INVESTIGATION OF VEHICLE COLLISIONS

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Progress in the Theoretical Investigation of Vehicle Collisions

(Fortschritte in der Berechnung von Fahrzeugzusammenstössen)

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Particular attention has been devoted in recent years to the investigation of the course of events in vehicle collisions, the two chief considerations being passenger safety and the establishment of scientific bases for legal proceedings. Developments in technique - especially for the latter purpose - are reviewed in this article.

1. Model Concepts

The collision between vehicles does not correspond to th simple, scholastic concepts of the laws of impact, which are mostly linked with the model of ivory billiard balls. The collision between such balls is (a) practically completely elastic; it occurs (b) without permanent deformations and according to a defined law of elasticity; in the impact (c) the duration of compression is extraordinarily short and (d) practically no side forces are present before and after the impact. Finally (e) a picture of the collision of two spheres cannot completely illustrate the progress of a collision in which both bodies, which may be reinforced in several ways, suffer an eccentric impact.

The collision between vehicles differs in all these respects from the model concept just described. In vehicle collisions the impact is (a) only very imcompletely elastic and at higher speeds almost completely inelastic; the coefficient of restitution k varies with the impact velocity and the design of the vehicle.(b). In almost every case permanent deformation and destruction of parts occurs; the relationship between force and collapse is unsteady, since in the course of the collision different parts of the vehicle which are of unequal elasticity, rigidity and deformation strength are successively deformed or destroyed. The duration of impact (c) is several times greater in a non-meshing ("grazing") collisi than in the limiting cases of the straight centred impact or of the head-on collision with meshing together of the two vehicles, so that the movements of the colliding structures which occur while the impact forces are effective can no long be simply neglected.

The applicability of the theorem of (linear) momentum is limited since (d) before and especially after the collision considerable forces can occur between tyre and roadway, these forces being influenced by the adhesiveness of the tyre on the road surface, the tyre tread, steer angle, braking or locking of the wheels, and defects in tyres or in the wheel suspensior The momentum theorem is, however, only valid on the assumptior of freely moving bodies uninfluenced by external forces. Finally (e) the general case of an oblique eccentric impact for both vehicles very frequently occurs, and this cannot be reproduced by the model employing spheres. It therefore necessitates expansion of the equation with the aid of the theorem of angular momentum.

2. Problems and Methods

For the evaluation of safety provisions (e.g. safety-belts) the chief point of interest is the time-pattern of dece erations of the vehicle and of its occupant, for the determin-ation of which the complete pattern of movement must be known in the greatest possible detail. In road construction the attempt has sometimes been made to establish a numerical value for the degree of danger existing at intersections between roads of equal importance and at acute-angled junctions. This is linked with the angle of impact and the deformations occurring. These considerations also can be verified with the aid of the findings meanwhile obtained, as is brought out in greater detail in Section 4.

In the case of the oblique eccentric impact dealt with in Section 3.2 the rotational movements induced obey the theorem of angular momentum. In the special case of the "meshed" or full impact*, the common speed of the two vehicles occurring at the point of impact at the end of the compression process can be introduced as an additional condition; this permits the determination of the direction of the impact force in Section 3.2.2.

In the general case, when the impact forces cease to operate, each of the two vehicles of mass m has a velocity of centre of gravity v_s' and an angular velocity ω' about the vertical axis passing through the centre of gravity of the vehicle with the moment of inertia J, and a corresponding residual energy of the magnitude

$$E' = \frac{m v_s'^2}{2} + \frac{J \omega'^2}{2}$$

* terms marked thus are literal renderings

The corresponding values at the beginning of impact can be derived from the velocities v_s' and ω' immediately after impact. These values are of the highest importance for the expert. The information available to him for their determination in photographs of the accident, however, generally consists only of the position of impact, the final position of rest of the vehicles, and possible tyre tracks. The expert therefore has the task first of all of determining the velocities v_s' and the angular velocities ω' of the vehicles involv from the path followed by the centre of gravity between these two positions, if it can be regarded as approximately rectilinear, and from the rotation of the longitudinal axes which has occurred on this path. The laws of impact are no longer applicable to those events occurring after impact. Rather mus he proceed, as shown in Section 5, from an energy balance or directly from the Newtonian law.

As regards the methods of calculation and their application especially in legal cases, economic considerations must be taken into account; the time expended and the costs of obtaining expert opinion must bear a reasonable relationship to the value of the claim; techniques must therefore be devel ed which are reliable and yet as rapid as possible. Bearing these points in mind, we shall collate in the following secti those individual findings of various authors which have contr uted or can contribute to the better understanding of these problems. Owing to limitations of space the treatment of each is necessarily brief; for the complete argument, reference must sometimes be made to the original works quoted as references.

3. Use of the momentum theorem and of the theorem of angular momentum

3.1. "True-centred" impact (centric impact)*

The use of the momentum theorem for the investigation of vehicle accidents goes back to the valuable work carried out by Brüderlin [1]. The equations presented by him admittedly also contain the theorem of angular momentum, but without consistent evaluation of the possibilities contained therein. Rather does his introduction of the concept of "virtual end-velocity", which is vague in definition and not entirely correct in derivation, bring the treatment of the eccentric impact back to the "true-centred" impact. Losagk's "vector diagram of forces" proceeds from the outset from Brüderlin's work. Both methods are accordingly to be considered under the heading "True-centred impact."

If one imagines a system of two point masses which are moving towards each other freely in a straight line and at a constant speed at any given moment, then the imaginary centre of gravity of the system lies at any given moment on the line connecting the two simultaneous loci of the point masses; it divides the line in inverse ratio to the individual masses. If only internal forces are effective during the collision, the momentum of the system is preserved according to the momentum theorem. We therefore have the equation

$$m_1 v_1 \hat{+} m_2 v_2 = (m_1 + m_2) v_g = m_1' v_1' \hat{+} m_2' v_2'$$

where v_g is the velocity of the imaginary centre of gravity of the system. The vector equation is so far valid, irrespecti of whether the impact follows an elastic or inelastic pattern. Its range of validity is however restricted to a period during which it can be safely assumed that the external adhesive, sid and brake forces are negligibly small as opposed to the internal impact forces. In the oblique centric collision the direc ion and magnitude of the respective velocities directly after impact depend essentially on the coefficient of elasticity, which is characterised by means of the coefficient of restitution k: for a completely elastic impact $k = 1$; for a complete inelastic impact $k = 0$.

This may be clearly demonstrated by means of the simple example of the right-angled impact of two point masses of equal magnitude and equal impact velocity. Under these assumptions, the directions of velocity are obviously interchanged in the case of a completely elastic impact, while the magnitud of the velocities remains unchanged. In the completely inelastic impact, on the other hand, the two bodies proceed in the direction v_g with the velocity $v' = v/\sqrt{2}$ as shown in Fig. 1a. While no loss of energy occurs with the completely elastic impact, the total residual energy in the case of the completely inelastic impact becomes

$$E' \equiv \frac{(m_1 + m_2)\, v_g'^2}{2} = \frac{m_1 v_1'^2 + m_2 v_2'^2}{2} = \frac{mv^2}{2}$$

This is half of the original total energy $E_o = mv^2$. The other half has been converted to deformation energy.

It is immediately obvious that the post-impact velocities in partly elastic collisions can be represented according to Fig. 1c. Eberan-Eberhorst [4] deals with this subject in detail and presents the equations

$$v_1' \cos\alpha' = \frac{(m_1 - km_2)\, v_1 \cos\alpha + (1 + k)\, m_2 v_2 \cos\beta}{m_1 + m_2}$$
$$v_2' \cos\beta' = \frac{(m_2 - km_1)\, v_2 \cos\beta + (1 + k)\, m_1 v_1 \cos\alpha}{m_1 + m_2}$$
$$v_1' \sin\alpha = v_1 \sin\alpha; \quad v_2' \sin\beta = v_2 \sin\beta.$$

The meanings of the terms used in these equations are as follows:

α = angle between v_1 and impact normal = $180^\circ - \beta$

α' = angle between v_1' and impact normal

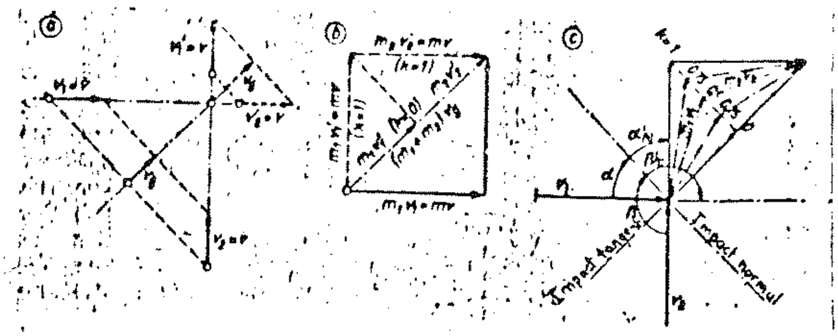


Fig. 1 Right-angled impact of point-masses, $m_1 = m_2$, $v_1 = v_2$, $0 = k = 1$ (a) General plan (b) Vector diagram of forces (c) vector diagram of velocities.

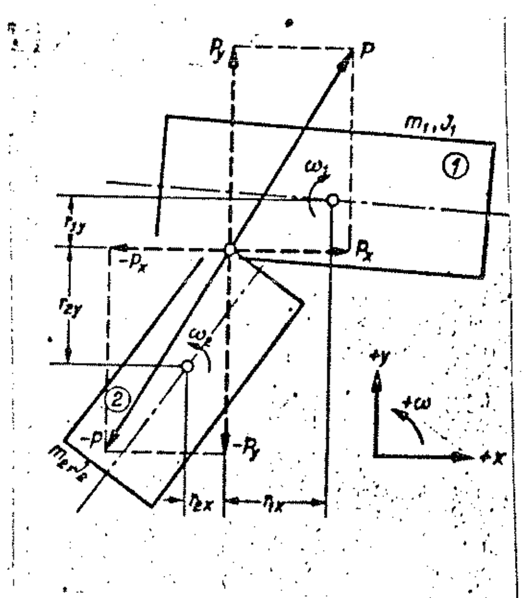


Fig. 2 Oblique eccentric collision between two vehicles

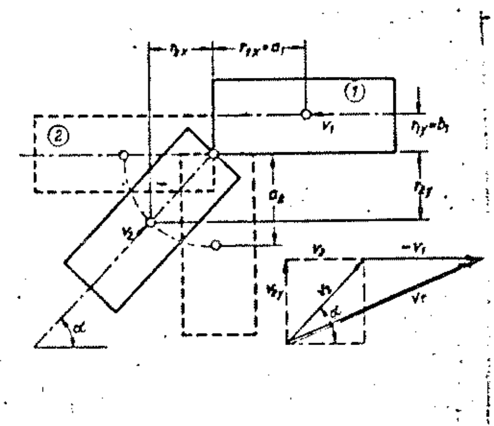


Fig. 3 Oblique "meshed" collision with different impac angles.

β = angle between v_2 and impact normal

β' = angle between v_2' and impact normal

In this work, Eberan-Eberhorst has determined by evaluat of American crash tests the coefficients of restitution k occurring in straight centred collisions between a heavy movi passenger car and a similar moving or stationary car ($G \approx 1,60$ kgf.) as a function of the relative velocity v_r. Under the conditions indicated, the coefficient of restitution k is $\rightarrow 1$ in non-violent impacts; it decreases even at v_r = 40.km./hr. to values of about $k = 0.2$, at higher speeds the impact becom almost completely inelastic ($k \rightarrow 0$). Accordingly, the consideration of partial elasticity in the case of more violent impacts represents a secondary refinement of the investigatio which can usually be omitted in favour of the more convenient assumption of an inelastic impact in view of the general uncertainty of specialist calculation.

If one can regard the post-impact path of two vehicles as being approximately rectilinear and uninfluenced by extern forces, and no essential rotational movement has occurred, th direction of velocities immediately after the impact is given approximately. In this case, the vector diagram of forces can be completed by the other data which may be available. On this occasion the coefficient of elasticity automatically enters into the equation, since it determines the direction of post-impact velocities.

Generally, however, the conditions are not so favourable Collisions occur in which both vehicles suffer eccentric impact and considerable spin. Side forces are brought into play by wheel-lock, unequal braking, and damage to individual tyres or wheel suspensions which have a marked effect on the post-impact path and its direction and thus make this method unreliable. The mode of investigation must therefore be improved and completed in two respects - firstly, with regard to the rotational movements in an eccentric impact (angular momenta) and, secondly, in respect of movements occurring during run-out until the vehicles' final position of rest is reached. The first of these two requirements leads to the us of the theorem of angular momentum in Section 3.2.; the secon must be solved in a completely different manner in Section 5.

3.2. Eccentric impact

3.2.1. General Case, collision without meshing together of th vehicles ("grazing collision").*

Fig. 2 shows a diagrammatic representation of an instantaneous situation during an oblique eccentric collision. The instantaneous force P working at the point of impact is meanwhile of unknown magnitude and direction. If it is resolved into the components $\pm P_x$ and $\pm P_y$, and the velocities of the centres of gravity of the two vehicles are similarly resolved into the components v_x and v_y , the momentum theorem accord ing to Newtonian law for a short time interval dt is

$$P_x\,dt \equiv m_1\,dv_{x1} = m_2\,dv_{x2}$$
$$P_y\,dt \equiv m_1\,dv_{y1} = m_2\,dv_{y2}.$$

and the theorem of angular momentum is

$$M_1\,dt \equiv (P_y\,r_{1x} - P_x\,r_{1y})\,dt = J_1\,d\omega_1$$
$$M_2\,dt \equiv (P_y\,r_{2x} - P_x\,r_{2y})\,dt = J_2\,d\omega_2$$

If during a "non-meshing" collision the point of impact of one or both vehicles moves, or if the effects of the impac forces persist for so long that the positions of the vehicles alter during the process, summary application of the momentum theorem and of the theroem of angular momentum to the whole impact event is no longer possible. It can nevertheless be shown [5] that under certain suppositions the typical patter of such an impact can be simulated step by step if one regard the components of force P_x and P_y as constant over very sho intervals of time Δt . The valid equations for each step are then

$$v_{1x} = (v_{1x})_0 - \frac{P_x}{m_1}\Delta t \quad \text{etc. and} \quad \omega_1 = (\omega_1)_0 \pm \frac{M_1}{J_1}\Delta t \quad \text{etc.}$$

The components of velocity at the point of impact are then of the magnitude

$$(v_x)_1 = v_{1x} \pm \omega_1 r_{1y}; \qquad (v_x)_2 = v_{2x} \pm \omega_2 r_{2y}$$
$$(v_y)_1 = v_{1y} \pm \omega_1 r_{1x}; \qquad (v_y)_2 = v_{2y} \pm \omega_2 r_{2x}.$$

The relative movements and the changes in compression respectively at the incident point of impact then work out at:

$$x = \left[(v_x)_1 - (v_x)_2 \right] \Delta t \quad \text{and} \quad y = \left[(v_y)_1 - (v_y)_2 \right] \Delta t$$

In order to carry out such calculations, however, further suppositions are necessary. A force-compression law $P_x = f_1 (x)$ and $P_y = f_2 (y)$ must be assumed and an assumptic must be made regarding the relative softness of the two vehicl at the point of impact, e.g., the body-compression distances of the two vehicles are taken to be half (equally soft) or in some other specific ratio. The variation of the lever arms r_x and r_y must also be decided on. The coefficient of elasticity can also be taken into account by means of the force-compression function.

A physically well-founded analysis of such cases which were not previously amenable to calculation thus appears basically feasible; the working specialist could not be expect to undertake the amount of work involved. Such investigations must rather remain the province of the research worker. The assumptions postulated still lack that practical confirmation which could be obtained by analysis of a very large number of collisions the course of which was reliably known.

3.2.2. "Meshed" or full impact*

The calculation of oblique eccentric impacts is immeasurably simpler, if one can assume that the duration of impact is very short, the point of impact is invariable, and the impact itself is inelastic. The impulse theorem and the theorem of angular momentum can then be applied to the whole of the compression event. Moreover, at the moment of maximum compression a speed u, which is common to both vehicles, develops at the point of impact, and this value u can be introduced to the calculation as a further condition. It can then be expressed in the form employed in the previous section, as

$$u_x \equiv v_{1x}' \pm \omega_1' r_{1y} = v_{2x}' \pm \omega_2' r_{2y}$$
$$u_y \equiv v_{1y}' \pm \omega_1' r_{1x} = v_{2y}' \pm \omega_2' r_{2x}$$

When the direction of the impact impulse $\int P\,dt$ is known approximately, a predominantly graphic procedure can be followed in many cases, and the vector diagram of velocities can be constructed in addition to the vector diagram of forces; if, for example, the masses and velocities of the colliding vehicles are very similar, there is very little difference between the direction of the impact impulse and the direction of relative velocity during impact, especially in the case of a right-angled collision. If one of the vehicles does not undergo any spin after impact, the direction of the impact impulse would pass through its centre of gravity. When the values involved in the calculation are only partly known, this graphic method permits a variation of these values with only a small expenditure of time and thus enables a general impression to be obtained.

Under the conditions indicated (meshed impact, short duration of impact, invariable direction of the impact impulse the direction of the impact impulse can be calculated exactly. The solution is indicated vectorially by Slibar [7] in the following form

momentum theorem
- for vehicle 1 $m_1 \mathfrak{v}_1' - m_1 \mathfrak{v}_1 = \mathfrak{S}$
- for vehicle 2 $m_2 \mathfrak{v}_2' - m_2 \mathfrak{v}_2 = -\mathfrak{S}$

theorem of angular momentum
- for vehicle 1 $m_1 i_1^2 \mathfrak{w}_1' = \mathfrak{r}_1 \times \mathfrak{S}$
- for vehicle 2 $m_2 i_2^2 \mathfrak{w}_2' = \mathfrak{r}_2 \times (-$

Generally, the velocity at the point of impact when the maximum compression is occurring has the following values:

for vehicle 1: $\mathfrak{u}_1' = \mathfrak{v}_1' + \mathfrak{w}_1' \times \mathfrak{r}_1$

for vehicle 2: $\mathfrak{u}_2' = \mathfrak{v}_2' + \mathfrak{w}_2' \times \mathfrak{r}_2$

In the case of the exact full impact these two velocitie: are equal to one another: in the completely inelastic impact ($k = 0$) they correspond to the common velocity at the point o: impact directly after the cessation of the force-effects, and we thus have

$$\mathfrak{u}_1' \equiv \mathfrak{u}_2' = \mathfrak{v}_1' + \mathfrak{w}_1' \times \mathfrak{r}_1 = \mathfrak{v}_2' + \mathfrak{w}_2' \times \mathfrak{r}_2$$

If one writes all the equations as component equations i: the rectangular coordinate system, the components of the impa: impulse result through evaluation as

$$\int P_x \, dt \equiv S_x = \frac{\alpha_x v_{21x} + \varkappa v_{21y}}{\alpha_x \alpha_y - \varkappa^2}$$

$$\int P_y \, dt \equiv S_y = \frac{\varkappa v_{21x} + \alpha_y v_{21y}}{\alpha_x \alpha_y - \varkappa^2}$$

using the abbreviations

$$\alpha_x = \frac{1}{m_1} + \frac{1}{m_2} + \frac{r_{1x}^2}{J_1} + \frac{r_{2x}^2}{J_2}$$

$$\alpha_y = \frac{1}{m_1} + \frac{1}{m_2} + \frac{r_{1y}^2}{J_1} + \frac{r_{2y}^2}{J_2}$$

$$\varkappa = \frac{r_{1x} r_{1y}}{J_1} + \frac{r_{2x} r_{2y}}{J_2}$$

and the relative velocity

$$v_{21x} = v_{2x} - v_{1x}; \qquad v_{21y} = v_{2y} - v_{1y}$$

The methods presented in this section are especially applicable to the investigation of approximately right-angled impacts. Before using them it is advisable to determine from an inspection of the damaged vehicles themselves or from film thereof whether in fact a meshing, full impact has occurred.

4. Danger coefficient of road junctions

In road construction the attempt has quite often been made to state a numerical value for the danger rate of right-angled and oblique-angled junctions; this depends essentially on the angle of impact of the vehicles in collisions. Probably the most recent work on this subject, by Künne [8], proceeds from the deformation energy (= loss of kinetic energy) in the case of a straight, "central" inelastic collision with

$$E_f = \frac{m_1 m_2}{m_1 + m_2} \frac{(v_1 - v_2)^2}{2}$$

and replaces the expression $v_1 - v_2$ by the vectorial relative velocity $v_1 \hat{-} v_2 = v_r$ during impact. With the aid of the derivations presented above the validity of this summary equation may be evaluated, at least within a range in which a "meshed" impact may be expected. There is a multiplicity of possibilities during the impact; only one example, Fig. 5, is therefore taken as an illustration, with equal vehicle masses $m_1 = m_2 = m$ and equal initial velocities $v_1 = v_2 = v$; the point of impact will always lie in the point 0 for all the different impact angles $0^\circ < \alpha < 90^\circ$.

One then obtains for example, according to the equations given in Section 3.2.2. with

$$v_r^2 = (v_1 + v_2 \cos\alpha)^2 + (v_2 \sin\alpha)^2 = (v_{1x} + v_{2x})^2 + (v_{1y} + v_{2y})^2;$$
$$i_1 = i_2 = i; \quad v_1 = v; \quad \lambda = a^2/i^2; \quad \beta = b/a; \quad \eta = v_2/v,$$

the velocities v' and the angular velocities ω' directly after impact from

$$S_x \equiv m_1 (v_{1x} - v_{1x}') = m_2 (v_{2x} - v_{2x}')$$
$$S_y \equiv m_1 (v_{1y} - v_{1y}') = m_2 (v_{2y} - v_{2y}')$$
$$J_1 \omega_1' = S_y r_{1x} - S_x r_{1y}; \qquad J_2 \omega_2' = S_y r_{2x} - S_x r_{2y}$$

The components of the impact impulse are

$$S_x = -\, m v \eta \, \frac{\frac{2+\lambda}{\eta} + 2\cos\alpha\,(1+\lambda) + \frac{\lambda}{\eta}\cos^2\alpha + \lambda\beta\sin\alpha}{4 - 2\lambda\,(2+\beta^2) + \lambda^2 (\sin\alpha - \beta\cos\alpha)^2}$$
$$S_y = +\, m v \eta \, \frac{\frac{\lambda}{2\eta}\sin 2\alpha + \frac{\lambda\beta}{\eta}(1+\beta\sin\alpha) + \lambda\beta\cos\alpha + (2+\lambda)\sin\alpha}{4 - 2\lambda\,(2+\beta^2) + \lambda^2 (\sin\alpha - \beta\cos\alpha)^2}$$

The total energy before the impact amounted to

$$E_1 + E_2 = \frac{m_1 v_1^2 + m_2 v_2^2}{2}$$

and the total residual energy after the impact was

$$E_1' + E_2' = \frac{m_1 v_1'^2 + m_2 v_2'^2}{2} + \frac{J_1 \omega_1'^2 + J_2 \omega_2'^2}{2}$$

The energy converted into deformation work in the case of the assumed meshed inelastic impact is accordingly

$$E_f - (E_1 + E_2) - (E_1' + E_2')$$

In the example worked out in Figs. 3, 4 and 5 it is assumed that

$a = 2.24$ m. $\eta = 1$ or 2

$b = 0.87$ m. $\beta = 0.388$

$i = 1.35$ m. $\lambda = 2.75$

There thus result, as a function of the impact angle α, the relative components of the impact impetus S_x /mv and S_y /m according to Fig. 4 and the relative residual energy $(E_1' + E_2'$ mv^2 in accordance with Fig. 5. In Fig. 5b the energy values are plotted in the direction of the applicable impact angle, although they are of course non-directional scalar values. The right-angled impact in the middle of the struck vehicle is also shown in Fig. 5 for purposes of comparison.

With the assumed width of the impact area (half the width of the vehicle) and within the range of impact angles considered, a full impact can be expected to occur. Under this assumption the deformation energy and hence the danger rate is considerably less than in Künne's equation when the angle of impact

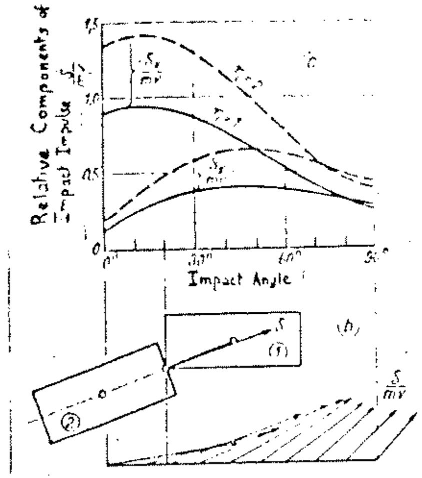


Fig. 4 Components of the impact impulse in Fig. 3

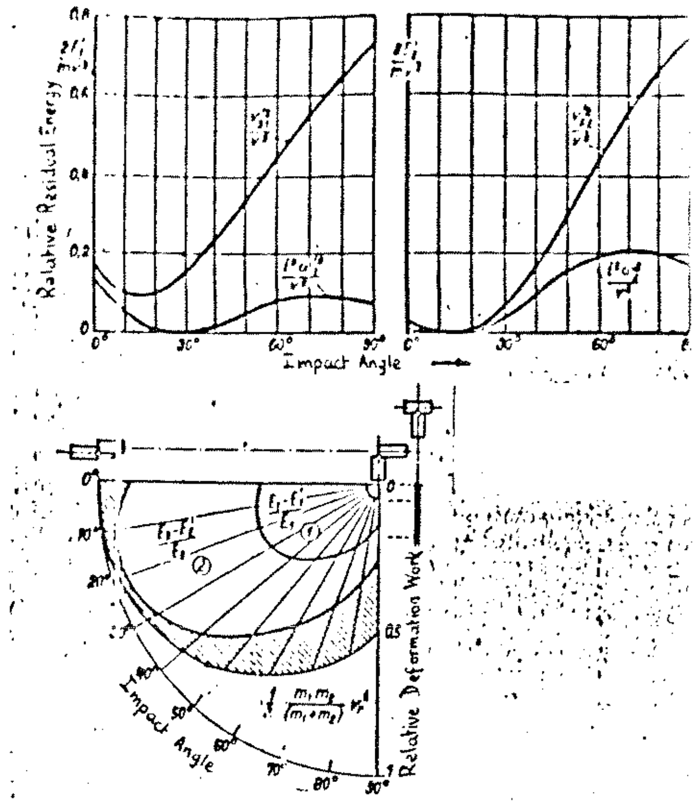


Fig. 5a and b Residual energy and deformation work in Fig. 3

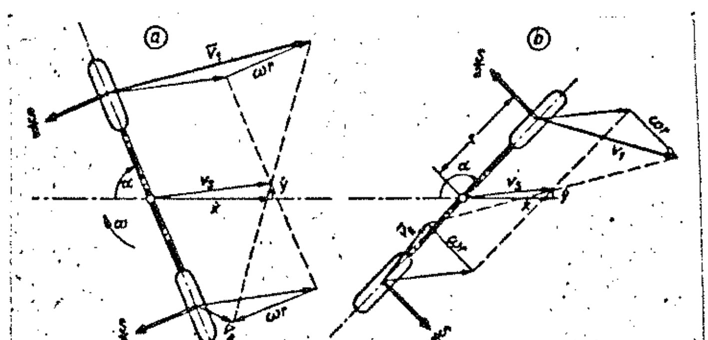


Fig. 6 Post-impact run-out, Model case 1, diagram

approaches 90°. This only applies to the [illegible] dir[illegible] by the collision. The large amount of residual energy persisting in approximately 90° collisions can lead to considerable additional damage if the vehicles proceed in an uncontrolled direction after the impact and meet further obstacles.

5. Post-impact path

Directly after the eccentric impact each of the vehicles involved has a velocity of centre of gravity v_s and an angular velocity ω around the centre of gravity, and a residual energy of $mv_s^2/2 + J\omega^2/2$. This latter is changed into resistance energy on the post-impact path leading to the final position of rest. The expert on accidents is interested in the velocities before impact; in suitable cases these can be calculated with the aid of the procedures indicated from the velocity and angular velocity directly after impact, for which, however, apart from tyre tracks, only the actual position of the collision and the resting position of the vehicles are generally known. It therefore becomes necessary to determine the vehicle speed and angular velocity at the beginning of run-out from the post-impact path s of the centre of gravity, which is imagined as following a straight line, and the rotation of the longitudinal axis of the vehicle $\Delta\alpha$.

Acceptable solutions have not yet been obtained; such solutions can however be approached through investigation of "model cases" whereby one proceeds initially from known values for v_s and ω, and calculates the motion of the vehicles until the final state of rest under certain assumptions, principally in respect of the resistance forces ("forward calculation"). After several examples have been worked out, approximations can be developed for "backward calculation". Two model cases have so far proved suitable for this work, both of which proceed from the simplifying assumption of a symmetrical vehicle with its wheels directed straight ahead and with intact tyres; the fourwheeled vehicle can in this case be replaced by a two-wheeled vehicle without any great inaccuracy.

In the first model case the rolling resistance in the direction of the track of the wheel planes is assumed to be zero; the slip resistance transverse to the wheel-plane is assumed to be constant and always directed against the corresponding component of velocity. These assumptions correspond more or less to the unbraked post-impact run-out. In the second model case, locked, braked wheels are presupposed, under the assumption that the contact-area of the tyre then opposes an equally great resistance to movement in any particular direction, the resisting force remains constant on each wheel and is always assumed to be directed against the direction of absolute velocity at the point of contact with the road surface. For the calculation of such cases it is further assumed that the road surface is level and its adhesion is constant.

5.1 First Model case; unbraked run-out

During run-out, two different phases of motion which can repeat themselves several times can be distinguished. In the first phase, Fig. 6a, the absolute velocities

$$\bar{v}_1 = v_t \hat{+} \omega r \quad \text{and} \quad \bar{v}_2 = v_t \hat{-} \omega r$$

of the road contact points are in the forward direction; the constant resistance forces S/2, which are directed transversel to the wheel plane, thus act in the same direction at both contact points. No resisting torque occurs. The equations fo the components of deceleration are:

$$\ddot{x} = -\frac{S}{m}\sin\alpha; \qquad \ddot{y} = -\frac{S}{m}\cos\alpha.$$

$$\ddot{\alpha} = -\frac{M}{J} = 0; \qquad \dot{\alpha} = \omega_0; \qquad \alpha = \alpha_0 + \omega_0 t$$

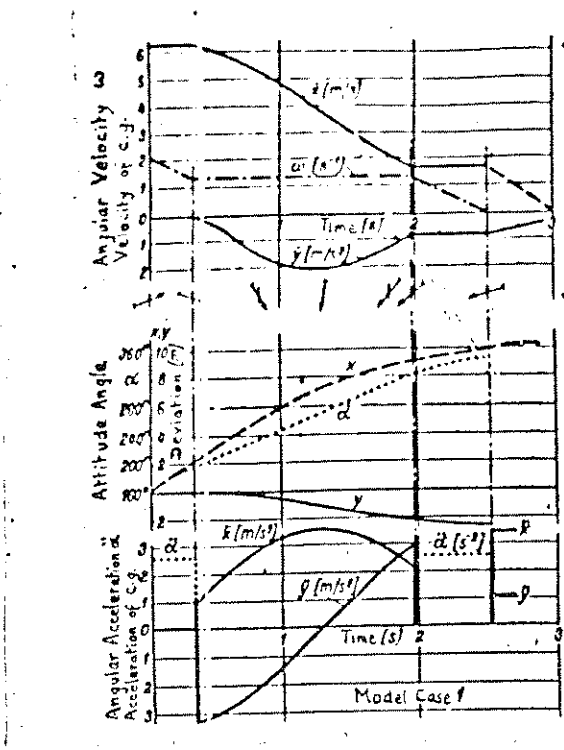
Thus in this phase only the sliding movement is braked, the angular velocity remaining unaltered.

If during rotation a position arises, however, as shown - Fig. 6b where the component oblique to the wheel is directed backwards by $\bar{v}_2$, then the resistance forces point in opposed directions, as might be assumed. Their x- and y-components cancel out, so that the sliding movement now proceeds unbraked the rotational movement, however, is braked by means of the resisting moment Sr. For this phase the valid equations are accordingly

$$\ddot{x} = \ddot{y} = 0;$$
$$\ddot{a} = -\frac{M}{J} = \text{const.}; \dot{a} = \omega_0 - \frac{Sr}{J}t; a = a_0 + \omega_0 t - \frac{Sr}{2J}t^2$$

The dynamic behaviour of the model may be calculated by means of double integration of the equations given. It shows the characteristic cornering qualities of the tyres during unbraked run-out with vanishing rolling resistance. Phases of constant rotational resistance meanwhile alternate with phases in which the sliding resistance in the x-direction follows a sinusoidal pattern. An example is given in Fig. 7. The post-impact path admittedly does not form a straight line, but the deviation in the y-direction is relatively small.

If one can neglect these y-components of the path of the centre of gravity, a rough back-calculation follows in accordance with the following considerations: in one phase $\ddot{\alpha}$ = const in the other phase the value $\ddot{x}$ = const. · sin α can be replaced by a mean value $\ddot{x}_m = \varkappa\ S/m$, whose coefficient $\varkappa$ is the mean value of an incomplete positive sinus curve and must therefore be between 0.7 and 1.0. Through combination of the segments of motion of similar phase and linearisation over their



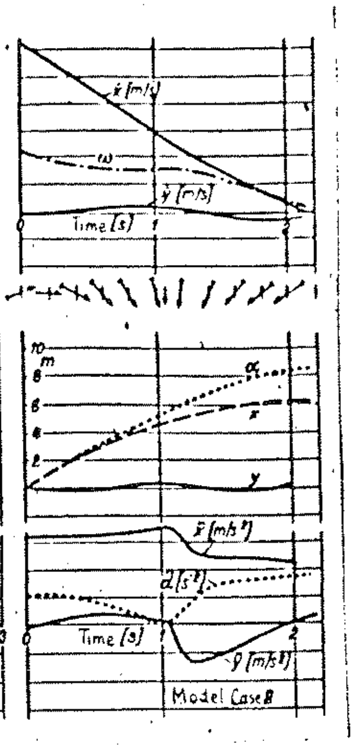


Fig. 7 Example for Fig. 6   Fig. 9 Example for Fig.8

1/3

4.8
1.8

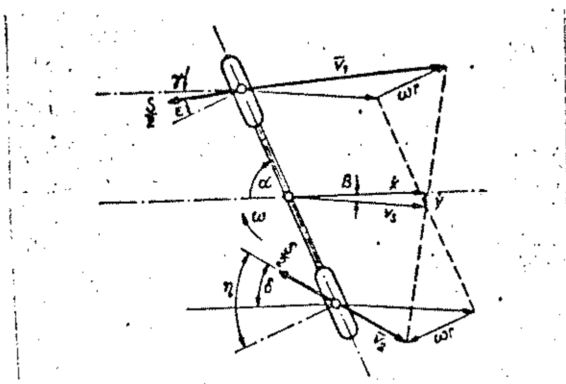


Fig. 8 Post-impact run-out, Model 2, diagram

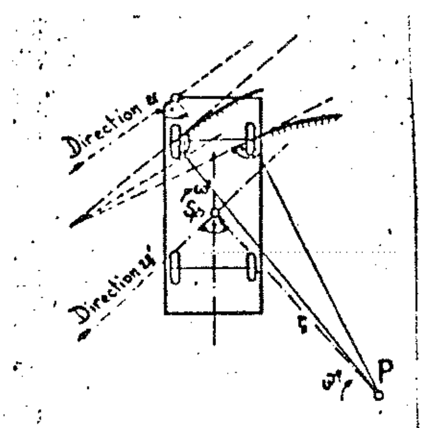
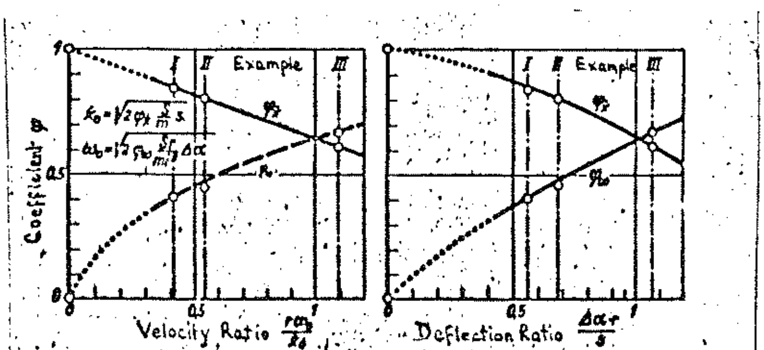


Fig. 11 Instantaneous pole of the motion directly after impact.



.7
.32

Fig. 10 Coefficients for backward calculation in Fig. 8. Wheels locked duration, we obtain as an approximation the equations

$$\omega_0^2 = \frac{A\,(\Delta\alpha)^2}{(\Delta\alpha)\,B + s} \quad \text{and} \quad \dot{x}_0 = \frac{\Delta\alpha}{\omega_0}\,A - \omega_0 B$$

where

$$A = 2\varkappa\, S/m \quad \text{and} \quad B = \varkappa\, i^2/r$$

The velocity of the centre of gravity $\dot{x}_0$ and the angular velocity ω_0 directly after impact can thus be approximately calculated from the estimated sliding deceleration S/m, from the path of the centre of gravity s which has already been found and is assumed to be rectilinear, and from the already-known rotation of the longitudinal axis of the vehicle. Sinc the mathematical approximation depends on a linearisation, appreciable errors develop especially if the beginning of the calculated process coincides with the beginning of one of the two phases indicated.

5.2. Second model case; run-out with locked wheels

In this case it is assumed that the sliding resistance is constant, i.e., independent of the speed and equal in each direction of motion, and is thus constantly directed against the instantaneous velocity $\bar{v}_1$ or $\bar{v}_2$ at the contact point with the road surface. This would happen with circular conta patches and treadless tyres.

The angles in Fig. 8 are given by

$$\varepsilon = 90^\circ - \alpha - \gamma; \qquad \eta = 90^\circ - \alpha + \delta;$$
$$\tan\gamma = \frac{\omega r \cos\alpha - \dot{y}}{\omega r \sin\alpha + \dot{x}}; \quad \tan\delta = \frac{\omega r \cos\alpha + \dot{y}}{\dot{x} - \omega r \sin\alpha}$$

The decelerations then work out at

$$\ddot{x} = -\frac{S}{2m}(\cos\gamma + \cos\delta); \; \ddot{y} = +\frac{S}{2m}(\sin\gamma - \sin\delta)$$
$$\ddot{\alpha} = -\frac{Sr}{2mi^2}\left[\sin(\alpha+\gamma) - \sin(\alpha-\delta)\right]$$

The step-by-step integration of these equations for investigation of the pattern of motion is quite practicable. It can be seen from Fig. 9 that under these assumptions the deviation of the path of the centre of gravity from the origi al direction of sliding is only slight and the decrease in velocity $\dot{x}$ with respect to time deviates only slightly from the linear. The steepness of the velocity drop depends essentially on the ratio

$$\left(\frac{r\omega_o}{\dot{x}_o} = \frac{\text{rotational velocity}}{\text{sliding velocity}}\right)_{\text{INITIAL}}$$

or on the ratio

$$\left(\frac{\Delta\alpha \cdot r}{S} = \frac{\text{rotational deflection}}{\text{sliding deflection}}\right)$$

Approximation equations can be derived from the forward calculation of various examples for the back-calculation in the form of the wellknown standstill-brake formula ($v^2 = 2bs$) namely

$$\dot{x}_o^2 = 2\varphi_{\dot{x}} \frac{S}{m} \cdot s \quad \text{and} \quad \omega_o^2 = 2\varphi_{\omega} \frac{Sr}{mi^2} (\Delta\alpha)$$

The coefficients $\varphi_{\dot{x}}$ and φ_{ω} were determined from the forward-calculated examples. They vary as a function of the deflecti[on] ratio $\Delta\alpha \cdot r/s$, rather as shown in Fig. 10.

5.3. Further model cases

"Forward calculation" is also feasible for other model cases; the following assumptions can be made in the first pla[ce]

Model case 3, rolling, braked, run-out. The two-wheeled symmetrical model can still be used in this case; in addition to the constant transverse resistance S, as in Model 1, a simila[r]ly constant rolling resistance R is introduced.

Model Case 4, unbraked run-out with a defective tyre. The calculation is based on a four-wheeled model; the resistance at three wheels is assumed to be as in Model 1, at the fourth defective tyre as in Model 2.

It has not so far proved possible, however, to derive simple rules for the backward calculation from the working o[ut] of corresponding examples. Even the approximation equations model cases 1 and 2 are reproduced with some reservations; their unrestricted use can only be advocated when the assumptions made can be confirmed by comparison with actual acciden of which the course is accurately known, or perhaps by model tests.

5.4. Wheel tracks during run-out

If tracks are produced on the roadway immediately after impact, the ratio of v_s' and ω' can readily be derived from kinematic relationships. As is shown in Fig. 11, the instantaneous pole P of motion directly after impact is given by the normals to the start of the track. One thus obtains also the direction of the velocity of the centre of gravity v_s' at this moment; it stands perpendicular to the radius vector r_S from the pole to the centre of gravity. Since the angular velocity ω_p' about the pole is equal to the angular velocity ω' about the centre of gravity, we obtain.

$$v_s' = r_s \omega_p' = r_s \omega',$$

and for an approximately straight path of the centre of gravity with $v_s \approx \dot{x}_o$ we obtain, in accordance with Fig. 10, the ratio

$$\frac{r\,\omega_o}{\dot{x}_o} \approx \frac{r\,\omega_o}{v_s'} = \frac{r}{r_s}$$

In this case it is tacitly assumed that the tyres behave "isotropically" as in Model 2.

The residual energy immediately after impact is

$$E' = \frac{m}{2}(v_s'^2 + i^2 \omega'^2) = \frac{m}{2}(r_s^2 + i^2)\,\omega'^2$$

Hence we have

$$\omega' = \sqrt{\frac{2E'}{m(r_s^2 + i^2)}} = \frac{v_s'}{r_s}$$

Calculation of the residual energy is essential in this case also for the determination of the velocity.

If a "meshed" collision or a head-on collision has indisputably taken place, the direction of the velocity **u** at the point of impact can also be determined to a good approximation for the tracks in Fig. 11 and used for the construction of the velocity diagram (see Section 3.2.2 and 6).

6. Deformation and Deformation Work

The basic expression for the energy balance of the collision, if the predominantly elastically stored energy components are omitted, is

$$E_{10} + E_{20} = E_1' + E_2' + E_{f_1} + E_{f_2}$$

where the impact energy without rotation is

$$E_{10} = \frac{m_1 v_1^2}{2}; \qquad E_{20} = \frac{m_2 v_2^2}{2}$$

and the values for residual energy immediately after impact

REFERENCES

[1] Brüderlin, Ad.: Die Mechanik des Verkehrsunfalls bei Kraftfahrzeugen, Zürich 1941

[2] Lossagk, H.: Der Zusammenstoß, BTÜ (Ber. d. Techn. Überwachungsvereine) 1958, p. 256/260

[3] —: Die Analyse von Fahrzeugzusammenstößen, Jahrbuch „Schiene und Straße" 1961

[4] Eberan-Eberhorst, R.: Über die Mechanik des Kraftfahrzeug-Verkehrsunfalls, Österr. Ing.-Zeitschrift 1962, p. 160/169

[5] Marquard, E.: Bewegungsstudien zur Beurteilung von Kraftfahrzeug-Zusammenstößen, Information BVSK 1964, p. 3/11

[6] —: Zur Mechanik des Zusammenstoßes von Fahrzeugen, ATZ 1962, p. 141/148

[7] Slibar, A.: Die mechanischen Grundsätze des Stoßvorganges und ihre Anwendung, Arbeitstagung der GUFU Bad Krozingen, Oct. 1964 und Arbeitstagung der GUFU Karlsruhe, Oct. 1964, appearing in Archiv für Unfallforschung

[8] Könne, H. D.: Die Schwere von Verkehrsunfällen in Abhängigkeit von den Fahrzeuggewichten und dem Auftreffwinkel, Forschungsarbeit aus dem Straßenbau, Neue Folge 457, Bad Godesberg 1964

[9] v. Vigier, Der Verkehrsunfall unter der Lupe, Automobil Revue, Bern 1964, Nr. 9

[10] Eberan-Eberhorst, R.: Dringliche Sicherheitsprobleme der Kraftverkehrstechnik, ATZ 1958, p. 299/308

are equal to the values for resistance energy ΣW_s on the path described until rest,

$$E_1' = \frac{m_1 v_{1s}'^2}{2} + \frac{J_1 \omega_1'^2}{2} = \Sigma W_1 s_1;$$
$$E_2' = \frac{m_2 v_{2s}'^2}{2} + \frac{J_2 \omega_2'^2}{2} = \Sigma W_2 s_2$$

If the resistance energies are determinable, the total permanent deformation work $(E_{f_1} + E_{f_2})$ results from the difference between the total impact energy and the sum of all the resistance works during run-out, apart from damping losses. No prediction regarding the distribution of deformation between the vehicles involved is, however, possible by this method. For this purpose, additional assumptions were necessary even in the considerations presented in Section 3.2.1.

It would be desirable if the deformation energies on each vehicle could be determined separately, perhaps from the permanent deformations which had arisen. There has been no lack of attempts to develop solutions of this nature, but they have not so far achieved conspicuous success. The most publicised advance in this direction [9] has in fact shown itself to be decidedly erroneous. Reliable values for the connection between deformation and deformation energy are only available in print for the American measurements previously mentioned (crash tests); these measurements are confined to a specific class of vehicle and to head-on collisions between a moving a[illegible] another moving or stationary vehicle.

The evaluation of the measured patterns of decelerations only affords a broad approximation for the functional connection between force P and deformation deflection x. In many cases the imp ct force acting on the mass centre of gravity can be regarded as having a constant mean value [10]. Often t[illegible] linear relationship $P = cx$, which corresponds to Hocke's law, can be used as an approximation for the compression[8]. Thus the spring constant c results for the crash tests in the order of magnitude $700 > c > 300$ kg./cm. These figures cannot however be generalised, since they are dependent on the design of the vehicles, on the size and position of the impact surfaces, e.g head-on crash as opposed to side impact against a tree, and on the elastic and frictional conditions at the point of impact.