EMPIRICAL CRASH INJURY MODELING
AND
VEHICLE-SIZE MIX

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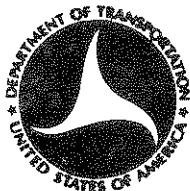
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EMPIRICAL CRASH INJURY MODELING AND VEHICLE-SIZE MIX

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16. Abstract

Crash injury prediction models were developed using data from the CPIR file for crashes which occurred since January 1, 1970, involving 1969 or newer cars, vans, and pickup trucks. Hostile and protective effects of vehicle size were separated in addition to injury severity increases with age, front seating position, and lack of restraints. Differences by crash configuration were also isolated. Elasticity of injury with respect to average vehicle weight change was computed using these models. Fuel cost decreases were compared with injury cost increases as vehicle weight decreases. Fuel cost savings exceed injury cost increases as vehicle weight is reduced. This conclusion assumes no change in the relationship between vehicle volume and vehicle weight. Injury reduction from larger and lighter vehicles and from improved vehicle design could increase the difference even more.

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EMPIRICAL CRASH INJURY
MODELING AND VEHICLE-SIZE MIX

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Executive Summary

This study developed a crash-injury prediction model and used that model to estimate the change in average injury as a function of average vehicle-size reduction. The cost of injury increase was compared with the fuel cost saving as vehicle size is reduced. This economic analysis indicated a substantial net cost savings from vehicle size reduction. The study also developed accident injury methodology and identified some important relationships between crash injury and variables measured by trained crash investigation teams.

The injury prediction model was developed using data from multidisciplinary crash investigations conducted since January 1, 1970, on vehicles beginning with model year 1969. Occupant-injury severity was estimated as the expected value of the injury distribution, measured by the overall Abbreviated Injury Scale (AIS). Expected AIS was predicted as a function of the change in impact velocity, occupant age, vehicle size (measured by vehicle weight), restraint usage, seating position, and crash configuration. Separate mathematical models were estimated for each subset of crashes identified by restraint usage and crash configuration. These separate models were combined using the percentage of vehicles in each crash configuration. Analysis of the combined model indicated that injury severity increased with occupant age, with reduction in vehicle size (measured by weight), and with nonusage of restraints. Occupants of rear seats sustained less severe injury. However, the strongest predictor in injury severity was the change in impact velocity.

To determine the relationship between average vehicle size and injury, it was necessary to separate two effects of vehicle size. First, when a heavier vehicle struck a lighter vehicle, the lighter vehicle had a greater change in impact velocity, which in turn increased the injury rate for occupants of that vehicle. This effect of weight, defined as the hostile effect, depended only on the relative weight of the two vehicles, not their absolute weight. Therefore, the hostile effect would remain the same if the weight of all vehicles was reduced proportionately. The second effect of vehicle size, called the protective effect, was related to factors such as increased energy absorption through crush, decreased passenger compartment intrusion, etc. In the vehicle population, vehicle weight was highly correlated with other measures of size, such as wheelbase, width, etc. Therefore, vehicle weight was used as a surrogate measure of vehicle size to estimate the protective effect. This protective effect will of course be reduced as vehicle size is reduced. In this study the protective effect was described by an estimated relationship between expected injury and vehicle weight. This relationship was then used to estimate injury increase as a function of vehicle weight decrease.

The relationship between injury and vehicle weight change was computed as the elasticity of injury with respect to vehicle weight. This elasticity is the ratio of percent change in injury to percent change in weight. The elasticity of injury was estimated to be −0.67 using the injury prediction model. Thus a 1% reduction

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in average vehicle weight increased injury severity by 0.67%. Using results presented in another study, the elasticity of fuel cost with respect to vehicle weight was estimated at 0.86. Thus, a 1% decrease in vehicle weight reduced fuel costs by 0.86%. Using the standard costs developed by the National Highway Traffic Safety Administration, the average injury cost per million miles of vehicle travel was estimated at $18,700. The fuel costs per million miles is $39,285, at an average price of $0.55 per gallon and an average yield of 14 miles per gallon. By combining these average costs and elasticities, an estimated net benefit of $213 per million miles of vehicle travel was obtained. Alternative assumptions could reduce this net benefit somewhat. However, no reasonable situation was identified for which the savings in fuel cost were less than the increased injury cost as vehicle weight decreased. Therefore, it was concluded that vehicle weight reduction will reduce overall vehicle travel costs.

This analysis assumed that the relationship between vehicle weight and size will remain the same in future vehicle populations. If, for example, lighter cars with the same volume and occupant protective characteristics are produced, the economic advantage in favor of reducing vehicle weight would be even greater.

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Introduction

The weight of automobiles is being substantially reduced with each new model that is introduced. The major force behind this effort is a desire for substantially improved fuel economy. However, an important and related crash-injury effect is anticipated. Various studies of the present accident population indicate that injuries suffered by occupants of lighter and smaller cars are of higher severity (for example, O'Day et al. 1973; Mela 1974; O'Neil 1974; Joksch 1976; and Reinfurt and Dutt 1977). Thus, there is a societal tradeoff between fuel saving and increased crash injury severity. The objective of the study reported here is to determine the relationship between vehicle weight and crash-injury severity. This relationship is then combined with the relationship between vehicle weight and fuel economy to obtain the societal trade off.

To understand the problem of vehicle weight and injury the reader needs to be aware of an important concept. In the present automobile population there is a high positive correlation between vehicle weight and measures of vehicle volume, such as wheelbase, overall weight, overall width, etc. For this reason vehicle size often refers to either vehicle weight or vehicle volume. Vehicle size has two effects on crash injury; these effects influence injury in opposite directions. In a two-vehicle crash, the heavier vehicle imposes larger decelerations on the smaller vehicle. These larger decelerations cause injuries of higher severity for occupants of the smaller vehicle. Alternatively, a vehicle with greater volume is able to absorb greater deformations. This results in injuries of lower severity to occupants of the larger car. Thus, when a large car strikes a small car, the occupants of the large car obtain the benefit of greater vehicle volume and avoid the penalty of lower vehicle weight. Occupants of smaller vehicles experience the negative of both of these effects. Thus, direct comparison of occupant injuries for small versus large cars in the present vehicle population results in overstating the inherent advantage of large cars in terms of occupant protection. If all vehicles are reduced in size proportionally, the occupant-protection capability associated with vehicle volume will be reduced. But the injury component associated with differences in vehicle weight will not be reduced. Therefore, any attempt to determine the relationship between vehicle weight and crash injury for the vehicle population must isolate these two components. This study has developed a methodology, based upon crash dynamics, for isolating these two components.

An injury prediction model has been developed that isolates the effect of the two components related to vehicle size and the effect of other significant variables. This model provides a direct measure of the average-injury change as a function of average-vehicle-weight change. This measure is then expressed in dollars by using conventional economic methods and the standard societal costs of motor vehicle accidents developed by NHTSA. The relationship between injury cost and average vehicle weight is then compared with the relationship between fuel cost and average vehicle weight. By this method the trade off between fuel cost and crash-injury cost can be compared in a common unit.

The injury prediction model developed in this study also has application to the general methodology of crash-injury analysis. For example, a method of computing crash severity, independent of injury severity, as a function of limited crash data has been developed. This method with some further refinements has potential application as a general crash statistic. Another useful result deals with the relationship between average AIS injury code and the probability distribution of AIS injury codes.

Discussion

The estimation of injury severity in future crash populations requires a careful identification of the relationship between injury severity and vehicle size. Joksch, O'Neill, and Haddon (1974) have proposed that vehicle size has two effects on injury severity. One effect is the "hostile" effect of a heavier car striking a lighter car. The lighter car always absorbs larger decelerations regardless of the impact velocity of each vehicle. This effect results from relative vehicle masses and not from absolute vehicle mass. Thus, if the weights of all cars are reduced proportionately, the "hostile" effect will not increase. A second effect is the "protective" effect of a vehicle's interior and exterior volume. The specific reasons for this protective effect are not as easy to define because of the complexities of vehicle construction and occupant crash dynamics. Larger cars provide more "crush distance" prior to passenger compartment intrusion and more room for intrusion without directly striking the passenger. Occupants also are located farther away from hostile interior components. Larger cars also have greater potential to absorb energy by crushing metal. This "protective" effect will be reduced if the volume of cars is decreased to achieve weight reduction. O'Neil et al. go on to recommend that manufacturers produce lower-weight cars with the same volume. This recommendation seems reasonable and is supported by many researchers, including this author, because it accomplishes fuel economy without increasing crash injury.

The above discussion indicates the importance of accurate estimates of the hostile and the protective effects of vehicle size. If the hostile effect is large and the protective effect is small, the vehicle volume reduction associated with vehicle weight reduction would result in a small crash-injury increase. Alternately, if the protective effect is large and the hostile effect is small, the reduction of vehicle volume to achieve weight reduction would result in a large crash-injury increase.

Any empirical study of the hostile and protective effect is confounded by the high correlation between weight and volume in the present vehicle population. For this reason, most studies have used some function of vehicle weight to measure both vehicle weight and vehicle size. Thus the measures of vehicle volume and vehicle weight are correlated. This correlation can produce misleading estimates of the hostile and protective effects.

Finally, it should be noted that studies that compare average injury by vehicle size are not suitable for estimating average injury by vehicle size in future crash populations. These studies obtain average injury, conditional upon crashes with vehicles in the present vehicle population. Thus they combine the hostile and the protective effect. If all vehicles are proportionately reduced in weight, the average hostile effect for a given vehicle weight is reduced and the average injury for occupants of lighter vehicles is reduced.

Crash-Injury Prediction Model

The study reported here develops models to predict expected occupant-injury severity as a function of crash parameters. It extends previously reported methodology (Carlson and Kaplan 1975; Carlson 1977) and develops improved models, based upon crash dynamics, which can be used to isolate the hostile and protective effects of vehicle size. In addition, these models can be used to control for crash severity when evaluating the effects of countermeasures designed to reduce injury.

The coefficients of the injury prediction models were estimated first by using the CPIR (Collision Performance and Injury Report). The structure of this file and its inherent biases have been discussed previously (Carlson and Kaplan 1975). In particular, the file overrepresents crashes with high severity.

To verify the methodology, coefficients were also estimated using data from the RSEP (Restraint System Evaluation Project) data file (Kahane and Mungenast). This file was designed to be a representative sample of crashes in defined geographic locations. As the geographic locations of the crash investigation teams were not selected randomly, the file contains certain biases. These are discussed by (Reinfurt, et al. 1976). Specifically the file overrepresents urban crashes. Thus, it is likely to overrepresent lower-severity crashes.

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This study estimates hostile and protective effects using uncorrelated measurements. Specifically, the measurement used to estimate the hostile effect is derived from a physical analysis of crash dynamics. This analysis develops a relationship that can be used to estimate the change in vehicle velocity, ΔV, as a function of reported impact velocities, impact angles, and both vehicle weights. The protective effect is estimated using vehicle weight, which is a surrogate for vehicle volume. Vehicle volume is not reported in the available crash data, and vehicle weight is highly correlated with size measures, such as wheelbase.

Development of the injury-severity prediction model used in this study began with previous results (Carlson 1977). In that work it was shown that injury severity in two-car crashes is a function of both vehicle weights and impact velocities, occupant age, restraint-system utilization, and occupant seating position.

The data used from the CPIR file were limited to crashes that occured after January 1, 1970. This was done to avoid biases that might result from using crashes that were investigated early in the Multi-disciplinary Accident Investigation (MDAI) program. These early cases tended to be exceptional in various ways, and the reported data were expected to suffer from extreme lack of uniformity. Vehicles beginning with model year 1969 were considered. Thus, they were produced under the modern motor vehicle safety standards. The analysis also included vans and pickups, which represented about 8 percent of the total sample of vehicles in crashes. For this reason, the resulting models have application to crashes involving cars, vans and pickups.

Injury Prediction Model Variables

A significant conceptual improvement over the previous model resulted from an analysis of crash dynamics. From this analysis it was shown that the crash force imposed on a vehicle occupant is proportional to the change in vehicle velocity, ΔV, resulting from the crash. In addition, it was shown that this velocity change can be estimated from the data reported in the CPIR file by using,[2]

$$\Delta V_1 = \frac{W_2}{W_1 + W_2} \sqrt{V_1^2 + V_2^2 + 2 V_1 V_2 \cos \alpha} \tag{1}$$

where

ΔV_1 is the estimated change in the case vehicle velocity as a result of the crash in miles per hour;

W_1 is the reported weight of the case vehicle in pounds;

W_2 is the reported weight of the other vehicle in pounds;

V_1 is the reported impact velocity of the case vehicle in miles per hour;

V_2 is the reported impact velocity of the other vehicle in miles per hour; and

α is the direction of the resultant velocity vector after impact, relative to the initial direction of the case vehicle.

The direction of the velocity vector, α, is estimated using, $\alpha = \theta_1 - \theta_2$ (2)

where

θ_1 is the reported direction of the impact force for vehicle 1 and

θ_2 is the reported direction of the impact force for vehicle 2.

Appendix A presents a detailed development of equation 1.

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[2]The analysis was developed with the able assistance of Dr. Russell Smith and Mr. Jerome Kossar of the NHTSA staff. See Appendix A for details of the analysis.

Several important insights result from equation 1. The effect of relative vehicle weight is determined by the ratio of the other vehicle weight to the sum of both vehicle weights. Thus ΔV increases with the weight of the other vehicle, given a fixed weight for the case vehicle. The magnitude of ΔV is also seen to depend upon the relative vehicle weights and not on the absolute weight of either vehicle. For vehicles of the same weight, ΔV is equal for both vehicles and does not depend upon the actual weight. However, the lighter vehicle always receives a larger fraction of the total impact velocity regardless of the relative velocity. For this reason ΔV includes a measure of the "hostile" effect of vehicle weight. A linear relationship between ΔV and injury assumes a similar occupant deceleration for all vehicles. However, different vehicles are designed to have widely different deceleration patterns for specific crash configurations. Using the data provided by crash investigation, it is not possible to separate different crash deceleration patterns. An even greater problem is the relationship between crash force and occupant dynamics. As indicated by Monk et al. (1977), there is considerable injury variation for a given ΔV even when the computation of ΔV is carefully controlled and crashes are selected to insure that they are similar in other characteristics.

The variable ΔV also includes the effect of impact velocity. Interpretation of ΔV can be improved by first considering a direct colinear head crash. For that crash, α is equal to zero and $\cos \alpha$ equals 1. Therefore, the velocity component is equal to $V_1 + V_2$ and,

$$\Delta V_1 = \frac{W_2}{W_1 + W_2} (V_1 + V_2).$$ (3)

Notice that in this case ΔV depends only on the sum of the impact velocities and not on the specific value of either. Similar analysis indicates that the velocity component for colinear rear-end crashes is equal to the difference in impact velocity, $V_2 - V_1$. For other crashes, the $\cos \alpha$ term provides the appropriate resolution of impact directions.

The protective effect of vehicle size was measured by the case vehicle weight, W_1. Since the protective effect is more likely to be related to vehicle volume, weight would appear to be an unsuitable variable. For example, a measurement such as length of wheelbase or overall width might be better. Alternatively manufacturer size classifications, such as subcompact, compact, intermediate, and full size, could also have been used. However, vehicle weight is highly correlated with wheelbase and with manufacturer size classifications in the present vehicle population. Thus weight is a usable surrogate for vehicle volume. The absolute value of the correlation between ΔV_1 and W_1 is less than 0.20 in the sample of crashes used to fit the injury prediction models. Therefore, the coefficients of ΔV_1 and W_1 in the injury prediction model are not influenced by multicollinearity between these variables. For that reason the hostile and protective effects can be uniquely identified.

Numerous studies have indicated that injury severity increases monotonically with occupant age. Our previous work indicated that the relationship was linear. But, for example, Preston (1975) found that the probability of death increased quadratically with age. In this analysis the use of a quadratic effect for age did not improve injury prediction. Thus, age was treated as a linear term in this study.

Previous work has also indicated that in a given crash front-seat occupants receive higher-severity injuries than do rear-seat occupants. For that reason, categorical (0, 1) variables were used to indicate that the occupant was either a driver or a right-front passenger. If the occupant is neither, then he is assumed to be in the rear seat.

The influence of crash configuration and usage of seat belts was included by fitting separate injury prediction models for each crash configuration by seat belt usage combination. Crash configurations were defined as head on, side impact, rear end, and single vehicle, with striking and struck vehicles treated separately. In addition, single-vehicle crashes were partitioned into rollover and striking-fixed-object subgroups. The data in the CPIR file included enough cases to provide reasonable injury-prediction models for restrained occupants. However, we stress that restrained occupants in the CPIR file typically used only

seat belts. Some occupants had upper-torso restraints; however, the data are insufficient to estimate the effect of these restraints in the CPIR file.

The measure of injury used in this study is the overall AIS code. Problems associated with the use of this measure have been discussed previously (Carlson and Kaplan 1975). It should be noted that other possible measures are also subject to criticism, such as, percent severe plus fatal injury, injury severity score, and percent injury plus fatal. The overall AIS utilizes the best judgment of total occupant injury by an investigator who has actually observed the vehicle occupant. Thus the investigator has all the available injury data, and we utilize the observer to integrate this data into a single injury-severity index.

For any crash of a given severity there is a distribution of injury severities. Occupants strike different objects at different angles, and occupants react differently to similar blows to their bodies. Some of these differences can be identified by using the injury prediction variables discussed previously. However, any analysis of the relationship between crash severity — measured by, for example, ΔV — and injury severity leads to a distribution of injury severities for a given crash severity.

For the study reported here we wanted to have a single number to represent this distribution of injury severity. By using overall AIS as the dependent variable in a least-squares multiple regression analysis, we obtain the expected value of the injury distribution conditional on the injury prediction variables. This expected value is equivalent to the distribution mean for a population of injury-severity scores. Since the models in this study predict average overall AIS, it is useful to understand how the distribution of injury severity changes with the mean of the distribution.

The relationship between mean overall AIS and the distribution of overall AIS was examined in several different ways. Figure 1, 2, and 3 show the relationship between mean overall AIS and the percentage of crashes with AIS greater than 1, 2, and 4. Each point on the graph represents a subset of occupants identified by specific intervals of ΔV, case vehicle weight, and occupant age. These conditioning variables were grouped into the following subsets:

Case Vehicle ΔV Subgroups	Interval
1	< 6 mph
2	6 – < 12 mph
3	12 – < 22 mph
4	22 – < 32 mph
5	≥ 32 mph

Case Vehicle ΔV Subgroups	Interval
1	< 3450 pounds
2	≥ 3450 pounds

Occupant Age Subroups	Interval
1	< 32 years
2	≥ 32 years

Fig. 1: Percent of Overall AIS Scores Greater than 1 versus Mean AIS for Crash Subsets Indexed by ΔV, Case Vehicle Weight, and Occupant Age
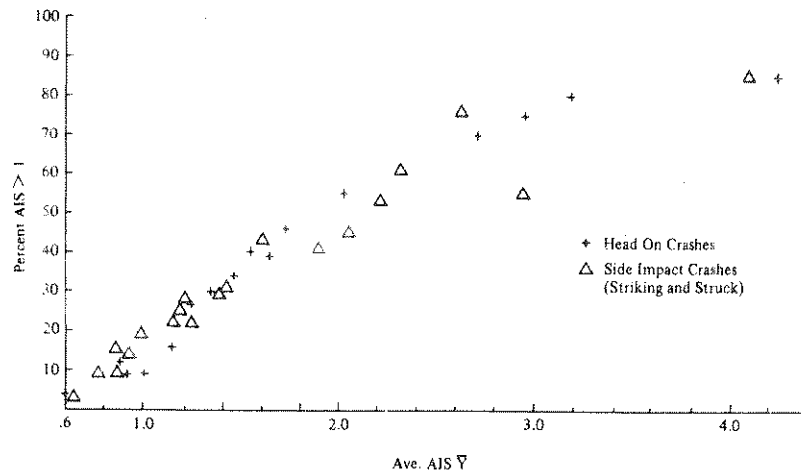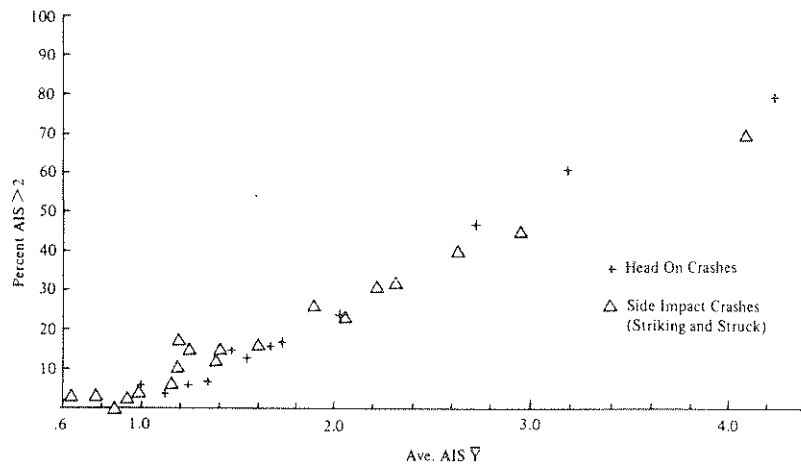

Fig. 2: Percent of Overall AIS Scores Greater than 2 versus Mean AIS for Crash Subsets Indexed by ΔV, Case Vehicle Weight, and Occupant Age
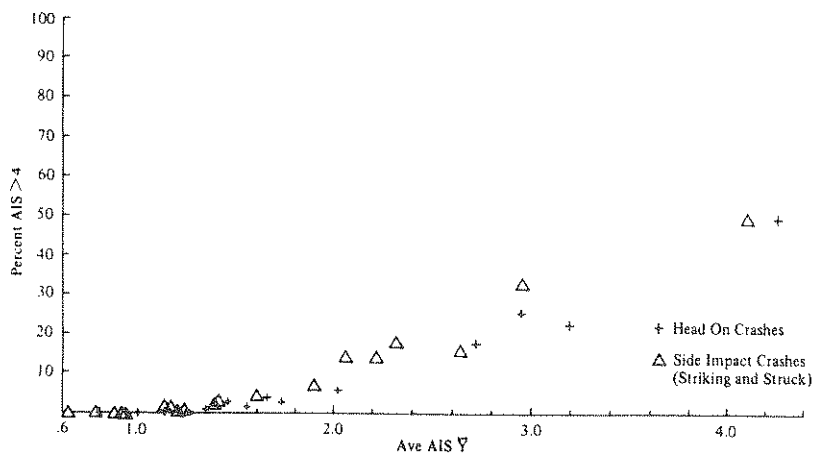

Fig. 3: Percent of Overall AIS Scores Greater than 4 versus Mean AIS for Crash Subsets Indexed by ΔV, Case Vehicle Weight, and Occupant Age

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The subsets are defined for all combinations of the three variable subgroups. Examination of these graphs clearly indicates the high correlation between average AIS and other measures of the distribution.

In a related study, Monk et al. (1977) conducted a detailed analysis of 173 cases in the CPIR file that involved the struck vehicle in side impact crashes. Only cases that had sufficient data to compute ΔV, using the CRASH (Calspan Reconstruction of Accident Speeds on the Highway), were used in their study. Figure 4 indicates the relationship between ΔV_1, computed using the CRASH II program, and mean AIS, for crashes grouped by ranges of ΔV_1. The high linear correlation is clearly evident from this graph. A least squares regression was also fit to the data, yielding the equation,

$$\overline{Y} = 0.86 + 0.099 \Delta V_1$$

$$R^2 = 0.86 \qquad (4)$$

where

$$\overline{Y} \text{ is the average AIS and}$$

$$R^2 \text{ is the percent explained variability.}$$

Comparison of $\overline{Y}$ and percent AIS greater than 2 for these ΔV subgroups indicated the same strong linear relationship between average AIS and other measures of the AID distribution as shown in figures 1, 2, and 3.

Structure of Model

The basic structure of the injury prediction models that were estimated using the CPIR data is;

$$\widehat{Y}_{ij} = \widehat{B}_{0ij} + \widehat{B}_{1ij}\Delta V_1 + \widehat{B}_{2ij}A + \widehat{B}_{3ij}D_1 + \widehat{B}_{4ij}R_F + \sum_{K=5}^{8} \widehat{B}_{Kij}\Delta W_K \qquad (5)$$

where,

$\widehat{Y}_{ij}$ is the predicted average AIS for crash configuration i and restraint usage j

$(i = 1, \cdots 8 ; j = 1, \cdot 2)$,

$j = 1$ indicates restraint usage and
2 indicates no restraint usage,

i is the crash configuration as shown in table 1,

$\widehat{B}_{Kij}$ are coefficients estimated using the crash data,

ΔV_1 is the change in impact velocity in miles per hour,

A is the occupant age in years,

$D_1 = 1$ occupant is driver
0 else

$R_f = 1$ occupant is right front seat
0 else

ΔW_K indicates the case vehicle weight group according to the following ranges:

	Vehicle weight range
$\Delta W_5 = 1$	2200 - 2899 pounds
$\Delta W_6 = 1$	2900 - 3599 pounds
$\Delta W_7 = 1$	3600 - 4299 pounds
$\Delta W_8 = 1$	greater than 4299 pounds
$\Delta W_K = 0$	else (K = 5, $\cdot \cdot$ 8)

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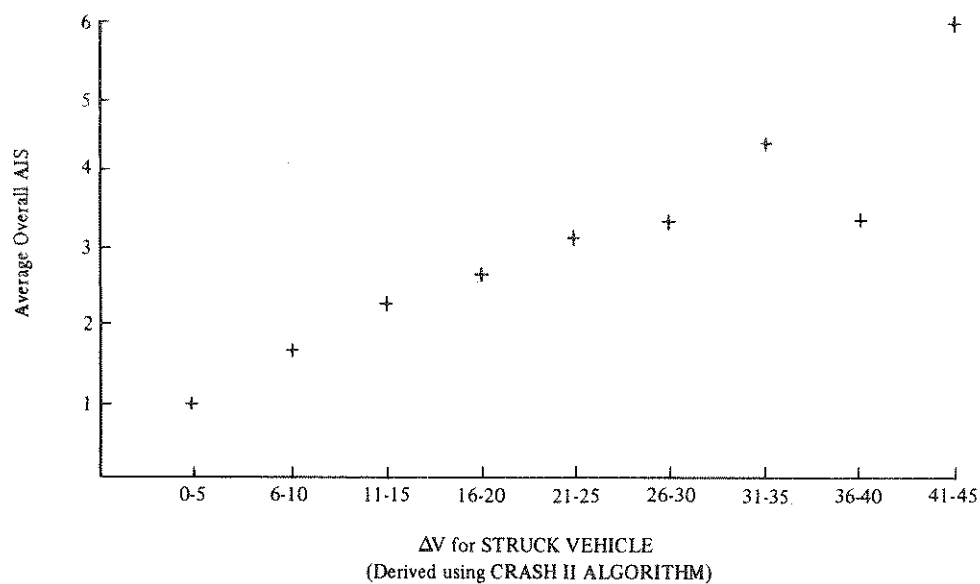
Fig. 4: Average Overall AIS vs ΔV for Side Impact Crashes

Table 1: Percentage of Vehicles in Crashes

Crash Configuration	Percent of Vehicles (α_i)
1. Head-on	13.2
2. Side-impact——striking	28.4
3. Side-impact——struck right	14.2
4. Side-impact——struck left	14.2
5. Rear-impact——striking	11.7
6. Rear-impact——struck	11.7
7. Single-vehicle——rollover	1.7
8. Single-vehicle——fixed object	4.8
	100.0

These percentages were obtained from the Restraint System
Evaluation Project (RSEP) data file. The sample contained in
this file was designed to be a representative sample of crashes;
however, the sample is biased toward urban crashes.

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To obtain an overall estimate of average occupant-injury severity, linear combinations of the subset models are obtained using,

$$\widehat{Y} = C \sum_{i=1}^{8} \alpha_i \widehat{Y}_{i1} + (1-C) \sum_{i=1}^{8} \alpha_i \widehat{Y}_{i2} \tag{6}$$

where

  Y   is the overall estimated average AIS as a function of the crash parameters,

  C   is the proportion of occupants wearing restraints, and

  α_1 is the proportion of vehicles in crash configuration i.

Specific values of α_1 obtained from an analysis of the RSEP file are shown in table 1.

Errors in Predictor Variables

The coefficients of the injury prediction models are biased if there are errors in the measurement of independent variables. In the data used for this study, it is likely that ΔV is measured with error. This error results from errors in the estimation, by field observers, of the impact velocity and the impact direction for each vehicle. Thus, the ΔV computed from equation 1 is assumed to have the structure

$$\Delta V_1 = \Delta V_2' + U \tag{7}$$

where

  $\Delta V_1'$ is the true change in velocity,

  ΔV_2 is the reported change in velocity, and

  U   is a random disturbance with mean zero and variance $\sigma^2(U)$.

It is well known that measurement error causes a negative bias in the estimated coefficient that is directly proportional to $\sigma^2(U)$ and inversely related to the sum of the squared deviations of the independent variable, ΔV (Kmenta 1971, page 309). The bias would tend to reduce the size of the coefficient, B_{1ij}, in equation 5. The reason for this bias can be seen intuitively by examining the idealized diagram in figure 5. If the measurement error for ΔV_1 is small compared to the range of ΔV and/or the number of observations is large, the coefficient bias will be small. As the measurement error is unknown, a possible negative bias in the estimated coefficient of ΔV_1 cannot be ruled out.

Fortunately it is possible to correct for the potential coefficient bias by using two-stage least squares. In Stage I, a regression model is fit with ΔV_1 as the dependent variable, and variables correlated with ΔV_1 are used as the independent variables. From the resulting least-squares model, the expected value, ΔV_1, is computed for each observation. These expected values are then used, instead of the original observed values of ΔV_1 to estimate the coefficients of the injury prediction model (equation 5). It has been shown that by using the expected value ΔV_1 the estimated coefficient will be unbiased (Kmenta 1971).

The application of two-stage least-squares is aided by the existence of a logical relationship between reported ΔV and other variables collected at the crash scene. Accident investigators typically use the extent and location of vehicle damage to estimate vehicle velocity. This fact combined with some analysis of the crash data led to the following Stage I model,
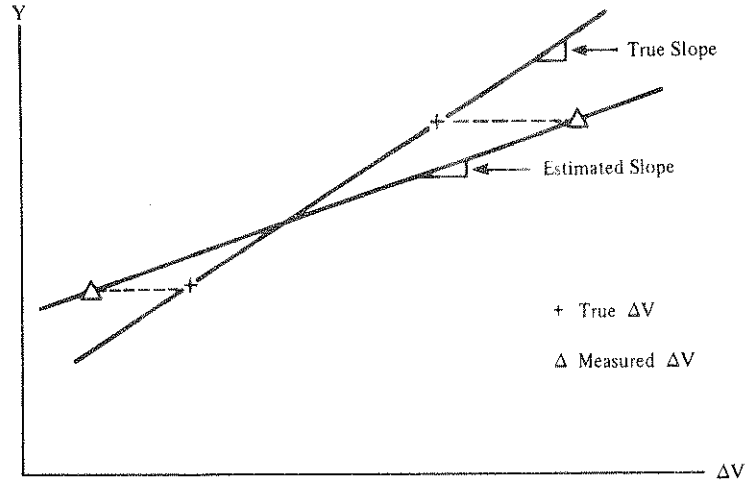
Fig. 5: The Effect of Measurement Error in ΔV on the Slope Coefficient

$$\Delta V_1 = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + \sum_{j=4}^{6} a_j x_j \qquad (8)$$

ΔV_1 is the expected change in velocity,

x_1 is the reported primary collision damage extent number,

x_2 is the vehicle model year,

x_3 is the weight of the other vehicle

x_4 = 1 damage is distributed over the impacted side of the vehicle,
    = 0 else,

x_5 = 1 damage is concentrated at the center of the impacted side of the vehicle,
    = 0 else

x_6 = 1 damage covers the center and one half of the impacted side of the vehicle,
    = 0 else.

This model was fitted for both the striking and struck vehicles in each crash configuration. Data from restrained and unrestrained occupants were combined, as the Stage 1 model is concerned with crash severity and/or vehicle damage and not occupant injury. Table 2 contains the coefficients and measures of goodness-of-fit for the Stage I models.

The injury prediction models were fitted using both measured ΔV_1 and expected ΔV_1 (that is, using ordinary least squares and two-stage least-squares). The estimated coefficient for ΔV_1 using two-stage least-squares was almost double the value estimated using ordinary least-squares.[3] The coefficient of ΔV from two-stage least-squares is also very close to the result obtained in equation 4 using the data from Monk et al. (compare equation 4 with equation 11). For these reasons, it seems apparent that ΔV has a large measurement error. In addition, this measurement error can cause a large bias in the estimated coefficient of ΔV if two-stage least-squares are not used.

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[3]The specific comparison refers to the model for all crashes obtained from the linear combination of the injury prediction submodels using equation 6.

Table 2: Coefficients for Stage I Models Which Predict ΔV

Crash Configuration	Constant	C.D.I. Extent	Vehicle Model Year	Other Vehicle Weight	Damage Area			R^2	S(Y|X)	N
					Distributed	Center	R-C LC			
1. Head-on	−2.32	4.40 (23.6)	−0.46 (−2.39)	0.37 (7.86)	8.47 (9.37)	−5.91 (−1.81)	4.25 (5.07)	0.47	10.46	905
2. Side-impact—striking	3.88	3.93 (16.8)	0.58 (3.88)	0.06 (2.00)	2.16 (3.99)	−5.04 (−2.11)	0.24 (0.40)	0.23	7.73	1134
3. Side-impact—struck	−0.56	3.57 (17.8)	−0.06 (−0.44)	0.18 (7.07)	2.78 (2.92)	−1.74 (−2.74)	2.10 (3.98)	0.28	7.22	1127
4. Rear-impact—striking	2.51	3.63 (14.7)	0.00 (0.00)	0.04 (1.28)	0.58 (0.76)	−2.84 (−1.28)	1.53 (2.06)	0.35	6.39	543
5. Rear-impact—struck	−3.38	2.40 (10.7)	0.62 (2.44)	0.13 (2.75)	2.16 (1.78)	−0.27 (−0.12)	0.45 (0.35)	0.29	7.66	398
6. Single-vehicle	19.01	3.83 (23.2)	−0.50 (−2.85)		10.05 (12.3)	−0.62 (−0.83)	3.58 (4.47)	0.23	14.11	2428

Note: The numbers below the coefficients are the coefficient student t statistics.

Estimated Coefficients For Injury Prediction Models Using CPIR Data

The coefficients estimated for each crash subset injury-prediction model are presented in tables 3 and 4. Occupant injuries that resulted from ejection were excluded from the analysis. This exclusion underestimates restraint effectiveness because the effect of restraints on reducing ejections was not included. Tables 3 and 4 also include some standard measures of goodness-of-fit, the proportion explained variability, R^2, and the standard error of the estimate, $S(Y/X)$. These measures provide an indication of the relative precision of the alternative models. Since the AIS is reported as integers over the range 0 to 6, R^2 is biased low, and $S(Y/X)$ is biased high (Carlson 1977). Thus, comparison of these statistics with R^2 and $S(Y/X)$ from other models is likely to be misleading.

Table 1 presents estimated proportions of vehicles by crash configuration. These were obtained from the RSEP file, which was designed to be a representative sample of tow-away crashes. It should be noted that this file contains an overrepresentation of urban crashes, thus the proportions for individual crash configurations may be in error. However, the proportions appear to be in general agreement with other sources. As a better source is not available, these proportions will be used in equation 6 to obtain the model for all crashes.

Using equation 6 and the proportions in table 1, injury-prediction models were obtained for occupants with and without seat belts. These models are:

1. Occupants with seat belts

$$\hat{Y}_1 = -0.66 + 0.102 \; \widehat{\Delta V}_1 + 0.0083A - 0.02D + 0.19R_f$$
$$\qquad\quad (15.9) \qquad\quad (4.20) \quad (-.18) \;\; (1.50)$$

$$-0.14\Delta W_5 - 0.21\Delta W_6 - 0.22\Delta W_7 - 0.35\Delta W_8$$
$$(-1.14) \quad\;\; (-1.84) \quad\;\; (-2.01) \quad\;\; (-2.61)$$

$$(9)$$

2. Occupants without seat belts

$$\hat{Y}_2 = -0.72 + 0.097\widehat{\Delta V}_1 + 0.014A + 0.22D + 0.24R_f$$
$$\qquad\quad (28.1) \qquad\quad (12.8) \quad (4.49) \quad (4.54)$$

$$-0.11\Delta W_5 - 0.22\Delta W_6 - 0.33\Delta W_7 - 0.40\Delta W_8$$
$$(-1.64) \quad\;\; (-3.62) \quad\;\; (-5.36) \quad\;\; (-5.60)$$

$$(10)$$

A graphic illustration of the estimated injury reduction from wearing seat belts is shown in figure 6. These graphs were constructed by assuming a 30-year-old driver in a 3,500-pound car. Examination of these graphs indicates that occupants wearing seat belts have a lower injury severity over the range of ΔV. This comparison does not include the effect of reduced occupant ejections for seat-belt wearers. Thus, the benefits from seat belts are understated.

By assuming a 20% restraint utilization, these two models can be combined to obtain a composite injury prediction model:

$$Y = -0.71 + 0.098\Delta V_1 + 0.012A + 0.17D + 0.23R_f$$
$$\qquad\quad (32.0) \qquad\quad (9.47) \quad (3.70) \quad (4.69)$$

$$-0.12\Delta W_5 - 0.21\Delta W_6 - 0.31\Delta W_7 - 0.39\Delta W_8$$
$$(-2.07) \quad\;\; (-3.89) \quad\;\; (-5.64) \quad\;\; (-6.19)$$

$$(11)$$

(The numbers below the coefficients are the coefficient student t statistics.)

A number of important crash injury results are contained in equations 9, 10, and 11. The $\widehat{\Delta V_1}$ statistic computed as described previously is the strongest predictor of occupant crash injury. Thus, it is a very useful variable for measuring differences in crash severity. Based upon the crash dynamics analysis and the empirical results, we conclude that $\widehat{\Delta V_1}$ controls for differences in crash severity and for the hostile effect of vehicle weight. Our analysis also indicates that the restraint system influence is fixed over the range of crash severity. This can be seen by examining the coefficients of $\widehat{\Delta V_1}$ in equations 9 and 10. In addition, the protective effect of vehicle size is also approximately the same for restrained and unrestrained occupants. Examination of the coefficients of ΔW_j (j = 5, . . . 8) shows this result.

Restraint systems have an interactive relationship with occupant age and seating position. For example, the coefficient of age is smaller for restrained occupants compared to unrestrained occupants. This implies that seat belts have greater injury-reducing benefits for older persons. It also should be noted that a quadratic effect for age was also tried in the models without reducing random error. Restraints also had greater influence on drivers compared to occupants of the rear seat. Comparison of the coefficients for D (driver = 1; other = 0) indicates this result. The restraint interaction does not appear to be as strong for right-front passengers, as indicated by the coefficients of R_f. That result appears to be counter intuitive given the restraint interaction for drivers. One possible explanation could be the smaller number of right-front occupants in the samples.

The reader should also notice that the standard error of the estimate $(S(Y/X))$ is relatively large compared to the expected value. There are two reasons for this. First, the reported AIS injury scores occur at discrete intervals. Thus, the reported AIS can be modeled as values rounded from a continuous injury scale. It is reasonable to anticipate that this rounding would lead to a positive bias in $S(Y/X)$. An analysis presented in Carlson (1977) shows that this bias does occur. A second reason for the large standard error is the fact that injuries vary considerably in the same crash. Many accident investigators have seen crashes in which one occupant was killed and another suffered minor or no injury. Because of this variance, large samples of crashes are required for estimating the models presented in this paper.
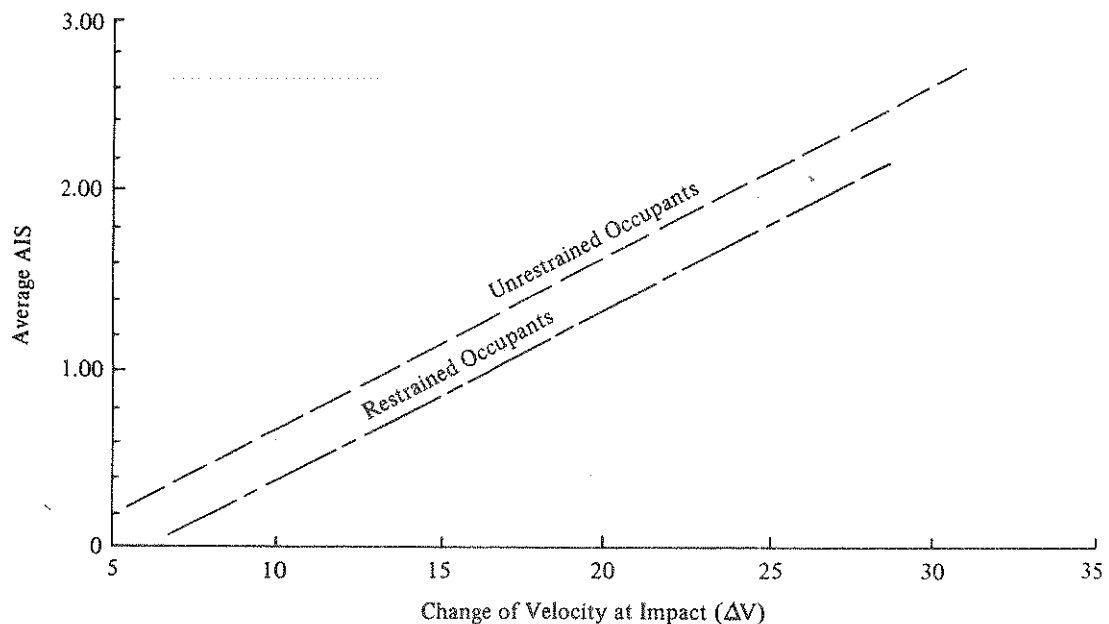

Fig. 6: Comparison of Expected Injury by ΔV for
Restrained and Unrestrained Models

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Table 3: Coefficients for Injury-Prediction Models — Restrained Occupants

Crash Configuration	Constant	ΔV	Occupant Age	Driver	Right Front	Vehicle Weight Group				R²	Sₑ(Y/X)	N
						ΔW_5	ΔW_6	ΔW_7	ΔW_8			
1. Head-on	-0.97	0.085	0.007	0.40	0.62	-0.29	-0.21	-0.17	-0.56	0.30	1.25	202
		(8.81)	(1.08)	(1.24)	(1.70)	(-0.73)	(-0.55)	(-0.47)	(-1.31)			
Coefficient Std. Error	.580	.00966	.00646	.323	.364	.394	.375	.368	.431			
2. Side-impact—striking	-1.10	0.129	0.005	-0.30	-0.02	0.24	0.12	0.12	-0.01	0.26	1.00	364
		(10.36)	(1.48)	(-1.28)	(-0.10)	(1.09)	(0.57)	(0.60)	(-0.05)			
	.360	.0124	.00347	.234	.249	.223	.213	.207	.227			
3. Side-impact—struck right	-0.50	0.107	0.012	-0.20	0.62	-0.62	-0.58	-0.48	-0.41	0.26	1.11	173
		(5.85)	(2.47)	(-0.69)	(1.94)	(-1.85)	(-1.78)	(-1.44)	(-1.02)			
	.505	.0183	.00498	.296	.321	.338	.325	.331	.401			
4. Side-impact—struck left	-0.73	0.160	0.010	-0.31	-0.69	-0.16	-0.49	-0.45	-0.48	0.27	1.16	149
		(6.27)	(1.65)	(-0.97)	(-1.86)	(-0.40)	(-1.39)	(-1.35)	(-1.12)			
	.677	.0256	.00626	.314	.369	.391	.351	.336	.434			
5. Rear-impact—striking	0.08	0.071	0.011	0.33	0.52	-0.49	-0.43	-0.63	-0.89	0.21	0.91	124
		(4.10)	(1.65)	(0.92)	(1.35)	(-1.42)	(-1.35)	(-2.01)	(-2.39)			
	.487	.0173	.00657	.359	.387	.345	.319	.314	.371			
6. Rear-impact—struck	0.31	0.034	0.008	0.18	0.22	-0.14	-0.14	-0.28	-0.39	0.13	0.51	101
		(2.50)	(2.13)	(0.75)	(0.85)	(-0.68)	(-0.67)	(-1.38)	(-1.42)			
	.346	.0136	.00381	.240	.257	.213	.209	.204	.272			
7. Single-vehicle—rollover	-0.46	0.065	0.010	-0.42	-0.36	-0.29	-0.32	-0.22	-0.69	0.25	1.18	135
		(4.02)	(1.19)	(-0.68)	(-0.56)	(-0.83)	(-0.93)	(-0.64)	(-1.37)			
	0.783	.0162	.00870	.622	.641	.352	.345	.349	.509			
8. Single-vehicle—fixed object	-1.70	0.059	0.008	0.75	1.00	0.39	0.23	0.14	0.26	0.18	1.25	300
		(5.85)	(1.38)	(1.58)	(2.01)	(1.39)	(0.90)	(0.52)	(0.67)			
	.590	.0100	.00560	.475	.498	.279	.258	.277	.392			
Combined	-0.66	0.102	0.008	-0.02	0.19	-0.14	-0.21	-0.22	-0.35			
		(15.9)	(4.20)	(-.18)	(1.50)	(-1.14)	(-1.84)	(-2.01)	(-2.61)			

Note: The numbers below the coefficients are the coefficient student t statistics.

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Table 4: Coefficients for Injury-Prediction Models — Unrestrained Occupants

Crash Configuration	Constant	ΔV	Occupant Age	Driver	Right Front	Vehicle Weight Group				R²	S(Y/X)	N
						ΔW_5	ΔW_6	ΔW_7	ΔW_8			
1. Head-on	-0.44	0.076	0.013	0.31	0.48	-0.21	-0.45	-0.67	-0.81	0.27	1.34	782
		(15.39)	(3.86)	(2.19)	(3.16)	(-0.98)	(-2.34)	(-3.40)	(-3.80)			
Coefficient Std Error	.251	.00497	.00326	.143	.153	.209	.194	.197	.214			
2. Side-impact striking	-0.70	0.098	0.013	0.07	0.15	-0.15	-0.21	-0.36	-0.36	0.23	0.93	1085
		(15.66)	(6.98)	(0.83)	(1.68)	(-1.30)	(-2.06)	(-3.43)	(-2.95)			
	.162	.00628	.00182	.0879	.0920	.115	.104	.104	.121			
3. Side-impact—struck right	-1.43	0.133	0.013	0.01	0.40	0.38	0.19	0.15	-0.02	0.26	1.16	609
		(12.30)	(4.86)	(0.10)	(2.81)	(2.16)	(1.19)	(0.93)	(-0.12)			
	.254	.0108	.00271	.133	.143	.178	.159	.165	.194			
4. Side-impact—struck left	-0.97	0.141	0.017	0.40	-0.03	-0.21	-0.50	-0.62	-0.60	0.27	1.30	460
		(10.40)	(4.47)	(2.27)	(-0.14)	(-0.86)	(-2.18)	(-2.62)	(-2.30)			
	.357	.0135	.00382	.176	.190	.244	.228	.238	.261			
5. Rear-impact—striking	-0.36	0.059	0.010	0.40	0.30	0.01	0.07	-0.01	-0.18	0.16	0.81	461
		(6.87)	(3.66)	(3.37)	(2.35)	(0.09)	(0.55)	(-0.09)	(-1.24)			
	.187	.00862	.00277	.117	.129	.149	.120	.130	.149			
6. Rear-impact—struck	0.05	0.076	0.011	0.21	0.15	-0.31	-0.36	-0.44	-0.54	0.22	0.91	338
		(7.38)	(3.44)	(1.52)	(0.97)	(-1.75)	(-2.11)	(-2.89)	(-2.50)			
	.225	.0103	.00310	.138	.152	.177	.169	.152	.215			
7. Single-vehicle rollover	-0.79	0.045	0.021	0.60	0.54	-0.34	-0.24	-0.38	-0.11	0.32	1.51	590
		(5.61)	(4.23)	(2.85)	(2.45)	(-1.38)	(-1.11)	(-1.65)	(-0.40)			
	0.390	.00804	.00490	.210	.219	.245	.220	.231	.290			
8. Single-vehicle fixed object	-1.45	0.082	0.022	0.35	0.46	-0.50	-0.30	-0.34	-0.43	0.24	1.37	1618
		(17.49)	(8.73)	(3.33)	(3.94)	(-3.50)	(-2.31)	(-2.49)	(-2.69)			
	.212	.00469	.00247	.106	.117	.144	.130	.136	.159			
Combined Model	-0.72	0.097	0.014	0.22	0.24	-0.11	-0.22	-0.33	-0.40			
		(28.1)	(12.8)	(4.49)	(4.54)	(-1.04)	(-3.62)	(-5.36)	(-5.60)			

Note. The numbers below the coefficients are the coefficient Student t-statistics.

Prediction of Injury Severity Conditional on Vehicle Size

By assigning appropriate values to the variables in equation 11, it would be possible to construct typical crash injury data. Consider a simplified example. Assume that the automobile population consists of an equal number of 3,100-pound and 3,800-pound cars. In addition, assume that crashes occur independently of vehicle size. Thus the probabilities of crashes by vehicle size are given by:

$$Pr(B \cap S) = Pr(B)\,Pr(S) = (.50)(.50) = .25$$
$$Pr(S \cap B) = Pr(S)\,Pr(B) = (.50)(.50) = .25$$
$$Pr(B \cap B) = Pr(B)\,Pr(B) = (.50)(.50) = .25$$
$$Pr(S \cap S) = Pr(S)\,Pr(S) = (.50)(.50) = .25$$

Where

B indicates Big (3,800-pound) cars and
S indicates Small (3,100-pound) cars.

The expected injury to a 30-year old driver in the big and in the small car was computed using equation 11. A key variable in this calculation is of course ΔV. For this example we assumed that the impact velocity part of equation 1 had a value of 24; therefore,

$$\Delta V_1 = \frac{W_2}{W_1 + W_2} \sqrt{V_1^2 + V_2^2 + 2V_1 V_2 \cos \alpha} = \frac{W_2}{W_1 + W_2} (24) \quad (12)$$

For example, a direct head-on crash with both vehicles impacting at 12 miles per hour would be presented by equation 12. From equation 12, ΔV would equal 12 when two small vehicles or two big vehicles collided. However, when a big vehicle collided with a small vehicle the values of ΔV would be

$$\Delta V_1 = \frac{3,100}{3,100 + 3,800} (24) = 10.78$$

for the big vehicle and $\Delta V_1 = 13.22$ for the small vehicle. Using these values of ΔV, the expected average AIS for drivers of big and small vehicles were computed as:

Vehicle 2

		Big (3,800 Lbs.)	Small (3,100 Lbs.)
Vehicle 1	Big (3,800 Lbs.)	$\hat{Y}_B = .69$ $\hat{Y}_B = .69$	$\hat{Y}_S = .91$ $\hat{Y}_B = .56$
	Small (3,100 Lbs.)	$\hat{Y}_B = .56$ $\hat{Y}_S = .91$	$\hat{Y}_S = .79$ $\hat{Y}_S = .79$

If the marginal distribution of driver injury were constructed from this hypothetical population, the results would be:

$$\bar{Y}_S = 0.85$$
$$\bar{Y}_B = 0.62$$
$$\bar{Y}_{combined} = 0.74$$

The injury difference between big and small cars is:

$$\text{Percent Difference in Average Injury Between Big and Small Cars} = 100 \left(\frac{0.85 - 0.62}{0.74} \right) = 31.1\%.$$

As the weight difference between big and small cars is 700 pounds, the injury-severity change per 100 pounds is 4.44% (31.1/7). This change is within the range found by other studies whose methodology was different. This percentage change includes both the hostile and the protective effect, as discussed above. If the weight of all vehicles is reduced proportionately, only the protective effect would change. This change in protective effect is given by the difference between the coefficients of ΔW_6 and ΔW_7 in equation 11. This change is

$$100 \left(\frac{-.31 - (-.21)}{.74} \right) = 13.5\%$$

for the 700-pound difference. Thus, the change in protective effect per 100 pounds is 1.93% (13.5/7).

Elasticity of Injury with Respect to Average Vehicle Weight

The conclusions from this analysis can also be expressed in terms of the elasticity of injury severity with respect to weight. Elasticity, e, in this situation is defined as the percent change of injury divided by the percent change of vehicle weight,

$$e_c = \frac{\Delta Y/Y}{\Delta W/W} = \frac{\Delta Y}{\Delta W} \frac{W}{Y} \tag{13}$$

where,

$\quad$ W $\quad$ is the vehicle weight and
$\quad$ ΔW $\quad$ is the change in vehicle weight.

Assuming that the average AIS is 0.74 and the average vehicle weight is 3,450 pounds, the elasticity at the crash population mean is,

$$e_c = -\left(\frac{(.10)}{(700)} \frac{(3450)}{(.74)} \right) = -.67.$$

Thus a 1% reduction of average vehicle weight would increase injury severity by 0.67%. This assumes that the relationship between vehicle volume and vehicle weight is similar to the present vehicle population. Since the elasticity is a point estimate, its value will vary with the vehicle weight and injury severity. Population estimates of average AIS and average vehicle weight can be estimated from several sources. The average AIS from the RSEP file is 0.61, which is expected to be low because of the bias toward urban crashes.

It is 1.53 from the CPIR file. This later value of 1.53 is known to be considerably higher than the national average, because the CPIR file overrepresents high-severity crashes. From the CPIR file the average vehicle weight is 3,450 pounds. This could be slightly low if we assume that smaller cars have crashes of higher severity.

To obtain an understanding of the possible range of elasticity, consider the elasticity for the following values of average weight and average AIS:

Average Vehicle Weight	Average AIS	Elasticity e_c
3300 pounds	0.60	−0.78
3300 pounds	0.80	− .59
3300 pounds	1.00	− .47
3600 pounds	0.60	− .86
3600 pounds	0.80	− .64
3600 pounds	1.00	− .52

These values are believed to be outer limits on the vehicle weight and injury measurements. Thus, the true elasticity should easily fall within the range of −0.47 to −0.86.

Analysis of Model Error

A fundamental assumption of multiple regression is that the unexplained variability has a uniform variance over the range of the model predictions. This is the property of homoscedasticity. If the error is heteroscedastic, the coefficients estimated using multiple regression will be unbiased, but the coefficient standard errors will be biased (Kmenta 1971).

The variance of the error for the expected AIS was found to increase with the expected value of the AIS. A correction for heteroscedasticity can be made if the random error is known to be functionally related to a known variable. For example consider the model,

$$Y_i = B_0 + \sum_{J=1}^{K} B_j X_{ji} + \epsilon_i \tag{14}$$

where

$$\epsilon_i = Z_i \epsilon$$
$$\epsilon \sim N(0, \sigma^2).$$

Bj's are coefficients, Y_i and X_{ji} are measured variables and, ϵ_i is a random distribance that increases with a measured variable Z_i. This model can be converted to a model that has a uniformly distributed disturbance by dividing each term in equation 1 by Z_i.

This leads to the model,

$$\frac{Y_i}{Z_i} = \frac{1}{Z_i} B_0 + \sum_{J=1}^{K} B_j \frac{X_{ji}}{Z_i} + \epsilon. \tag{15}$$

The coefficients can be estimated from equation 2 by "weighting" each observed variable by the factor $1 \div Z_i$. For that reason the method is often called, "weighted least squares."

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For the injury-prediction models, two different assumptions concerning Z_i were made. First, the error was assumed to be linearly related to ΔV_i. Thus all variables were weighted by $1/\Delta V_i$ and the coefficients were estimated. Under this assumption, crashes with higher impact velocity have occupant injuries with greater variability. For example, occupant trajectory, occupant-striking angle, and interior surface struck are assumed to have greater influence on injury severity as impact velocity increases. As these factors cannot be adequately estimated from postcrash investigation, they lead to increased error. A second assumption was that the error was linearly related to occupant age. Based upon this assumption all data were weighted by 1, divided by occupant age. This assumed that, with increasing age, variables such as physical condition, bone structure, and general health are related to injury severity. As these factors cannot be measured and as they vary more with increasing age, the injury variability increases with age.

If either of these assumptions are true, the unweighted multiple regression should have a positive bias for the coefficient standard errors. Equations 11, 16, and 17 are the overall injury-prediction models whose coefficients were estimated using unweighted two-stage least squares, two-stage least squares weighted by $1/\Delta V_i$, and two-stage least squares weighted by 1 divided by occupant age.

1. Two-stage least squares, unweighted:

$$\widehat{Y} = -0.71 + 0.098\widehat{\Delta V_1} + 0.012A + 0.17D + 0.23R_F$$
$$\phantom{\widehat{Y} = -0.71} (.0031) \phantom{0.098\widehat{\Delta V}} (.00094) \ (.046) \ (.049)$$

$$-0.12\Delta W_5 - 0.21\Delta W_6 - 0.31\Delta W_7 - 0.39\Delta W_8$$
$$(.058) (.054) (.055) (.063)$$

(11)

2. Two-stage least squares, weighted by $1/\Delta V_i$:

$$\widehat{Y} = -0.52 + 0.086\widehat{\Delta V_1} + 0.009A + 0.18D + 0.26R_F$$
$$\phantom{\widehat{Y} = -0.52} (.0030) \phantom{0.086\widehat{\Delta V}} (.00078) \ (.039) \ (.0.41)$$

$$-0.02\Delta W_5 - 0.13\Delta W_6 - 0.21\Delta W_7 - 0.29\Delta W_8$$
$$(.050) (.044) (.045) (.051)$$

(16)

3. Two-stage least squares; weighted by 1, divided by occupant age:

$$\widehat{Y} = -0.66 + .078\widehat{\Delta V_1} + 0.016A + 0.26D + 0.29R_F$$
$$\phantom{\widehat{Y} = -0.66} (.0027) \phantom{.078\widehat{\Delta V}} (.0013) \ (.041) \ (.025)$$

$$-0.10\Delta W_5 - 0.09\Delta W_6 - 0.20\Delta W_7 - 0.27\Delta W_8$$
$$(.043) (.042) (.042) (.052)$$

(17)

(The numbers below the coefficients are the estimated coefficient standard errors.)

Examination of the coefficient standard errors indicates that they are numerically smaller under both of the weighting schemes. These results are in agreement with the assumed error structure. All three methods provide unbiased estimates if the model assumptions are true. The estimated coefficients are reasonably close numerically. Thus we conclude that the distribution structure of the errors is not having excessive influence on the overall models. We do observe some compensating differences between the coefficient of ΔV_1 and the model constant. This negative correlation between these two model coefficients is expected from regression theory.

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Another comparison of the models can be made by computing their predicted injury severity and their elasticity of injury severity with respect to weight. By assuming a 30-year-old driver in a car weighing 3,450 pounds and a ΔV_1 of 12, the expected injury from the three models are:[4]

	Expected Injury
Model I	0.74
Model II	0.79
Model III	0.87

As shown above, the elasticity of injury with respect to vehicle weight is, $e = \dfrac{W}{Y} \dfrac{\Delta Y}{\Delta W}$

where W is vehicle weight,
   Y is expected injury, and ΔW, ΔY are changes in
   vehicle weight and injury severity.

By assuming an average vehicle weight of 3,450 the elasticities from the three models are:

	Elasticity
Model I	−0.67
Model II	− .50
Model III	− .62

Thus, we have some indication of the potential range of the elasticity. It also should be noted that the expected injury and the elasticity from models I and III are much closer, compared to model II. Comparison of expected injury shows that models I and II are closer. Based upon this result and the pattern of the coefficients it is not possible to choose between the three models. However, any one of the three could be used without changing the conclusions of this study.

Crash Injury Models From RSEP Data

The basic structure of the models estimated using the RSEP data is:

$$\widehat{Y}_{ij} = \widehat{B}_{0ij} + \widehat{B}_{1ij} \Delta V_1 + \widehat{B}_{2ij} A + \sum_{K=3}^{5} \widehat{B}_{Kij} \Delta X_K \tag{18}$$

where,

Y_{ij} is the expected most severe AIS for crash configuration i and restraint usage j
  $(i = 1, \ldots 8; j = 1, \ldots 2)$,

$j = \dfrac{1 \text{ indicates restraint usage and}}{2 \text{ indicates no restraint usage,}}$

---

[4]The computation of expected injury and elasticity uses the procedure shown in detail in a previous section.

i   is the crash configuration as shown in table 1,

$\hat{B}_{Kij}$ are the coefficients estimated from the crash data,

ΔV_1 is the change in impact velocity in miles per hour,

A   is the occupant age in years and,

ΔX_K indicates the case vehicle size group according to the following groups.

$$\Delta X_3 = 1 \quad \text{Compact}$$
$$\Delta X_4 = 1 \quad \text{Intermediate}$$
$$\Delta X_5 = 1 \quad \text{Full size}$$
$$\Delta X_K = 0 \quad \text{Subcompact } (K = 3, 4, 5)$$

The specific vehicles in each size group are defined in Kahane and Mungenast (1977).

The overall injury prediction model was constructed using the same procedure as described above for the previous model. The injury measure used for this model was the most severe-occupant AIS. The most-severe AIS is expected to be smaller numerically than the overall AIS used from the CPIR data for an occupant with similar injuries. The reason for this difference is that the overall AIS may include the combined result of several different injuries. Another difference between the two data files is the method used to obtain a value for ΔV. In the RSEP file, ΔV was estimated from the vehicle damage data using the CRASH program.[5] Finally the RSEP file contains substantially fewer high severity crashes and therefore coefficient estimates will have larger errors. For these reasons it was not expected that the estimated coefficients would be the same as these obtained using the CPIR. This expectation proved to be correct. The coefficients for ΔV are somewhat smaller in this model compared to the first. However, ΔV was the strongest predictor of injury severity in this model.

The final overall model, which assumes 20 percent restraint utilization, is,

$$\hat{Y} = 0.18 + 0.039\Delta V + 0.0045A - 0.07\Delta X_3 - 0.09\Delta X_4 - 0.07\Delta X_5 \tag{19}$$
$$\quad\quad\quad (32.5) \quad\quad (7.5) \quad\quad (-2.6) \quad\quad (-3.3) \quad\quad (-2.7)$$

(the numbers below the coefficients are the coefficient student t statistics.)

This model indicates that the only protective effect of vehicle weight occurs between subcompacts and all other cars. Specifically, subcompacts have an expected most-severe injury that is 0.07 greater than the injury for compacts and full-sized automobiles and 0.09 greater than the injury for intermediate-sized automobiles. This result is considerably different from that obtained using the CPIR data. We can use equation 19 to compute the expected value of the most-severe injury for a specific crash assumption. For example, assuming a 30-year-old driver in an intermediate vehicle with a ΔV of 12, the expected most-severe injury is 0.70. This is in contrast to an expected overall AIS of 0.74 to 0.87 for the same assumptions by using the model from

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[5]Specifically, the variable Energy Equivalent Velocity was used. This variable was found to be a better predictor of injury than the Barrier Equivalent Velocity, which was also computed.

the CPIR data. Recall that we expect overall AIS to be larger than maximum AIS for a given injured occupant. In addition, the computed ΔV in the two data files may be different because of the different methods used for computation.

As a final comparison, the elasticity of injury with respect to vehicle weight was computed for the two models under the same conditions used to compute the expected injuries discussed above. The elasticity for the RSEP model is $-.32$ in contrast to an elasticity of $-.50$ to $-.67$ for the CPIR model. I believe that this large difference indicates a fundamental problem with the data used to estimate the coefficients of the RSEP model. The RSEP file does not contain a large enough fraction of severe injuries to provide good estimates of the coefficients. In contrast, the CPIR file has a bias toward higher-severity crashes. Regression theory indicates that coefficient estimates have smaller variance if the observations are distributed uniformly over the range of the independent variables. As the RSEP file is a representative sample, it contains many low-injury observations and fewer high-injury observations. Based on regression theory I believe that this has led to an estimated elasticity that is too small from the RSEP file.

As a final analysis of the RSEP injury-prediction model, consider the submodels for restrained and unrestrained drivers.

1. Restrained drivers

$$\hat{Y} = 0.12 + 0.031\Delta V - 0.0002A + 0.0\Delta X_3 - 0.04\Delta X_4 + 0.03\Delta X_5$$
$$\quad (18.2) \qquad (-.2) \qquad (0.0) \qquad (-1.3) \qquad (0.8)$$
(20)

2. Unrestrained drivers

$$\hat{Y} = 0.19 + 0.041\Delta V + 0.0057A - 0.09\Delta X_3 - 0.11\Delta X_4 - .10\Delta X_5$$
$$\quad (27.3) \qquad (7.9) \qquad (-2.8) \qquad (-3.4) \qquad (-3.1)$$
(21)

The numbers below the coefficients are the coefficient student t statistics. By assuming a 30-year-old driver in an intermediate car, the comparable models for expected most-severe injury are,

1. Restrained drivers

$$\hat{Y} = 0.08 + 0.031\Delta V$$

2. Unrestrained drivers

$$\hat{Y} = .25 + 0.041\Delta V$$

For example, with ΔV equal to 12, the expected most-severe injury for restrained drivers is 0.45 and for unrestrained drivers, 0.74. This difference of 0.29 $(0.74 - 0.45)$ indicates the injury reduction from using restraints. The model also indicates that the injury reduction benefit increases with increasing ΔV.

Economic Effect of Vehicle Weight Change

The elasticity developed in this study was used to obtain crash-injury cost as a function of average vehicle weight. First, we computed the average injury cost per million miles driven. Next, the estimated elasticity was used to determine the relationship between injury cost and average vehicle weight. A similar computation was performed for fuel cost. From these two results the trade off was expressed as a cost difference per million miles of travel as a function of change in average vehicle weight.

A recent study (Dutt and Reinfurt 1977) presented driver crash injury rates per million miles traveled in North Carolina. These are:

Fatal crashes	0.024 per million miles
Serious injury crashes	.116 per million miles
Minor injury crashes	.670 per million miles

Standard costs have been developed for use in cost-benefit studies and for safety program decisions by NHTSA managers. These costs are:

Fatal crashes	$287,715
Serious injury crashes	$ 30,336
Minor injury crashes	$ 2,463

Therefore, the expected driver-injury cost per million vehicle miles traveled is:

$$(0.024) \times (\$287,715) + (0.116) \times (\$30,336) + (0.670) \times (\$2,463) = \$12,074.$$

From an analysis of all crashes in the CPIR file it was determined that there are 1.55 occupant-plus-driver injuries per driver injury. Therefore, the total expected crash-injury loss per million miles traveled is:

$$(1.55) \times \$12,074) = \$18,700.$$

The fuel cost analysis was based upon results presented by McGillivary (1976). In that study the marginal cost of fuel as a function of vehicle weight change is $0.001 per mile per 100 pounds. This result assumes an average price of $0.55 per gallon and an average consumption of 1/14 gallons per mile (14 miles per gallon). The elasticity, e_f, of fuel cost with respect to vehicle weight is,

$$e_f = \frac{\Delta C/C}{\Delta W/W} = \frac{\Delta C}{\Delta W} \cdot \frac{W}{C}$$

where ΔC is the change in fuel cost per 100 pound change in vehicle weight per mile,

C is the average fuel cost per mile, and

W is the average vehicle weight in pounds.

By using the previously stated values, we found that,

$$e_f = \frac{.001}{100} \cdot \frac{3450}{.55/14} = .86 .$$

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The elasticity of injury with respect to vehicle weight was $e_c = -.67$, from Model I. Thus fuel costs are influenced much more by changes in vehicle weight than are occupant crash injuries. To provide a direct cost comparison we first computed the fuel cost per million miles as,

$$(.55/14) \times 10^6 = \$39,285$$

Observe that the fuel costs are 110% higher than injury costs per mile traveled

$$\left((39,285 - 18,700) / (18,700) \times (100) \right).$$

Using the above elasticities and average costs, we found that a 1% decrease in vehicle weight will increase injury cost by,

$$(-0.67) \times (-0.01) \times (18,700) = \$125,$$

per million miles of vehicle travel. Similarly, fuel costs will change by,

$$(0.86) \times (-0.01) \times (39,285) = \$338.$$

Thus, the net benefit of a 1 percent change in average vehicle weight is \$213 per million miles of vehicle travel. Our previous analysis also indicated that with an extreme set of assumptions the elasticity of injury with respect to vehicle weight could be as large as -0.86. Given these extreme assumptions, the net benefit of a 1% change in vehicle weight would be,

$$\$338 - (0.86) \times (0.01) \times (18,700) = \$177.$$

Therefore, the positive benefits of vehicle size decrease are robust with respect to assumptions concerning the vehicle population.

Finally, by using an estimate of $1,391 \times 10^9$ total[6] vehicle miles per year, the total annual cost savings for each 1 percent change in vehicle weight are \$296 million. Based upon this analysis, we conclude that encouragement of vehicle weight reduction by the Department of Transportation has a positive cost benefit. The modeling effort presented in this paper assumed that the present relationship between vehicle weight and injury would remain as vehicle weight is reduced. However, manufacturers have already undertaken programs to reduce vehicle weight while maintaining the interior volume for passengers. These programs have been undertaken for marketing reasons. However, it is anticipated that such changes will reduce the absolute value of elasticity of injury with respect to vehicle weight. Thus the benefits of future weight reduction are expected to be even higher than indicated by this study. In addition, benefits from any other improved vehicle design that reduces crash injury have not been included.

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[6]Highway Statistics 1976, Federal Highway Administration.

APPENDIX A: DETERMINATION OF ΔV FOR TWO-VEHICLE CRASHES USING DATA IN CPIR FILE

It is well known that the change in velocity, ΔV, of the center of gravity for a crashing vehicle is directly related to the force absorbed by an occupant of a crashing vehicle. By definition

$$\Delta V = \int_0^T \alpha(t) \, dt$$

where   t   is the time from start of crash,

         $\alpha(t)$ is the deceleration of the vehicle center of gravity at time t and

         T   is the time duration of the crash.

$$\bar{\alpha} = \frac{\Delta V}{T}$$

Vehicle occupants receive forces that are monotonically related to deceleration. The mechanical properties of a restraint system will mitigate this force. Similarly the force imposed on an unrestrained occupant will be somewhat mitigated if he strikes an interior surface before the crash cycle is completed.

This of course assumes an idealized case where crash forces pass through the center of mass for both vehicles, and thus there are no rotational forces.

Based upon the above argument it was hypothesized that ΔV would be a good predictor of occupant injury. Of course occupant injury also depends upon occupant dynamics, which does not directly correspond to vehicle dynamics, especially for unrestrained occupants. Thus, there will be an error term associated with the measurement of occupant ΔV. If that error is random it is possible to use Two-Stage Least Squares multiple regression to obtain an estimate of the expected value of injury as a function of ΔV.

As ΔV is a potentially important variable for injury prediction, we need a method for computing ΔV from available crash data. The method used for this study utilized the reported impact velocities and clock direction of vehicle damage vectors as reported by the field investigators who collected the CPIR data.

The x and y directional components of ΔV are given by,[7]

$$\Delta V_1^x = \frac{M_2}{M_1 + M_2} (V_1 + V_2 \, \cos \alpha)$$

$$\Delta V_1^y = \frac{M_2}{M_1 + M_2} (V_2 \, \sin \alpha)$$

where

         ΔV_1^y , ΔV_1^x are the components of V_1

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colinear with the direction of the case vehicle (i.e., vehicle 1); ΔV_1^y is the component of ΔV_1 that is perpendicular to the direction of the case vehicle; M_1, M_2 are the masses of vehicle 1 and vehicle 2 repectively; V_1, V_2 are the impact velocities of vehicle 1 and vehicle 2, as reported by the field investigators', and α is the direction of the resultant velocity vector with respect to the case vehicle. By referring to figure A-1 which is an idealized schematic of a crash, it can be seen that,

$$\alpha = \theta_1 - \theta_2 \tag{3}$$

where

θ_1 is the clock direction of the principle impact for vehicle 1, and

θ_2 is the clock direction of the principle impact for vehicle 2.

Equations 1 and 2 can be used to determine ΔV_1, by applying the basic principles of vector addition,

$$\Delta V_1 = \sqrt{(\Delta V_1^x)^2 + (\Delta V_2^y)^2} \tag{4}$$

Equation 4 becomes

$$\Delta V_1 = \frac{M_2}{M_1 + M_2} \sqrt{V_1^2 + V_2^2 + 2V_1 V_2 \cos(\alpha)} \tag{5}$$

by substitution of equations 1 and 2 followed by appropriate mathematical manipulations.

Equation 5 was used to compute ΔV_1 for each case in the CPIR file.

The ΔV_1 computed in the above manner is of course an approximation based upon the reported field observations. An important shortcoming occurs in the use of θ_1, and θ_2 to determine α. Impact directions, θ_1 and θ_2, are reported as clock directions and are thus rounded to the nearest $30°$. Thus, for example, if the field investigator has determined that the impact direction is, $\theta_1^* = 14°$, it would be reported as 12 o'clock or $0°$. However, if the impact direction has been determined to be, $\theta_1^* = 16°$, it would be reported as 1 o'clock or $30°$. Thus, a small error in measurement by a field investigator could lead to a large difference in the reported value of θ_1. If compensating errors are made in the determination of θ_1 and θ_2 then the computed angle, α, is the same. In that case the only problem is that of rounding. These errors are assumed to be random, and thus they are assumed to be part of the random error in the measurement of ΔV. Finally, consider the sensitivity of ΔV_1 to changes in α as shown in the following table. These results are based upon the assumption that $M_1 = M_2$ and that $V_1 = V_2 = 30$.

α	$\cos \alpha$	ΔV_1
0	1	30
$30°$	0.866	28.98
$60°$	0.50	25.98
$90°$	0	21.22
$120°$	−0.50	15.00
$150°$	−0.866	7.77
$180°$	−1.00	0

In the above analysis it can be seen that errors in α would have a lesser effect on ΔV_1 in frontal-type crashes compared to rearward-type crashes. Since frontal crashes have higher ΔV_1, the effect of errors in measuring θ_1 and θ_2 would have a greater influence on injury estimation for less severe crashes.
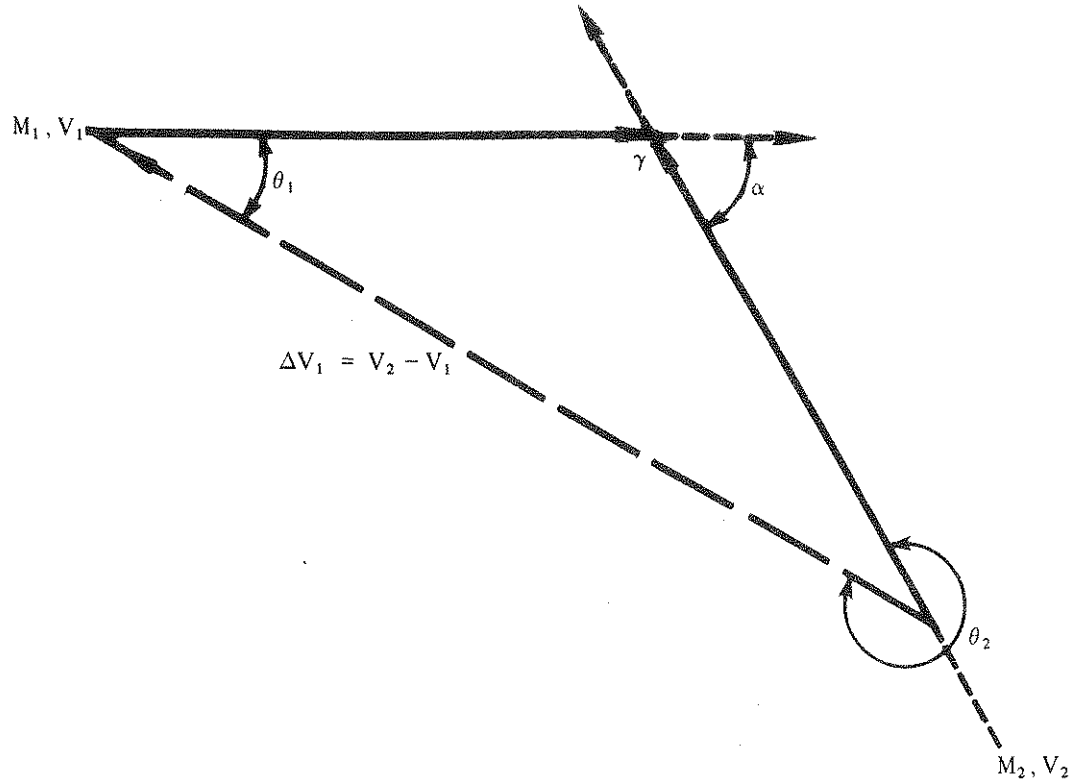
At the time of impact:

Vehicle 1 is traveling at Velocity V_1 and Vehicle 2 is traveling at Velocity V_2 .

The principle damage direction for Vehicle 2 is θ_1 and for Vehicle 2 it is θ_2 . There the resultant ΔV is as indicated. To determine ΔV we use α where,

$$\alpha = \pi - \gamma$$

$$\gamma = \pi - (\theta_1) - (2\pi - \theta_2)$$

$$\alpha = \theta_1 - \theta_2 + 2\pi = \theta_1 - \theta_2$$

$$\Delta V_1^x = \frac{M_2}{M_1 + M_2} (V_1 + V_2 \ \text{Cos} \ \alpha)$$

$$\Delta V_1^y = \frac{M_2}{M_1 + M_2} (V_2 \ \text{Sin} \ \alpha)$$

Figure A-1: Schematic Diagram of Typical Impact Velocity Vectors

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