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INTRODUCTION

The EDCRASH computer program is used to estimate the impact speed of one or two vehicles involved in a crash. The calculation procedures are relatively simple and straight-forward and can be accomplished with a hand calculator. An overview of these procedures is provided by the flow chart (figure 1) and in the following paragraphs.

Figure 1 - Flow chart of EDCRASH processing phase
Scope of Analysis

Two independent phases can be analyzed. At a minimum, the impact phase is analyzed. The results of this analysis, called the damage analysis, use measurements of vehicle damage to estimate the change in velocity, delta-V, for each vehicle. At a maximum, if accident site data is supplied, then, in addition to the impact phase, the impact-to-rest phase is analyzed. Accident site data is necessary if an estimate for impact speed is desired because the separation velocity must be known.
If accident site data is supplied, the delta-V can be calculated from linear momentum, without using damage data. This trajectory-based approach works well for oblique (intersection and other non-head-on) collisions (figure 2). However, if the pre-impact velocity directions are within +/- 10 degrees of a collinear (head-on) configuration at impact (figure 3), the momentum calculation becomes very sensitive to the measurement of this angle. The difficulty arises from a lack of precise knowledge of the exit angle. Therefore, the damage-based delta-V is automatically used in this case, as observed in the EDCRASH output under the heading "Basis of Results" (see figure 4).

Figure 3 - Typical collinear collisions
Introduction

SUMMARY OF EDCRASH RESULTS

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85 Ford P/U

WARNING MESSAGES: NO MESSAGES

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Figure 4 - EDCRASH output. Notice the basis of the results (damage or momentum) is displayed. Zero velocity indicates that method was not used.

As shown in the flow chart (figure 1), there are five major calculation procedures which occur during the EDCRASH processing phase. These are:

- DAMAGE
- SEPARATION VELOCITIES
- COMMON VELOCITY CHECK
- TRAJECTORY SIMULATION
- OBLIQUE IMPACT

Each of these procedures will be discussed briefly. Then, the two major procedures, DAMAGE and SEPARATION VELOCITIES, will be described in detail.
Figure 5 - EDCRASH damage-only results

Since the mid-seventies, information on the structural stiffness of vehicles has been gained from barrier testing. This information allows the analysis of delta-V based only on vehicle damage (Figure 5). Most crash tests were head-on. Therefore, the damage-based analysis is best-suited to collinear (head-on) impacts. The damage-based and trajectory-based analyses nicely complement each other. However, the damage-based results also have limitations. The major limitation stems from a lack of structural stiffness data for all vehicles, especially the newer front-drive cars. (This information is expected to be supplied soon.) Another limitation is the assumption of a uniform crush stiffness along the entire side, or end, of the vehicle. It is obvious that the wheel region will offer more resistance to crush forces than a quarter-
Introduction

panel region. A frontal or rear impact, which uses the bumper to distribute the impact force, provides a more uniform crush resistance - another reason the damage-based analysis is best-suited to collinear impacts. A third limitation is tied to the lack of information describing the crush at various elevations. The most frequent example of this occurs as a result of "bumper override", wherein the stiff bumper and frame portions of one vehicle strike above the bumper and frame of the other vehicle. The crush data used by EDCRASH assume the higher stiffness associated with the bumper and frame. When these structures are not crushed, but the softer sheet metal above them is, the crush is greater than expected and the delta-V is over-estimated.

SEPARATION VELOCITIES

The separation, or post-impact, velocity of each vehicle is calculated based on the user's accident site data (impact and rest positions, impact-to-rest path, and tire-road resistances). The procedure is very flexible, allowing and accounting for the effects of a curved path, rotation of the vehicle while skidding, and rollout after skidding.

COMMON VELOCITY CHECK

Before impact speeds are computed, EDCRASH checks the separation velocity of each vehicle to confirm they are compatible with each other. This confirmation is called the common velocity check. It is performed by computing the earth-fixed velocity at the damage centroid of each vehicle (fig. 6). It is necessary because the analysis of the impact phase is based on the common velocity assumption; that is, at the moment of separation, the region of the vehicles in contact with each other have mutually engaged so that damage centroids have earth-fixed velocities (speed and direction) which are nearly equal.
Figure 6 – Common Velocity Check. The velocity of each vehicle's damage centroid is determined relative to the earth-fixed coordinate system.

Because the calculation for each vehicle is independent from the other, it would only be a coincidence if the velocities of the damage centroids were identical. However, if the scene data is correct, the velocities should be roughly equal. If the velocities are not equal, but are within 10 percent of each other, the common velocity assumption is
satisfied. If not, then the velocity of the slower vehicle is increased by 10 percent, the velocity of the faster vehicle is reduced by 10 percent, and the common velocity check is again made. If the resulting velocities are still not within 10 percent of each other, EDCRASH aborts and issues a common velocity error. If the velocities are within 10 percent of each other, a message is issued to notify the user that an adjustment has been made and calculations proceed.

![Diagram of Trajectory Simulation](image)

**Figure 7 - Trajectory Simulation.** The simulated path (1) is compared to the measured path data (2).

**TRAJECTORY SIMULATION**

EDCRASH includes the provision for simulating the impact-to-rest phase of the accident. This is called
a trajectory simulation. After the common velocity check confirms the separation linear and angular velocities of each vehicle are valid, these velocities are used, along with the impact position and heading, as the initial conditions for the simulation. If the separation velocities are correct, then the simulated vehicle should come to rest at the measured rest position.

When a trajectory simulation is complete, the simulated rest position and heading are compared to the measured rest position and heading (figure 7). An error term, based on the difference between simulated and measured results, is computed for each measured path position and heading. When the error terms are sufficiently small, the trajectory simulation is said to have "converged" on the proper separation linear and angular velocities. If any of the error terms is not sufficiently small, the separation linear and angular velocities are adjusted and the trajectory simulation is performed up to four more times (five tries in all) seeking to converge on acceptable values. If the trajectory simulation fails to converge after five tries, the velocities which produced the smallest error terms are used as the separation linear and angular velocities.

OBLIQUE IMPACT

After the separation velocity of each vehicle has been determined, the delta-V is added to the separation velocity in order to determine the impact velocity. The damage-based delta-V is known from prior calculations. As discussed previously, the conservation of linear momentum can also be used to determine the delta-V, and will be used to determine the impact velocity if the impact velocity vectors are more than +/- 10 degrees from collinear.

The conservation of linear momentum simply states the momentum of the system (both vehicles) at the begin-
Introduction

ning of the impact is equal to the system momentum at the end of the impact. The concept is shown graphically and mathematically in figure 8. This calculation is performed after the separation conditions are determined because the forward and lateral separation velocities must be known before the impact velocities can be determined.

Figure 8 - Conservation of linear momentum solution for oblique impacts.
CALCULATION DETAILS

The preceding section described the general procedures used by EDCRASH to estimate impact speed and delta-V. The foundation of the program is provided by the separation velocity and damage portions of the program. These procedures will now be described in detail.

Damage

The DAMAGE procedure estimates the change in longitudinal velocity ($\Delta V_x$) and lateral velocity ($\Delta V_y$) for each vehicle during the impact phase (figure 9).

![Diagram](image)

**Pre-Impact**  
**Post-Impact**

Figure 9 - Delta-$V_x$ (change in longitudinal velocity) and Delta-$V_y$ (change in lateral velocity)
The basis of the DAMAGE calculation procedure is Newtonian physics and the conservation of linear momentum, not the conservation of energy. The linear impulse shared between the vehicles is computed from the amount of crush sustained during the crash. From basic physics, it is known the delta-V of the vehicle (or any object) is equal to the impulse divided by its mass. Since the mass of a vehicle is known or easily found in tables, the goal of DAMAGE is really to determine the linear impulse.

**DETERMINATION OF THE LINEAR IMPULSE**

\[ \dot{\mathbf{b}} = \mathbf{b}_0 - \mathbf{b}_c \]

**Figure 10** - Central collision between two spring/mass particles
Referring to figure 10, which describes the collision between two particles having masses $M_1$ and $M_2$ attached to linear springs $K_1$ and $K_2$ along the X-axis, Newton's laws of motion apply as follows.

From Newton's 2nd Law ($\Sigma F = Ma$) applied to particle no. 1, the external force to cause a displacement in the linear spring from its initial length, $X_1 - X$, is

$$-K_1 (X_1 - X) = M_1 \frac{d^2X_1}{dt^2}$$

and for particle no. 2 is

$$-K_2 (X - X_2) = M_2 \frac{d^2X_2}{dt^2}$$

Letting $\delta = X_1 - X_2$, these equations can be written in the form

$$\frac{d^2\delta}{dt^2} + \frac{K_1K_2}{(K_1 + K_2)} \frac{M_1 + M_2}{M_1 M_2} \delta = 0$$

The above equation is the differential equation for an undamped simple harmonic oscillator having the general solution $\delta = A \sin(\omega t)$, where

- $A$ = the amplitude of the oscillation
- $\omega$ = the circular frequency of the oscillation
  $$\omega = \sqrt{\frac{K_1K_2}{(K_1 + K_2)} \frac{M_1 + M_2}{M_1 M_2}}$$
- $t$ = period of the oscillation
  $$t = \frac{2\pi}{\omega}$$

The period of the oscillation, $t$, is the entire period associated with loading and unloading the mass. Since no tension is allowed between the masses (vehicles) while unloading (separating), the force is assumed to disappear after $\delta$ reaches the maximum amplitude or maximum engagement. Figure 11 shows an example of the actual time history which is used.

To apply this solution to the case of vehicle collisions, observe the initial conditions are known. The rate of spring deflection at the beginning of contact is simply equal to the closing velocity.
between the particles,

\[ \frac{d\delta}{dt}_{t=0} = V_{10} - V_{20} \]

where \( V_{10} \) and \( V_{20} \) are the initial particle velocities. From this observation, the value of the amplitude is

\[ A = (V_{10} - V_{20}) \sqrt{(M_1 M_2 / (M_1 + M_2)) (K_1 + K_2) / (K_1 K_2)} \]

The maximum amplitude is reached when \( M_1 \) and \( M_2 \) reach a common velocity. This collision occurs according to the conservation of linear momentum,

\[ (M_1 + M_2) V_{\text{com}} = M_1 V_{10} + M_2 V_{20} \]

Each spring deflects independently from the other. The total deflection of both springs, \( \delta_{\text{max}} \), is related to the deflection of each spring by the ratio of the spring constants, \( K \). Mathematically,

\[ \delta_1 = \left( \frac{K_2}{(K_1 + K_2)} \right) \delta_{\text{max}} \]

and

\[ \delta_2 = \left( \frac{K_1}{(K_1 + K_2)} \right) \delta_{\text{max}} \]

Figure 11 - Period of oscillation. The entire period ends at (1). However, the collision is modeled to end at (2), after maximum penetration (crush or deflection) is reached.
Since \( \delta_{\text{max}} = \delta_1 + \delta_2 \), the above equations may be combined as

\[
K_1 \delta_1^2/2 + K_2 \delta_2^2/2 = K_1 k_2 \delta_{\text{max}}^2/2(K_1 + K_2)^2 + K_2 k_1 \delta_{\text{max}}^2/2(K_1 + K_2)^2
\]

and by rearranging, we get

\[
(K_1 k_2/2(K_1 + K_2))\delta_{\text{max}}^2 = k_1 \delta_1^2/2 + k_2 \delta_2^2/2
\]

Here is a critical observation: \( k_1 \delta_1^2/2 \) and \( k_2 \delta_2^2/2 \) are the energies absorbed by springs \( k_1 \) and \( k_2 \), respectively, at the moment of common velocity, \( v_{\text{com}} \). Defining these energies as \( E_1 \) and \( E_2 \), we can finally write the linear impulse as

\[
I = \sqrt{2(E_1 + E_2)(M_1 M_2/(M_1 + M_2))}
\]

and, since \( I = M_1 \text{delta-}v_1 = M_2 \text{delta-}v_2 \), the speed changes for each vehicle are

\[
\text{delta-}v_1 = v_{10} - v_{\text{com}} = \sqrt{2(E_1 + E_2)(M_1 M_2/(M_1 + M_2))}/M_1
\]

and

\[
\text{delta-}v_2 = v_{\text{com}} - v_{20} = \sqrt{2(E_1 + E_2)(M_1 M_2/(M_1 + M_2))}/M_2
\]

This derivation is based on the conservation of linear momentum and Newton's laws of motion. Although the spring deflections are expressed by their stored energies, the above development is not based on, or related to, the conservation of energy. Scrutinizing the results of an EDCRASH damage-based run will, in fact, show that mathematically, energy is not generally conserved.
NON-CENTRAL IMPACT

The model developed in the preceding section was based on the assumption of a central impact, i.e., the motion of both particles was along a straight line (the X-axis); the line of action (impulse) was also along this straight line, through the center of mass of each particle or vehicle (see figure 12).

Figure 12 - Central vs. non-central collisions
EXCRASH Training Manual

Since this is rarely the case when two vehicles collide, a correction factor, $\Psi$, is used to account for a non-central impact. This correction factor is described below.

A vehicle has a rotational (yaw) moment of inertia, $I_{zz}$, which defines its resistance to changes in angular rotation rate. The radius of gyration of the vehicle having mass, $M$, is $K_{gy} = \sqrt{I_{zz}/M}$.

The radius of gyration is the radius of a hoop having the same mass and moment of inertia as the vehicle. For a non-central collision where the impulse acts a distance, $h$, from the center of mass, the effective mass of the vehicle is reduced by the factor $\Psi$, where

$$\Psi = \frac{k^2}{g_y} \left( \frac{k^2}{g_y} + h^2 \right)$$

If the collision impulse does not act through the center of mass, the vehicle will tend to rotate, reducing its delta-$V$ as follows

$$\begin{align*}
\text{delta}-V_1 &= V_{10} - V_{\text{com}} \\
&= \sqrt{2 \left( E_1 + E_2 \frac{V_1 M_1 V_2 M_2}{(V_1 M_1 + V_2 M_2)} \right) / V_1 M_1}
\end{align*}$$

and

$$\begin{align*}
\text{delta}-V_2 &= V_{\text{com}} - V_{20} \\
&= \sqrt{2 \left( E_1 + E_2 \frac{V_1 M_1 V_2 M_2}{(V_1 M_1 + V_2 M_2)} \right) / V_2 M_2}
\end{align*}$$

Note that $h = 0$ and, therefore, $\Psi = 1.0$ for central collisions.

PRINCIPAL DIRECTION OF FORCE

The impulse acts on the vehicle at the Principal Direction of Force, PDOF. Therefore, the delta-$V$ has longitudinal and lateral components related to the PDOF by

$$\text{delta}-V_{\text{long}} = \text{delta}-V \cos(\phi)$$

Page 24
and

\[ \delta V_{lat} = \delta V \sin(\phi). \]

This is the equivalent to observing the \( \delta V \) occur in the direction of principal force (figure 13). This fact is also illustrated in figure 8.

![Figure 13 - Principal Direction of Force (PDOF) and relationship to Delta-V](image)

**Determination of Damage Energy**

As was discovered previously, the linear impulse can be determined if the energy stored in the spring deflection can be determined. This energy is the same as the crush energy. The process used by EDCRASH to model the crush energy follows.
The model is based on the assumption that the exterior surface of the vehicle resists inward crush (displacement) like a linear spring. The outer body of the vehicle can be thought of as being surrounded by such springs, as described by figure 14. Each spring has a linear spring constant, B, which has a different value for the front end, rear end and sides (refer to the EDCRASH Program Manual, Table 4). B has the units of lb/inch per inch of damage width, or simply lb/in². Note the free length of these springs actually extends out beyond the outer surface of the vehicle. This allows the model to account for impact energy which does not damage the vehicle. Constant, A, is used to account for this effect. A (units, lb/in) is the preload force per inch of damage width required to deflect the spring an amount equal to the free length, A/B.

The width and depth of crush determines the damage energy. The damage energy is determined by the spring deflections. Therefore, it is necessary to describe the spring deflections. This is done using the vehicle crush profile as shown in figure 15.

The damage profile has a total width, W. Each increment of the damage width, dW, has a measured crush depth, C, and a spring deflection, δ = C + A/B. In order to determine the amount of damage energy associated with the total damage profile, the spring deflections are integrated over the total damage width. For any linear spring, the energy stored in the spring due to deflection, x, is $E = \int \frac{1}{2}kx^2dx$. When this formula is applied to our vehicle model,

$$E = \int \left( \frac{B}{2} \left( \frac{A}{B} + C \right) \right)^2 dW$$

$$= \int \left( \frac{A^2}{2B} + AC + \frac{C^2B}{2} \right) dW$$

$$= \int \left( \frac{A^2}{2B} \right) dW + \int AC dW + \int \left( \frac{B}{2} \right) C dW$$

Integrating over the damage width yields

$$E = \left( \frac{A^2}{2B} \right) W + ACW + \left( \frac{B}{2} \right) C W$$

$$= \left( \frac{A^2}{2B} \right) W + \left( A + B\delta \right) \text{Area}$$
Figure 14 - Vehicle damage model

In addition to the constants, A and B, the above solution is seen to depend upon the width of the damage profile, the area (in plan view) of the damage, and the distance, $\hat{x}$, from the centroid of the damage area to the exterior surface of the vehicle.

During a non-central collision, the force of impact does not act through the center of mass. Therefore, non-central collisions always produce rotation of the vehicle. For non-central collisions, the centroids of the damage regions are assumed to reach a common velocity prior to separation (refer to the earlier discussion of COMMON VELOCITY CHECK). In order to account for non-central collision (see figure 16), the distance from the line of action of the impulse to the centroid must be determined. Therefore, it is necessary to specify the location of the damage profile relative to the center of mass. $\hat{D}$ is the specified distance from the center of the damage profile to the vehicle centerline. Based on this distance, the distance, $\hat{y}$, from the damage centroid to the vehicle centerline can be computed. Thus, the distance from the line of action of the impulse to the center of mass is

$$h = (X - \hat{x})\sin\theta - \hat{y}\cos\theta$$
NON-PERPENDICULAR CRUSH

A last correction factor is required before our model is complete. This correction factor accounts for collision forces which are applied at a non-perpendicular angle to the surface of the vehicle.

Crush is always measured perpendicular to the undeformed surface. When the collision force acts at an angle, \( \theta \), the distance through which the force acts is greater by a factor equal to \( 1/\cos \theta \) than the measured crush. Since the terms involving crush are
squared (refer to the crush energy equation developed previously), the correction must take on the form
$1 / \cos^2 \theta = 1 + \tan^2 \theta$. Therefore, the computed
ergies are multiplied by a factor of $1 + \tan^2 \theta$.

$$E = [(A^2/2B)W + (A + BE)Area](1 + \tan^2 \theta)$$

This factor has a significant influence on the
damage-based energy. This is demonstrated by
observing that when $\theta = 45$ degrees, $1 + \tan^2 \theta = 2.0$,
effectively doubling the estimated damage energy.
This factor is not allowed to exceed 2.0

Figure 16 - Non-central Collision
**Separation Velocities**

The objective of the impact-to-rest phase analysis is to determine the velocities of each vehicle at separation. The velocity vector at separation is resolved into longitudinal and lateral components with respect to the vehicle heading angle (figure 17).

![Diagram](image)

- $v_s \equiv$ SEPARATION VELOCITY
- $U_{vel} \equiv$ FORWARD (LONG.) SEPARATION VELOCITY
- $V_{vel} \equiv$ LATERAL SEPARATION VELOCITY
- $V_h \equiv$ SEPARATION HEADING ANGLE (also $\psi_s$)

Figure 17 - Longitudinal and lateral separation velocities
Separation Velocities

To determine separation velocities requires information obtained from an accident site inspection. The minimum information required by this analysis is the positions and heading at impact and rest, the direction of vehicular rotation, the tire-ground friction coefficient, and the deceleration or rolling resistance at each wheel. Additional information may be supplied in order to refine the results. This information may include a pre-impact slip angle, rotational skidding, the position and heading at an end of rotation which is followed by rollout, and an intermediate position between impact and rest which defines a curved path.

Based on the information supplied about the impact-to-rest phase, EDCRASH will determine the linear and angular velocities at the instant of separation. The linear velocity vector has a direction at separation which depends on the amount of information. At a minimum, the direction will be defined by the straight line between the impact and rest positions (figure 18). For curved paths, the direction may be defined by the tangent to the circular arc which passes through the position at impact, an intermediate point on the curved path, and the rest position (figure 19). Finally, the direction may be defined by a straight or curved path between impact and the end of rotation, if one is supplied (figure 20).

PATH DESCRIPTIONS

The manner in which each of these paths and separation angles is determined will now be developed.

Straight Path

The geometry of a straight path (figure 18) is determined from simple geometry. The length of the path is the distance between the two points which define the impact and rest positions. The separation angle
is the arctangent of the Y component of the path length divided by the X component of the path length.

\[
y = \arctan\left(\frac{Y_{\text{rest}} - Y_{\text{impact}}}{X_{\text{rest}} - X_{\text{impact}}}\right)
\]

Figure 18 - Straight path between impact and rest

Curved Path

The geometry of a curved path (figure 19) is also determined geometrically. The path is defined as a portion of a circular arc which begins at the separation position, goes through a user-entered intermediate point on the curved path, and ends at
the rest position (or end of rotation - see below). The process is simply one of determining the length of the arc.

\[ v_y = v_f \]

Figure 19 - Curved path between impact and rest
The first step in the process is to draw a chord, D, from the separation position to the rest or end of rotation position and determine its length. Then, the perpendicular distance, d', from the chord to the point on curve is determined. If this distance is less than one inch, the path is assumed to be straight. If not, the approximate radius is computed by assuming the perpendicular distance is the maximum radial offset. (This radius is approximate because the distance, d', will only be the maximum radial offset if the intermediate point is selected half way along the arc.) If the approximate radius is greater than 35000 inches (2917 feet), then the path is assumed to be straight.

If the path is found to be curved, then the next step is to compute its actual radius. This is done by finding the coordinates of the center of the arc. These coordinates can be found by finding the intersection of the perpendicular bisectors of the two chords: one which runs from the impact position to the intermediate point and the other which runs from the intermediate point to the rest or end of rotation position.

Finally, the arc length is computed from the calculated radius and the angular interval from $\gamma_1$ to $\gamma_2$. The separation angle is simply $\gamma_1$.

End Of Rotation

The geometry of a path which includes rollout following an end of rotation (figure 20) is determined by the adding together of two path segments computed in the manner described above. The straight rollout portion is always computed as above using the end of rotation and rest position. If the path between separation and end of rotation is also straight, the straight skidding portion of the path will be handled in the same way. If the path is curved, as indicated by a user-entered intermediate point on the curved
Separation Velocities

path, then the calculations for determining the curved arc length and separation angle are used.

At this point, the length of the path or path segments and the separation angle have been determined for each vehicle. The next step is to determine the motion of the vehicle over this path.

Figure 20 - Impact-to-rest path including an end of rotation
MOTION-RESISTING FORCES

Motion-resisting forces are forces which slow (decelerate) the vehicle during the impact-to-rest phase. EDCRASH assumes all motion-resisting forces occur at the wheels in the form of rolling resistance. Aerodynamic drag is ignored.

Two methods of modeling motion-resisting forces are available. The first (and recommended) method is to enter the fraction of lockup at each individual wheel, thus requiring four entries per vehicle (figure 21). The second method is to enter a single value equal to the average vehicle deceleration (figure 22).

If the individual wheel option is selected, a value of 0.0 is the equivalent to free-wheeling. A value of 1.0 is equivalent to locked-wheel braking. Note that a rolling tire always has rolling resistance. Note, also, that rolling resistance effectively increases when impact-damaged metal is in contact with a tire. Deceleration is based on the average wheel lockup. During a trajectory simulation, however, the individual wheel lockups determine the force at each wheel.

If the average deceleration option is selected, the deceleration value must be less than or equal to the coefficient of friction. Also, the trajectory simulation option is not allowed.
Separation Velocities

\[ \theta = \frac{\Sigma R}{4} \]

Figure 21 - Individual wheel lock-up method of determining motion-resisting forces

\[ \theta = \frac{\text{DECEL}}{\mu} \]

Figure 22 - Average deceleration method of determining motion-resisting forces
TYPES OF MOTION

There are three types of motion the vehicle may take along its path: (1) rollout, (2) non-spin skid, and (3) rotating skid.

\[ V = \sqrt{2g\theta \mu S} \quad \text{(in/sec)} \]

where
- \( g \) = acceleration of gravity
- \( = 386.4 \text{ (in/sec}^2) \)
- \( \theta \) = average wheel lock-up
- \( \mu \) = friction coefficient
- \( S \) = path length (in)

Figure 23 - Rollout motion between impact and rest

Rollout

Rollout is the term used to describe a vehicle which is tracking perfectly, that is, its rear tires are following the front tires. The technical definition of rollout is vehicle motion such that the velocity vector is pointed in the same direction as the heading vector - the sideslip angle, \( \beta \), is equal to zero (figure 23). Any vehicle motion with a zero sideslip angle is rollout.
Rollout may begin at the moment of separation. It may also begin at the end of a curved or rotating path.

Rollout does not mean free rolling; there may be braking during rollout. In fact, the wheels may be locked. When a vehicle decelerates during rollout, its initial velocity may be calculated by traditional energy means (as shown in figure 23) where the average wheel lockup is multiplied by the tire-road friction coefficient to determine the deceleration force.

Non-spin Skid

Rollout and non-spin skid are identical with one exception: The sideslip angle is non-zero. This means the vehicle is sliding sideways. As shown in figure 24, a non-spin skid may have a straight path or a curved path. Although the heading angle changes while traveling a curved path, the sideslip angle, $\beta$, remains constant.

When a vehicle decelerates during a non-spin skid, its initial velocity may be calculated by traditional energy means, just as in the case of rollout, except that the skid angle is included. The average wheel lockup, $RF_{avg}$ and constant sideslip angle, $\beta$, are combined with the tire-ground friction coefficient to produce the effective friction coefficient according to the Mouk-Burgett formula (as shown in figure 24) to determine the deceleration force.
V = \sqrt{2g_{eff}S} \quad \text{(in/sec)}

where
\begin{align*}
g & = \text{acceleration of gravity} \\
& = 386.4 \text{ (in/sec}^2) \\
\mu_{eff} & = \text{effective friction coef.} \\
& = \mu \sqrt{(\sin^2 \beta + \mu^2 \cos^2 \beta)} \\
S & = \text{path length (in)}
\end{align*}

Figure 24 - Non-spin skid motion between impact and rest

Rotating/Spinning Skid

A rotating/spinning skid is typically referred to as a "spin out", a condition described by vehicle rotation about its vertical axis. The path may be straight or curved. As shown in figure 25, a rotating/spinning skid is different from a non-spin skid because the sideslip angle, \( \beta \), changes along the path.
The separation velocity of a spinning vehicle cannot be calculated accurately by traditional energy means unless all wheels are locked. The difficulty is due to the fact that the deceleration force, which is constant for rollout and non-spin skid, is not constant for a rotating/spinning skid. The force varies as the sideslip angle varies.
The solution procedure was first developed by Marquard in 1966 (2). This procedure was refined by McHenry (3) for use in the CRASH program. Both of these developments are given below, followed by the calculations used by EDCRASH.

Consider the case of a spinning vehicle with freely rotating wheels (figures 25 and 27). Paying close attention to the directions of the velocity vector and heading vector, one notes there are times when the two vectors are pointed in the same direction and times when the two vectors are perpendicular to each other.

During the instant when the vehicle is moving in a forward direction (figure 26), the velocity vector is aligned with the heading vector ($\beta = 0$). The longitudinal force against the vehicle is small, so the linear velocity is reduced very little. However, since the vehicle is spinning, there are lateral forces at the front and rear tires. These forces are approximately equal in magnitude and are opposite in direction! As a result, during this instant, the amount of spinning (angular velocity) is being reduced at a maximum rate.

Now, consider the instant when the spinning vehicle is moving in a lateral (sideways) direction (figure 27). In this case, the velocity vector is perpendicular (90 degrees) to the direction of the heading vector (i.e., $\beta = 90$ degrees). The force against the direction of travel (which is now lateral) is at its peak, so the linear velocity is being reduced at the maximum rate. However, the angular velocity is no longer being reduced significantly because the forces at the front and rear wheels due to vehicle spinning are no longer opposite in direction. This critical observation is rather subtle and can be deduced by looking at figure 27. As long as the linear velocity is great enough to overcome the angular velocity, all wheels will be sliding in the same direction, so the force is in the same direction! As a result, there
Figure 26 - Instant of Rotating/spinning skid when velocity vector is aligned with heading vector

Figure 27 - Instant of Rotating/spinning skid when velocity vector is perpendicular to heading vector
is no motion-resisting torque applied to the vehicle and angular velocity is not reduced.

As shown in figure 28, the linear and angular velocities cycle between these two extreme conditions constantly as the vehicle spins until it either stops spinning or else comes to rest. Note that the two phases (linear and angular velocity reductions) occur independently. That is, the linear velocity decreases while the angular velocity remains constant and the angular velocity decreases while the linear velocity remains constant. The vehicle ultimately comes to rest. The total time to come to rest is equal to the sum of the time required to arrest the linear velocity and the time required to arrest the angular velocity.

Figure 28 - Linear and angular velocity histories
Figure 29 - Idealized velocity histories

The areas under the linear and angular velocity curves shown in figure 28 are equal to the linear and angular displacements, respectively. By assuming the areas can be approximated by triangles formed by straight lines (figure 29), the following relationships apply.

\[ \Delta \omega \equiv \left(\frac{\dot{\omega}_0}{2}\right) T_1 \]  \hspace{1cm} (1)
\[ S \equiv \left(\frac{\dot{s}_0}{2}\right) T_2 \]  \hspace{1cm} (2)

where

- \(\Delta \omega = \) total angular rotation
- \(S = \) total distance from impact to rest
- \(t_1 = \) time to arrest angular velocity
- \(t_2 = \) time to arrest linear velocity
During the time intervals of angular deceleration, the magnitude of deceleration is approximately

\[ \omega = \frac{\mu g}{k^2} \left( \frac{a+b}{2} \right) \]  

(3)

where

- \( \mu \) = nominal tire-ground friction coefficient
- \( g \) = acceleration of gravity
- \( k^2 \) = radius of gyration squared (in yaw)
- \( a + b \) = wheelbase

From equation (3), the actual time for angular deceleration is

\[ t_1 = \frac{\psi_s}{\omega} = \frac{2\psi_s k^2}{\mu g (a+b)} \]  

(4)

The linear deceleration force changes as the sideslip angle changes. If the average value of the cosine of the sideslip angle equals 0.85 during spinning, then the average linear deceleration is approximately

\[ \ddot{S} = 0.85 \mu g \]  

(5)

and the associated time for linear deceleration is

\[ t_2 = \frac{\dot{S}_S}{\ddot{S}} = \frac{\dot{S}_S}{0.85 \mu g} \]  

(6)

The total time required to come to rest is the sum of the time required to arrest the linear and angular velocities,

\[ T = t_1 + t_2 = \frac{2\psi_s k^2}{\mu g (a+b)} + \frac{\dot{S}_S}{0.85 \mu g} \]  

(7)

If both phases end at the same time, the from (1) and (2),

\[ \frac{2\Delta \psi}{\psi_s} \equiv \frac{2S}{\dot{S}_S} \equiv T \]  

(8)
Separation Velocities

Therefore,

$$\frac{\dot{S}_s}{\dot{u}_s} = \frac{S}{\Delta w}$$  \hspace{1cm} (9)

Substituting (8) and (9) into (7) yields

$$\frac{2\Delta w}{\dot{u}_s^2} = \frac{2k^2}{(a+b)\mu g} + \frac{S}{0.85\mu g \Delta w}$$  \hspace{1cm} (10)

and the solution for \( \dot{u}_s \) is

$$\dot{u}_s = \sqrt{\frac{\mu g \Delta w^2}{k^2} - \frac{|\Delta w| + S}{(a+b) \frac{\text{sgn}(\Delta w)}}}$$  \hspace{1cm} (11)

From (7) and (8),

$$\dot{S}_s = 1.7 \left( \frac{\mu g (\Delta w)}{\dot{u}_s} - \frac{k^2 |\dot{u}_s|}{(a+b)} \right)$$  \hspace{1cm} (12)

The above solution assumes a free-wheeled (no wheel lock-up) spinout. In order to account for braking or wheel damage resulting in partial or total wheel lock-up, the rolling resistance, \( RR \), at each wheel is used to compute the average wheel lock-up, \( \Theta \), where

$$\Theta = \frac{RR_{r/l} + RR_{l/l} + RR_{r/l} + RR_{l/l}}{4}.$$  \hspace{1cm} (13)

For free wheeling, \( \Theta = 0 \); for locked wheels, \( \Theta = 1 \). For partial braking or partial wheel lock-up, \( 0 < \Theta < 1 \).

As a result of this modification, the linear velocity at time \( t_1 \) is reduced to

$$\dot{S}_1 = \dot{S}_s - 0.85\mu g t_1$$  \hspace{1cm} (13)

The total time available for linear deceleration is reduced to

$$t_2 = \frac{\dot{S}_1}{0.85\mu g} = \frac{\dot{S}_s}{0.85\mu g} - \Theta t_1$$  \hspace{1cm} (14)
The total time required to stop both the linear and angular motions becomes

$$T = t_1 + t_2 = \frac{\dot{S}_g}{0.85\mu g} + (1-\theta) \frac{2\dot{\psi}_g k^2}{(a+b)\mu g}$$

(15)

Therefore, when partial wheel lock-up is considered, equations (11) and (12) become

$$\dot{\psi}_g = \sqrt{\frac{\mu g(\omega)^2}{\left|\frac{k^2}{a+b}\right| |\dot{\psi}_b| (1-\theta) + \frac{S}{1.7} \text{sgn}(\omega)}}$$

(16)

$$\dot{S}_g = 1.7 \left( \frac{\mu g(\omega)}{\dot{\psi}_g} - \frac{k^2 |\dot{\psi}_b| (1-\theta)}{a+b} \right)$$

(17)

The preceding solution for linear and angular separation velocities does not include a provision to include a residual linear velocity which frequently exists after the spinout phase. Therefore, equations (8) and (9) may not apply. In addition, simulation experiments of spinout trajectories reveal the assumption that the area under the velocity vs time curves (figure 29) approximated the linear and angular displacements is somewhat defective. Finally, the slopes of linear and angular velocity time histories do not change abruptly. Rather, the changes are more gradual, as shown in figure 28. The resulting decelerations were over-predicted by using the idealized plots shown in figure 29.

An important observation was also made as a result of the trajectory simulation experiments: The shape of the velocity vs time history plots is a function of the ratio of initial linear and angular velocities.

McHenry modified Marquard's procedure by allowing the provision for a residual linear velocity at the end of rotation. The procedure is based on the ratio of initial linear-to-angular velocities observed in the simulation experiments. A fifth-order polynomial function was developed from the simulations which
Separation Velocities

includes empirical coefficients, \( a_1 \) through \( a_5 \), based on a range of velocity ratios (see figure 30). Since, for a reconstruction, the velocity ratio at separation is initially unknown, several trial values are obtained from an approximation using these empirical coefficients. Five solutions are obtained. The solution which most closely matches its trial value of the velocity ratio is used. The derivation follows.

The total time required to arrest angular motion is approximately

\[
T_1 = a_1 \frac{\Delta \omega}{\psi_s} = t_1 + t_2 \quad (18)
\]

The actual time of angular deceleration is

\[
t_1 = \frac{2\omega_k k^2}{(a+b) \mu g a_2} \quad (19)
\]

and the actual time of linear deceleration is

\[
t_2 = \frac{(S_s - \dot{S}_1)}{\alpha_4 g} - \frac{\alpha_3 \beta_4 t_1}{\alpha_4} \quad (20)
\]

The change in linear velocity during time \( T_1 \) is approximately

\[
S_1 = \left( \frac{S_s + \dot{S}_1}{\alpha_5} \right) T_1 \quad (21)
\]

From equations (18) and (21),

\[
a_1 \frac{\Delta \omega}{\psi_s} = a_5 \frac{S_1}{(S_s + \dot{S}_1)} \quad (22)
\]
EDCRASH Training Manual

From equations (18), (19), and (20),

\[ a_1 \frac{\Delta \omega}{\omega_s} = \frac{2w_s k^2}{(a+b)\mu g \sigma_2} \left( 1 - \frac{a_3 \theta}{a_4} \right) + \frac{\dot{S}_g - \dot{S}_1}{a_4 \mu g} \]  (23)

From (22),

\[ \dot{S}_g - \dot{S}_1 = \frac{a_4}{a_1} \left( \frac{S_1 \omega_s}{\Delta \omega} \right) - 2S_1 \]  (24)

Substituting (24) in (23),

\[ \dot{\omega}_s^2 + B\dot{\omega}_s + C = 0 \]  (25)

where

\[ B = \frac{\dot{S}_1 |\Delta \omega|}{D} \]  (26)

\[ C = \frac{a_1 a_4 \mu g (\Delta \omega)^2}{2D} \]  (27)

\[ D = \frac{a_4 k^2 |\Delta \omega| \left( 1 - \frac{a_3 \theta}{a_4} \right)}{a_2 (a+b) \sigma_2} + \frac{a_3 S_1}{2 \sigma_1} \]  (28)

Using the quadratic formula and equations (26), (27), and (28), equation (25) can be solved directly for separation angular velocity, \( \dot{\omega}_s \). Then, from equation (23), the separation linear velocity is

\[ \dot{S}_g = \dot{S}_1 + 2a_4 \left( \frac{a_1 \mu g}{a_2 \omega_s} \frac{\omega_s k^2 \left( 1 - \frac{a_3 \theta}{a_4} \right)}{(a+b) \sigma_2} \right) \]  (29)

The detailed solution procedure is aimed at solving equations (25) through (29). This procedure, as used by EDCRASH to determine the "best" linear and angular separation velocities, will now be described.
SOLUTION PROCEDURE

The following data are inputs required for the solution:

\[ X'_{c1}, Y'_{c1}, \psi_1 \] = Position and orientation at end of rotation (feet, degrees)
\[ X'_{cs}, Y'_{cs}, \psi_s \] = Position and orientation at separation (feet, degrees)
\[ S_1 \] = Residual linear velocity at end of rotation (ft/sec)
\[ a + b \] = Vehicle wheelbase (inches)
\[ k^2 \] = Vehicle yaw radius of gyration squared (in^2)
\[ \mu \] = Nominal tire-ground friction coeff.
\[ \theta \] = Vehicle average wheel lock-up for all wheels
\[ g \] = Acceleration of gravity
\[ (386.4 \text{ in/sec}^2) \]

The step-by-step solution procedure follows:

1. \[ S_1 = \text{rotating/skidding path length (inches)} \]
   \[ = 12 \sqrt{(X'_{c1}-X'_{cs})^2 + (Y'_{c1}-Y'_{cs})^2} \]

2. \[ \Delta \psi = \text{heading change (radians)} \]
   \[ = (\psi_1 - \psi_s)/57.3 \]

3. \[ \psi_s = \text{separation angle (radians)} \]
   \[ = \tan^{-1}\left(\frac{X'_{c1}-Y'_{cs}}{X'_{c1}-X'_{cs}}\right) \]

4. \[ \rho' = \text{approximate value of ratio of linear-to-angular velocity} \]
   For \[ \theta = 1.0, \]
   \[ \rho' = 1.408 \left( \frac{S_1}{|\Delta \psi|} \right) - 32 \]
For $\Theta < 1.0$,

\[
\rho' = \frac{-B + \sqrt{B^2 - 4AC}}{2A}
\]

where \[A = (1-\Theta)8.52 \times 10^{-4}, B = 0.94 - 0.23\Theta, C = 40.64 - 8.64\Theta - (S_1/|\omega|)\]

5. The trial values for the linear-to-angular velocity ratio, $\rho_j = S_1/|\omega|$, are

\[
\begin{align*}
\rho_1 &= 0.70\rho' \\
\rho_2 &= 0.85\rho' \\
\rho_3 &= 1.00\rho' \\
\rho_4 &= 1.15\rho' \\
\rho_5 &= 1.30\rho'
\end{align*}
\]

6. For each trial value $\rho_j$, calculate $a_i$ through $a_5$ according to the following formula by using the empirical coefficients found in the following table:

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>2.58</td>
<td>0.88</td>
<td>0.2417</td>
<td>0.671</td>
<td>1.223</td>
</tr>
<tr>
<td>$c_1$</td>
<td>-7.44e-3</td>
<td>-3.92e-3</td>
<td>4.85e-3</td>
<td>1.4772e-3</td>
<td>1.7307e-2</td>
</tr>
<tr>
<td>$c_2$</td>
<td>1.504e-5</td>
<td>8.00e-6</td>
<td>-9.667e-6</td>
<td>-4.50e-6</td>
<td>-1.053e-4</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5.8e-9</td>
<td>1.993e-7</td>
</tr>
<tr>
<td>$K$</td>
<td>1.66</td>
<td>0.40</td>
<td>0.85</td>
<td>0.85</td>
<td>2.08</td>
</tr>
</tbody>
</table>

If $0 < \rho_j < 250$, $a_{ij} = c_{i0} + c_{i1}\rho_j + c_{i2}\rho_j^2 + c_{i3}\rho_j^3$

If $\rho_j \geq 250$, $a_{ij} = k_i$

A graph which displays the relationship between the various coefficients, $a_i$, and the trial linear-to-angular velocity ratio, $\rho_j$, is shown on the next page (figure 30).
Figure 30 - $\alpha_i$ vs $\rho_j$

7. $D_j = \frac{\alpha_j k^2 |\Delta\psi| \left(1 - \frac{\alpha_{3j}}{\alpha_{4j}}\right)}{\alpha_{2j}(a+b)} + \frac{\alpha_{5i} S_1}{2\alpha_{1j}}$

8. $B_j = \frac{12S_1 |\Delta\psi|}{D_j}$

9. $C_j = \frac{\alpha_{3j} \mu g(\Delta\psi)^2}{2D_j}$

10. $\dot{\omega}_{S_j} \left(\text{rad/sec}\right) = \frac{B_j}{2} + \frac{1}{2} \sqrt{B_j^2 + 4C_j} \text{ sgn}(\Delta\psi)$

11. $\dot{S}_{S_j} \left(\text{in/sec}\right) = 12S_1 + 2\alpha_{4j} \left(\frac{\alpha_{1j} \mu g(\Delta\psi)}{2\omega_{S_j}} - \frac{|\dot{\omega}_{S_j}| k^2 \left(1 - \frac{\alpha_{3j} \phi}{\alpha_{4j}}\right)}{(a+b)\alpha_{2j}}\right)$

12. $\beta_j = \frac{\rho |\dot{\omega}_{S_j}| - 1}{\dot{S}_{S_j}}$
13. Letting $n$ be the value for which $\beta_1$ is smallest, return with the conditions at separation:

\[
\hat{\psi}_s = 57.3 \hat{\psi}_{sn} \text{ (degrees/sec)}
\]
\[
\hat{s}_s = \hat{s}_{sn} \text{ (inches/sec)}
\]
\[
\hat{u}_s = \hat{s}_s \cos(\gamma_s - \omega_s) \text{ (inches/sec)}
\]
\[
\hat{v}_s = \hat{s}_s \sin(\gamma_s - \omega_s) \text{ (inches/sec)}
\]

The above values, along with the separation angle calculated in step 3, provide the separation conditions necessary to determine the post-impact momentum in OBLIQUE (see figures 8, 17, and 31).

Figure 31 - Separation Conditions determined by EDCRASH
Separation Velocities

Summary

In the preceding materials, it is seen that the EDCRASH program is modular. Certain sections are devoted to certain calculation procedures. The DAMAGE section computes the delta-V from crush measurements. The SEPARATION VELOCITIES section computes the linear and angular velocities and angle at separation from accident site measurements. COMMON VELOCITY CHECK compares the separation conditions for both vehicles to insure the results are compatible with the common velocity assumption. TRAJECTORY SIMULATION performs a simulation of the impact-to-rest phase of the accident, using the calculated separation velocities, to confirm the results. Finally, OBLIQUE computes the delta-V from the accident site measurements for oblique collisions.

Processing is now complete. All the EDCRASH output has been computed. Program control is passed to the OUTPUT section and the results are displayed. For further details about the EDCRASH output and instructions for program use, refer to the EDCRASH Program Manual.
CASE STUDIES

This section of the manual illustrates the application of EDCRASH to staged collisions and real-world crashes. The illustrations are in the form of case studies which have been selected to demonstrate certain program features.

Each case study is presented by first describing the crash and the information desired from EDCRASH. The vehicle and accident site data are presented and converted to EDCRASH input. Then, the output is presented and discussed.

CASE NO. 1 - DAMAGE-ONLY ANALYSIS

This case involved a head-on collision between a 1980 Dodge Colt (Veh #1) and a 1978 Chevrolet Camaro (Veh #2). Accident site data were not available, so impact speeds could not be determined. The severity of impact was desired in order to assess the occupants' exposure to injury. This was obtained by using EDCRASH to determine the delta-V.

The accident scene diagram is shown on the following page. Veh #2 attempted to enter the highway and proceed southbound while Veh #1 was traveling northbound on the highway approaching the intersection.

The vehicle data are shown below.

Vehicle Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Veh #1</th>
<th>Veh #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Category</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Weight</td>
<td>2180</td>
<td>3640</td>
</tr>
<tr>
<td>CDC/PDOF</td>
<td>12/30</td>
<td>12/20/10</td>
</tr>
<tr>
<td>Stiffness Category</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Damage Width</td>
<td>62</td>
<td>58</td>
</tr>
<tr>
<td>Crush Profile</td>
<td>19,20</td>
<td>28,25,22,19,14,8</td>
</tr>
<tr>
<td>Damage Offset</td>
<td>0</td>
<td>-8</td>
</tr>
</tbody>
</table>

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Case Studies

Figure 32 - Accident Scene Diagram for Case No. 1

The results are shown below, followed by a display of the vehicle Damage Profiles (figure 33).

SUMMARY OF ECRAH Results

LIT. USER: Engineering Dynamics
S/N: C123456
Date: 02-22-1980
Case No. 1 - Damage Only Analysis

WARNING MESSAGES: NO MESSAGES

SPEED CHANGE (DAMAGE)

<table>
<thead>
<tr>
<th>VEH #1</th>
<th>TOTAL</th>
<th>LAT.</th>
<th>AVE.</th>
</tr>
</thead>
<tbody>
<tr>
<td>39.6 MPH</td>
<td>-39.6 MPH</td>
<td>0.0 MPH</td>
<td>39.6 MPH</td>
</tr>
<tr>
<td>22.5 MPH</td>
<td>-22.5 MPH</td>
<td>4.1 MPH</td>
<td>22.5 MPH</td>
</tr>
</tbody>
</table>

ENERGY DISSIPATED BY DAMAGE: VEH #1 566415.0 FT-LB VEH #2 94414.0 FT-LB
Figure 33 - Case No. 1, Vehicle Damage Profiles

For purposes of comparison, consider that FMVSS 208 requires that a vehicle not expose its occupants (simulated by a test dummy) to levels of force beyond human tolerance (4) during a 30 mph barrier crash. The results indicate the delta-V for the Dodge Colt was nearly 40 mph, a rather severe impact. Fortunately, both occupants survived because the occupant compartment remained intact. Had restraints been used, injuries may have been reduced substantially. The delta-V of the Pontiac Firebird was substantially less due to the mass ratio of the colliding vehicles, an illustration of why "heavier is better" during a collision.
CASE NO. 2 - RICSAC6

This staged collision was part of the RICSAC study (5) (Research Input for Computer Simulation of Automobile Collisions) used to validate the CRASH and SMAC programs. RICSAC6 was an oblique front-to-side impact between a 1974 Chevrolet Chevelle (Veh #1) and a 1975 VW Rabbit (Veh #2). Both vehicles were traveling 21.5 mph at impact. The measured delta-V for Veh #1 (the striking vehicle) was 9.2 mph; the delta-V for Veh #2 was 11.9 mph. These data were compared to EDCRASH results to estimate program accuracy.

During the 60° impact, Veh #1 struck Veh #2 in the passenger door. As a result, Veh #1 was slightly redirected while Veh #2 was spun around clockwise. The accident scene diagram is shown below.

![Accident Scene Diagram for Case No. 2]

Figure 34 - Accident Scene Diagram for Case No. 2
Vehicle Data

<table>
<thead>
<tr>
<th>Class Category</th>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>4300</td>
<td>2623</td>
</tr>
<tr>
<td>CDC/PDOF</td>
<td>112dew1/30</td>
<td>02rdew1/30</td>
</tr>
<tr>
<td>Stiffness Category</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Damage Width</td>
<td>54.5</td>
<td>77</td>
</tr>
<tr>
<td>Crush Profile</td>
<td>5,5,1,3,1,5,1,8, 4,12,17,8,19,3, 2.3, 17,8,3</td>
<td></td>
</tr>
<tr>
<td>Damage Offset</td>
<td>9.8</td>
<td>-3.3</td>
</tr>
</tbody>
</table>

Accident Site Data

| Impact Positions | 0,0,0       | 11.1,2.67,120 |
| Rest Positions   | 60,11,15    | 20,21,242     |
| Rotating Skidding? | no      | yes          |
| Rotation Direction | cw      | cw           |
| Tire-ground Friction | .87     | .87          |
| Rolling Resistances | .01,.01,2,2 | .01,.01,2,2 |

The EDCRASH results are shown below, followed by the Site Drawing (figure 35) and Damage Profiles (figure 36).

**Summary of EDCRASH Results**

1. User: Engineering Dynamics
2. SUN: CRASHIN
3. Date: 02-23-1986
4. Case No.: 2

**Warning Messages:**

Magined estimates for Magnitude of Principal Force greatly violate Newton's third law of energy. Review the output to determine required corrections to Damage Data and adjust as necessary.

The Magnitudes of Principal Force for Vehicles 1 and 2 should be approximately equal. NOTE: The difference may be due to bumper over-ride, a type of vehicle-to-vehicle engagement not within the scope of the Damage Analysis.

**Common Velocity Warning:** An adjustment of vehicle separation conditions was performed in order to be consistent with the common velocity assumption. The adjustment did not exceed 10 percent.
Case Studies

IMPACT SPEED (TRAJECTORY AND CONSERVATION OF LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th></th>
<th>VEH #1</th>
<th>VEH #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORWARD</td>
<td>24.4 MPH</td>
<td>24.3 MPH</td>
</tr>
<tr>
<td>LAT.</td>
<td>0.0 MPH</td>
<td>0.0 MPH</td>
</tr>
</tbody>
</table>

SPEED CHANGE (DAMAGE)

<table>
<thead>
<tr>
<th></th>
<th>VEH #1</th>
<th>VEH #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LON.</td>
<td>15.4 MPH</td>
<td>25.6 MPH</td>
</tr>
<tr>
<td>LAT.</td>
<td>-13.5 MPH</td>
<td>-22.1 MPH</td>
</tr>
<tr>
<td>ANG.</td>
<td>7.6 MPH</td>
<td>-12.8 MPH</td>
</tr>
</tbody>
</table>

SPEED CHANGE (LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th></th>
<th>VEH #1</th>
<th>VEH #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LON.</td>
<td>14.5 MPH</td>
<td>23.8 MPH</td>
</tr>
<tr>
<td>LAT.</td>
<td>-13.6 MPH</td>
<td>-18.4 MPH</td>
</tr>
<tr>
<td>ANG.</td>
<td>5.1 MPH</td>
<td>15.1 MPH</td>
</tr>
</tbody>
</table>

ENERGY DISSIPATED BY DAMAGE: VEH #1 14289.5 FT-LB VEH #2 124990.5 FT-LB

RELATIVE VELOCITY DATA

SPEED ALONG LINE THRU CGS (LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th></th>
<th>VEH #1</th>
<th>VEH #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LON.</td>
<td>23.7 MPH</td>
<td>6.9 MPH</td>
</tr>
</tbody>
</table>

SPEED ORTHOG. TO CG LINE (LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th></th>
<th>VEH #1</th>
<th>VEH #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LON.</td>
<td>-5.7 MPH</td>
<td>-33.5 MPH</td>
</tr>
</tbody>
</table>

CLOSING VELOCITY (LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th>VEH #1</th>
<th>VEH #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>40.0</td>
</tr>
<tr>
<td>Y</td>
<td>10.0</td>
</tr>
<tr>
<td>PSI</td>
<td>242.2</td>
</tr>
</tbody>
</table>

UNITS: mph, ft, deg

10 ft intervals

Figure 35 - Case No. 2, ECORASH Site Drawing
Two warning messages are displayed. The first tells us the damage data is suspect. The reason is the large difference in the magnitudes of principal force (see figure 36 for these values). Although vehicle photos are not available, the damage profiles strongly suggest bumper override - it appears the front bumper of Veh #1 may have ridden over the rocker panel and entered the occupant compartment of Veh #2. Since the damage-based results assume engagement of both vehicle structures, the magnitude of principal force and delta-V for Veh #2 are over-estimated.

The second warning message indicates the separation velocities were adjusted to conform with the common velocity assumption.

The output shows good agreement between the actual and predicted impact speeds.
Case Studies

CASE NO. 3 - RICSAC7

This staged collision, RICSAC7, was a 60° front-to-side impact between a 1974 Chevelle Malibu (Veh #1) and a 1975 Volkswagen Rabbit (Veh #2). Both vehicles were traveling 29.1 mph at impact. The delta-V for Veh #1 (the striking vehicle) was 12.0 mph; the delta-V for Veh #2 was 16.5 mph.

This crash was very similar to RICSAC6, a 60° impact, Veh #1 striking Veh #2 in the passenger door. As a result of the higher speeds, Veh #2 spun further around and had rollout after the end of rotation. The accident scene diagram is shown below.

Figure 37 - Accident Scene Diagram for Case No. 3
Vehicle Data

<table>
<thead>
<tr>
<th>Class Category</th>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>3700</td>
<td>1700</td>
</tr>
<tr>
<td>CDC/PDF</td>
<td>11/6/11/30</td>
<td>02/6/11/30</td>
</tr>
<tr>
<td>Stiffness Category</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Damage Width</td>
<td>66</td>
<td>105</td>
</tr>
<tr>
<td>Crush Profile</td>
<td>0,1,3,2,3,8,5,6.3</td>
<td>0,11,17,8,21,21.3,7,3</td>
</tr>
<tr>
<td>Damage Offset</td>
<td>4</td>
<td>-8.5</td>
</tr>
</tbody>
</table>

Accident Site Data

| Impact Positions | 0,0,0 | 10,7,3,45,120 |
| Rest Positions   | 84.5,18.2,16.5 | 22.9,41.4,262 |
| Rotating Skidding? | no | yes |
| End of Rotation  | n/a | 22,30,250 |
| Rotation Direction | cw | cw |
| Tire-ground Friction | .87 | .87 |
| Rolling Resistances | .01,.01,.2,2 | .01,.01,1,2 |

The EDCRASH results are shown below, followed by the Site Drawing (figure 38) and Damage Profiles (figure 39).

SUMMARY OF EDCRASH RESULTS

Lic. User: Engineering Dynamics
S/N: CRAC3008
Case No. 7 - RIC3AC7

WARNING MESSAGES:

Damage-based velocities for Magnitude of Principal Force very small, determine whether required corrections to Damage Data are not necessary. The Small Magnitude of Principal Force for Vehicles 1 and 2 should be approximately equal. NOTE: The difference may be due to bumper override, a type of vehicle-to-vehicle engagement not within the scope of the Damage Analysis.

COMMON VELOCITY WARNING: An adjustment of vehicle separation conditions was performed in order to be consistent with the common velocity assumption. The adjustment did not exceed 10 percent.

The Damage-based DELTA-V component differ from the Momentum-based DELTA-V State by more than 10 percent. Review the Speed Changes displayed in the Summary of Results.

If the user entered some data (particularly the angles at impact and the positions at impact and rest) is correct, then the user-entered PDF's or Damage Data may be suspect. The difference may also be the result of bumper override, a type of vehicle-to-vehicle engagement not within the scope of the Damage Analysis.)
Case Studies

Impact Speeds (Trajectory and Conservation of Linear Momentum)

Forward Lateral

VEH #1 79.9 MPH 4.1 MPH
VEH #2 74.7 MPH 6.8 MPH

Speed Change (Damage)

TOTAL  LAT.  ANG.

VEH #1 19.7 MPH 17.1 MPH 0.9 MPH -50.0 DEG.
VEH #2 43.8 MPH 37.2 MPH -21.5 MPH 20.0 DEG.

Speed Change (Linear Momentum)

TOTAL  LAT.  ANG.

VEH #1 14.8 MPH -12.3 MPH 0.5 MPH -26.1 DEG.
VEH #2 32.8 MPH -26.7 MPH -17.9 MPH 33.7 DEG.

Energy Dissipated by Damage: VEH #1 251108.8 FT-LB VEH #2 196487.1 FT-LB

Relative Velocity Data

Speed Along Line Thru CGS (Linear Momentum)

VEH #1 24.7 MPH
VEH #2 7.3 MPH

Speed Dirng. To CG Line (Linear Momentum)

VEH #1 0.0 MPH
VEH #2 55.9 MPH

Closing Velocity (Linear Momentum)

51.9 MPH

Figure 38 - Case No. 3, EDGRASH Site Drawing
Figure 39 - Case No. 3, EDCRASH Damage Profiles

Just as in Case No. 2, the damage data is suspect and the separation velocities were adjusted (see page 62 for the explanation).

A third message is displayed. It results from a disagreement between the delta-V computed from the damage measurements and the delta-V computed by momentum (which is based on separation velocities computed from accident site data). In this case, it is probably due to bumper override. There is also reason to believe the stiffness coefficients are too high, since the damage-based delta-V's were over-estimated for both RICSAC6 and RICSAC7 and both tests used similar vehicles.

The results reveal the impact speed for Veh #1 is under-estimated while for Veh #2 it is over-estimated. The momentum-based delta-V estimate for
Case Studies

Veh #1 agrees well with the actual data. However, the agreement is poor for Veh #2, possibly because of its high separation angular velocity.

CASE NO. 4 - OBLIQUE COLLISION

This case involved a head-on collision between a 1977 Jeep Cherokee (Veh #1) and a 1977 Ford Mustang (Veh #2). The use of EDCRASH was secondary to the actual need: EDCRASH was used to determine the speed of the vehicles to determine the controllability of the Jeep, based on its speed, while negotiating a curve of known radius (the controllability was analyzed using EDSVS, the Single Vehicle Simulator). The severity of impact for the Mustang was also important because its occupants were injured and the use of restraints was questioned.

While traveling eastbound on a two-lane highway with packed snow, Veh #1 had lost control in a shallow right curve. The vehicle spun counter-clockwise and slid sideways into the oncoming outside lane. At the same time, Veh #2 was westbound, entering the curve. The vehicles collided with Veh #2 striking the passenger door of Veh #1. The accident scene diagram is shown on the next page.

Vehicle Data

<table>
<thead>
<tr>
<th></th>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Category</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Weight</td>
<td>4000</td>
<td>3200</td>
</tr>
<tr>
<td>CID/PDIF</td>
<td>03rzew2/85</td>
<td>11fdew3/-0</td>
</tr>
<tr>
<td>Stiffness Category</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Damage Width</td>
<td>70</td>
<td>68</td>
</tr>
<tr>
<td>Crush Profile</td>
<td>0,8,12,25,23,8</td>
<td>36,31,15,11,15,19</td>
</tr>
<tr>
<td>Damage Offset</td>
<td>-50</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 40 - Accident Scene Diagram for Case No. 4

<table>
<thead>
<tr>
<th>Accident Site Data</th>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact Positions</td>
<td>19, 0, 90</td>
<td>10, -1, 0</td>
</tr>
<tr>
<td>Rest Positions</td>
<td>32, 13, 140</td>
<td>18, 6, 120</td>
</tr>
<tr>
<td>Sideslip Angle</td>
<td>70</td>
<td>0</td>
</tr>
<tr>
<td>Rotating Skidding?</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Rotation Direction</td>
<td>cw</td>
<td>cw</td>
</tr>
<tr>
<td>Tire-ground Friction</td>
<td>.15</td>
<td>.15</td>
</tr>
<tr>
<td>Rolling Resistances</td>
<td>.1, .1, .2, .2</td>
<td>.01, .2, .2</td>
</tr>
</tbody>
</table>
Case Studies

The EDCRASH results are shown below, followed by the Site Drawing (figure 41) and Damage Profiles (figure 42).

**SUMMARY OF EDCRASH RESULTS**

Lilienthal Engineering Dynamics  
S/N: 114142RM  
Date: 02-22-1986  
Case No. 6 - Oblique Collision

**WARNING MESSAGES:**

COMMON VELOCITY WARNING -- An adjustment of vehicle separation conditions was performed in order to be consistent with the common velocity assumption. The adjustment did not exceed 10 percent.

**IMPACT SPEED (TRAJECTORY AND CONSERVATION OF LINEAR MOMENTUM):**

| VEH #1 | 22.4 MPH | 2.0 MPH | 32.4 MPH |
| VEH #2 | 32.4 MPH |

**SPEED CHANGE (LINEAR MOMENTUM):**

<table>
<thead>
<tr>
<th>TOTAL</th>
<th>LONG.</th>
<th>LAT.</th>
<th>ANG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEH #1</td>
<td>22.4 MPH</td>
<td>-2.0 MPH</td>
<td>44.4 MPH</td>
</tr>
<tr>
<td>VEH #2</td>
<td>32.4 MPH</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**ENERGY DISSIPATED BY DAMAGE:**  
VEH #1: 9753.1 FT-LB  
VEH #2: 9783.1 FT-LB

**RELATIVE VELOCITY DATA**

**SPEED ALONG LINE THRU CG (LINEAR MOMENTUM):**

| VEH #1 | 22.4 MPH |
| VEH #2 | 33.4 MPH |

**SPEED ALONG, TO CG LINE (LINEAR MOMENTUM):**

| VEH #1 | 22.4 MPH |
| VEH #2 | 33.4 MPH |

**CLOSING VELOCITY (LINEAR MOMENTUM):**  
31.4 MPH

The only warning message was the common velocity warning, indicating the separation velocities were adjusted.

The results show Veh #1 was traveling 20 mph at impact (6.8 mph forward velocity and 18.6 mph lateral velocity). Additional analysis of the pre-impact
Figure 41 - Case No. 4, EDCRASH Site Drawing

Figure 42 - Case No. 4, - EDCRASH Damage Profiles
path revealed the vehicle was negotiating the curve at approximately 30-35 mph when it lost control. This speed was far below the critical speed. The loss of control was apparently initiated when the driver drove onto the shoulder and over-corrected. The delta-V for Veh #2 was approximately 30 mph, very close to the 30 mph velocity used in FMVSS 208. A restrained occupant would have a very good chance of survival without serious injury. A child in a properly-used infant restraint would also do well, especially in the back seat.

Note the damage-based and momentum-based delta-V's are in close agreement. Note the PDOF and angle of the impulse computed from the momentum analysis are also in close agreement. These signs of compatibility are useful when assessing the validity of the results (refer to the EDCRASH output).

CASE NO. 5 - INTERSECTION COLLISION

This case involved a collision between a 1976 Camaro (Veh #1) and a 1974 AMC Matador (Veh #2). Veh #2 was approaching a shallow left curve but had the option of proceeding straight onto a side road. At the same time, Veh #1 was on the side road approaching the curve. The driver of Veh #1 apparently did not realize Veh #2 was going to negotiate the left curve in front of her and she proceeded to drive straight into the oncoming car. The purpose of the analysis was to determine the speed of Veh #1 to find out if she had stopped at the stop sign.

Accident site information revealed the crash may have occurred two different ways. The 20° angle of impact was either due to the natural angle formed by Veh #2 in the curve or it was due to pre-impact braking by Veh #2 and a resulting 20° sideslip angle. Each of these scenarios was analyzed. The accident scene diagram is shown on the next page.
Figure 43 - Accident Scene Diagram for Case No. 5

Vehicle Data

<table>
<thead>
<tr>
<th></th>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Category</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Weight</td>
<td>3920</td>
<td>3960</td>
</tr>
<tr>
<td>CDC/PDOP</td>
<td>12fdew4/0</td>
<td>01fdew6/25</td>
</tr>
<tr>
<td>Stiffness Category</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Damage Width</td>
<td>76</td>
<td>77</td>
</tr>
<tr>
<td>Crush Profile</td>
<td>18,22,25,30,38,46</td>
<td>11,11,21,43,63,83,0</td>
</tr>
<tr>
<td>Damage Offset</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Case Studies

Accident Site Data

<table>
<thead>
<tr>
<th></th>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact Positions</td>
<td>27,18.5,0</td>
<td>39.5,19,155</td>
</tr>
<tr>
<td>Rest Positions</td>
<td>80,14,158</td>
<td>98,35,88</td>
</tr>
<tr>
<td>Sideslip Angle</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rotating Skidding?</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>End of Rotation</td>
<td>72,16,163</td>
<td>n/a</td>
</tr>
<tr>
<td>Rotation Direction</td>
<td>cw</td>
<td>cw</td>
</tr>
<tr>
<td>Tire-ground Friction</td>
<td>.41</td>
<td>.41</td>
</tr>
<tr>
<td>Rolling Resistances</td>
<td>1,0,1,2,2</td>
<td>1,4,2,2</td>
</tr>
</tbody>
</table>

The EDCRASH results are shown below, followed by the Site Drawing (figure 44) and Damage Profiles (figure 45).

SUMMARY OF EDCRASH RESULTS

Lic. User: Engineering Dynamics  S/N: CRASHAN  Date: 02-22-1986
Case No.5: Intersection Collision

WARNING MESSAGES:

The Damage-based DELTA-V(5) differ from the Non-impact-based DELTA-V(5)
by more than 10 percent. Review the speed changes displayed in the
SUMMARY OF RESULTS.

If the user-entered scene data (particularly the angles at impact and the
positions at impact and rest) is correct, then the user-entered PDOF's
or Damage Data may be suspect. The difference may also be the result
of bumper over-ride (a type of vehicle-to-vehicle engagement not within
the scope of the Damage Analysis).

IMPACT SPEED (TRAJECTORY AND CONSERVATION OF LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th></th>
<th>FORWARD</th>
<th>LATERAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEH #1</td>
<td>52.5 MPH</td>
<td>0.0 MPH</td>
</tr>
<tr>
<td>VEH #2</td>
<td>11.0 MPH</td>
<td>0.0 MPH</td>
</tr>
</tbody>
</table>

SPEED CHANGE (DAMAGE)

<table>
<thead>
<tr>
<th></th>
<th>TOTAL</th>
<th>LONG.</th>
<th>LAT.</th>
<th>ANG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEH #1</td>
<td>46.0 MPH</td>
<td>-46.0 MPH</td>
<td>6.0 MPH</td>
<td>-22.0 DEG.</td>
</tr>
<tr>
<td>VEH #2</td>
<td>46.0 MPH</td>
<td>-46.0 MPH</td>
<td>6.0 MPH</td>
<td>-22.0 DEG.</td>
</tr>
</tbody>
</table>

SPEED CHANGE (LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th></th>
<th>TOTAL</th>
<th>LATERAL</th>
<th>ANG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEH #1</td>
<td>33.2 MPH</td>
<td>-33.2 MPH</td>
<td>-1.1 MPH</td>
</tr>
<tr>
<td>VEH #2</td>
<td>33.2 MPH</td>
<td>-33.2 MPH</td>
<td>-1.1 MPH</td>
</tr>
<tr>
<td>ENERGY DISSIPATED BY DAMAGE: VEH #1</td>
<td>229146.5 FT-LB</td>
<td>VEH #2</td>
<td>378149.9 FT-LB</td>
</tr>
</tbody>
</table>

RELATIVE VELOCITY DATA

SPEED ALONG LINE THRU CGS (LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th></th>
<th>VEH #1</th>
<th>VEH #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEH #1</td>
<td>32.5 MPH</td>
<td>10.5 MPH</td>
</tr>
</tbody>
</table>

SPEED ALONG LINE (LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th></th>
<th>VEH #1</th>
<th>VEH #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEH #1</td>
<td>-2.1 MPH</td>
<td>-5.1 MPH</td>
</tr>
</tbody>
</table>

CLOSING VELOCITY (LINEAR MOMENTUM)

<table>
<thead>
<tr>
<th></th>
<th>VEH #1</th>
<th>VEH #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEH #1</td>
<td>65.2 MPH</td>
<td></td>
</tr>
</tbody>
</table>
Figure 44 - Case No. 5, EDCRASH Site Drawing

Figure 45 - Case No. 5, EDCRASH Damage Profiles
The results revealed Veh #1 essentially entered the intersection at 53 mph and had not stopped for the stop sign. Veh #2 was negotiating the curve at 12 mph, a rather slow speed considering the advisory speed was 20 mph.

A message was displayed resulting from a disagreement between the delta-V computed from the damage measurements and the delta-V computed by momentum (which is based on separation velocities computed from accident site data). The cause was probably defective damage data for Veh #2, which had its bumper fall off without absorbing much energy. As a result, the damage measurements reflected crush greater than actually occurred.

The case was rerun, changing only the pre-impact sideslip angle for vehicle #2 to account for the possibility its driver may have foreseen the crash and attempted to brake after beginning the curve (pre-impact skid marks were observed, but their association with this accident could not be confirmed).

Revised Accident Site Data

<table>
<thead>
<tr>
<th></th>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sideslip Angle</td>
<td>0</td>
<td>20</td>
</tr>
</tbody>
</table>

The EDCRASH results associated with the revised input are shown on the next page, followed by the Site Drawing (figure 46). Since the damage data were not changed, the Damage Profiles remain the same as displayed in figure 45.

The impact speed of Veh #1 increased to 66 mph, still indicating it had failed to stop at the stop sign. The speed of Veh #2 is now 25 mph, probably a reasonable speed after skidding approximately 50 feet prior to impact (indicating the vehicle entered the curve at approximately 35 mph).
EDCRASH Training Manual

SUMMARY OF EDCRASH RESULTS

LIC. User: Engineering Dynamics  S/N: [Redacted]  Date: 02-22-1980
Case No.5 - Collision w/Sideslip (Head-on)

WARNING MESSAGES:

User-entered sideslip angle $\beta$ was adjusted for compatibility with components of separation velocity and specified direction of principal force. If adjusted value of $\beta$ is not consistent with available physical evidence, basis for impact should be reviewed for possible adjustment and rerun.

IMPACT SPEED (TRAJECTORY AND DAMAGE)

| VEH #1 | 40.1 MPH | 4.1 MPH |
| VEH #2 | 24.5 MPH | 4.7 MPH |

SPEED CHANGE (DAMAGE)

| TOTAL | 46.8 MPH | -46.8 MPH | 0.0 MPH | 0.0 DEG. |
| VEH #1 | 46.3 MPH | -42.0 MPH | 19.8 MPH | 25.0 DEG. |

ENERGY DISSIPATED BY DAMAGE: VEH #1: 3791 lb-ft-lb VEH #2: 3791 lb-ft-lb

Figure 46 - Case No.5, EDCRASH Site Drawing with pre-impact sideslip

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Inspection of the output reveals the momentum results are no longer displayed. This is because the impact went from oblique (20° impact angle) to collinear due to the introduction of a 20° sideslip angle. Note, also, the sideslip angle was adjusted according to the warning message. In a sense, the equations become "over-determined" by supplying too much information. Therefore, an adjustment is necessary for compatibility between the PDOP's and the separation angles.

The increased speeds provided by this analysis are suspect because the damage data upon which it is based are suspect (the validity of the damage data was questioned earlier). If the damage data was revised downward to achieve compatibility with the momentum-based results before rerunning with the revised sideslip angle, the results might be improved.
REFERENCES


