

**DEVELOPMENT OF A COMPUTER PROGRAM
TO AID THE INVESTIGATION
OF
HIGHWAY ACCIDENTS**

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FOREWORD

This report summarizes the results of the first year of a program of research aimed at the development of a computer program to aid the investigation of highway accidents. The specific objectives are (1) to develop mathematical formulations and a computer program for processing and evaluating data transmitted from the accident scene in highway crash investigations and (2) to evaluate the accuracy achieved in reconstructions produced by the computer program by means of trial applications to staged collisions.

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SUMMARY

A computer program and an associated optical measurement system are described which have been developed to aid the investigation of highway accidents. The described developments constitute the major components of a system which will provide a capability of processing and evaluating data, via radio contact with the operator of a time-sharing computer terminal, while the investigators are at the accident scene.

Reconstructions produced by the computer program are compared with the results of staged collisions in the literature. The instrumented optical system is described in detail and results of trial applications are discussed.

Plans for completion and evaluation of the overall system, which could not be implemented within the first year of effort, are presented and discussed.

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1.0 INTRODUCTION

1.1 Background

The reporting of automobile accidents, which serves a number of purposes (i.e., law enforcement, litigation and research), varies widely in detail and accuracy. Minor accidents are frequently reported without any investigation being performed. In cases where serious injuries and/or substantial property damage occur, an investigation including measurements and scene photographs may be performed. Also, in recent years, a number of accident research teams have generated detailed reports on investigations of selected accidents (e.g., References 1, 2 and 3).

In the case of detailed investigations, the manner of treatment of the physical evidence, in relation to speed estimates or complete accident reconstructions, is made nonuniform by the necessity both for selection of appropriate assumptions and approximations and for applications of judgement. The existing calculation techniques are generally limited to manual or slide rule procedures, and they involve the use of many simplifying assumptions. The time required for the manual calculations precludes complete reconstruction at the accident scene, while the physical evidence is intact and readily available. Rather, in cases where reconstruction calculations are planned, the investigators must attempt to recognize and measure all critical items of information prior to leaving the scene. Thus, the quality and the completeness of data taken at the scenes of accidents and the application of such data in the generally iterative and simplistic reconstruction calculations each tend to vary with the expertise and persistence of the individual investigators.

If analytical reconstruction procedures could be uniformly applied to large numbers of actual highway accidents, they could provide a logical basis for refining the categorization of occupant exposures in studies of crash injuries. They could also provide valuable information on the roles of vehicle and/or highway defects and of driver judgement in accident causation.

With refined categorization of occupant exposures, actual highway accidents are seen as being potentially the best available source of improved information on the injury thresholds of humans in the automobile-crash type of exposure (i.e., partial or no restraint, including impact on the vehicle interior) and of measures of the effectiveness of protective devices. This viewpoint is based on the facts that experimental research with volunteers must be run substantially below injury thresholds, and that the interpretation of results obtained with cadavers and animals, in terms of corresponding results for living humans, is not a straightforward and validated procedure. Thus, there is no direct means of generating improved tolerance data for this type of exposure, and the development of improved protective devices for use in automobiles is hampered by the limited extent of applicable human tolerance information.

The primary items of data used in accident reconstruction are tracks, skid marks, damage to vehicles and obstacles, debris, terrain properties (i.e., friction coefficients, slopes, obstacles, etc.) and testimony of witnesses and drivers. The physical laws are applied in successive attempts to fit the total body of evidence. Since a perfect fit generally cannot be achieved, it is necessary to select the more significant and reliable items of evidence.

Because improved training of investigators and the use of more detailed reporting formats are seen as means of achieving only limited improvements in the quality and completeness of accident data for use in reconstructions, the presently reported research program has been

aimed at the development of an on-scene investigation aid with which a general upgrading might be achieved.

1.2 Overall Concept of This Research

A program of research performed by the Cornell Aeronautical Laboratory, Inc. (CAL) during the period 1966-1971 (Reference 4), under support of the Federal Highway Administration, provided convincing evidence of the feasibility of achieving accurate reconstructions of the dynamics of violent maneuvers and collisions of automobiles through the use of a computer program. This development, combined with an awareness of the problems involved in achieving uniformity in the detail and accuracy of accident data, led to the concept of processing and evaluating such data while the investigators are at the scene, via radio contact with a reconstruction computer program.

While the specific computer programs described in Reference 4 may ultimately be applied for this purpose, the presently reported research program has made use of relatively simplified analytical representations to achieve compatibility with available time-sharing computer facilities and to simplify the requirements for input data. However, the selected analytical representations, which take advantage of the use of computer solutions, include significant improvements in accuracy over those generally applied in manual calculations.

The computer program of reconstruction calculations that has been developed will process data transmitted from the accident scene to an operator of a time-sharing digital computer terminal. Both a vocal channel and a data channel will be used in the communications link, so that numerical data can be transmitted directly to a paper-tape recorder adjacent to the computer terminal. Subsequent to the transmission of dimensional and damage data from the scene and initial processing by the computer program, a conversational mode of program operation will

request specific items of information (e.g., repeats of measurements, additional measurements, evidence to support introduction of damage effects or control inputs, etc.) as repeated attempts are made to reconcile any inconsistencies that might exist among the various measurements and in the testimony of witnesses and/or drivers. A guide book, containing photographs of representative types of accidents and evidence and sketches illustrating measurement techniques, is planned for use by the investigators to simplify the task of description, to provide clarity in instructions and to permit the use of multiple choice answers. To evaluate the accuracy achieved in reconstructions and the convenience of applications, trial applications to staged collisions are also planned.

The reporting format for accidents processed through a computer program can be quite comprehensive. In the final form of the presently reported computer program, it is planned to include a computer-graphics display of the scene with superimposed sequential displays of the reconstructed event. A tabulation of all items of physical evidence upon which the reconstruction is based can readily be included. Items of testimony, or indirect evidence, that are rejected can be listed with statements regarding the basis for rejection. An error analysis can also be performed, so that a summary of probable error ranges in the reconstruction can be included.

A summary statement can be produced regarding contributing factors and the principal cause of the accident. If desired, a summary of indicated violations can also be provided on the basis of applicable traffic laws.

For purposes of injury and human tolerance studies, a display of the reconstructed acceleration environment can also be generated. When medical reports are available, injury information can be added for correlation with the defined exposures.

It should be noted that the presently reported first year effort has been aimed at achievement of a fully operational system, including the reconstruction computer program and the associated measurement and communications systems, so that trial applications could be performed on staged collisions. While encouraging results have been achieved with both the computer program and an optical measurement system, it has not been possible, within the time and funding constraints of this first-year effort, to achieve a fully operational system. The developmental difficulties that were encountered and the current status of the research project are discussed in Section 3.0.

1.3 Organization of This Report

Conclusions and recommendations based on results of the first year efforts of this research are presented in Section 2.0. The results of the research are summarized in Section 3.0.

In Section 4.0, the computer program, in its present form, is described in detail. References are listed in Section 5.0.

In Section 6.0, detailed equations and logic for the computer program, including operational extensions that could not be implemented within the first year, are presented.

2.0

CONCLUSIONS AND RECOMMENDATIONS

2.1 Conclusions

2.1.1 The first year of this exploratory research program has produced encouraging results that support the feasibility of immediate processing and evaluation of data transmitted from the accident scene during an investigation.

The degree of success achieved in comparisons of analytical predictions with data from staged collisions (Section 3.3) provides convincing evidence of the validity and generality of the developed computer program. Also, the system developed for measurement and coding of scene dimensions is considered to constitute a significant advance toward the goal of data transmission for remote processing.

The relatively ambitious analytical approach (Section 3.2) includes approximations of the effects of vehicle deformations and the corresponding extent of damage. The locations and magnitudes of permanent deformations are included as part of the program output information, both in tabular form and in the form of a Vehicle Deformation Index (based on SAE J224a). This unique analytical feature, after extensive developmental problems, has been demonstrated to be quite successful (Section 3.3). Such damage information permits valuable additional correlation with the physical evidence and, thus, it is considered to constitute a significant advance in the state of the art of accident reconstruction.

It is unfortunate that development of the "collision" routine (Section 6.1) greatly exceeded original estimates for time and effort. However, it is generally recognized that the extent of the "debugging" phase of the development of computer programs is highly variable and it is, therefore, difficult to estimate accurately. The unanticipated development problems delayed progress to the extent that it was necessary to reduce the scope of the first year effort. It is hoped that the unavoidable slip in schedule, and the corresponding underestimate of effort, will not jeopardize future support for this research effort, which is considered to potentially constitute a means of achieving a major advance in the quality of accident data.

2.1.2 The selected analytical approach (Section 3.2) is quite adequate for the purposes of the present research program.

The detailed correlation that has been achieved between analytical predictions and the results of staged collisions in the literature (Section 3.3) actually exceeds expectations, in view of the many simplifications. The specific comparisons that are presented were limited, both by the availability of adequately defined experiments and by the relatively recent achievement of an operational "combined" program (i.e., collision and trajectory calculations applied within a single computer routine, as discussed in Sections 3.2 and 6.4), to a total of six 90° intersection-type collisions on flat, horizontal terrain. However, the excellent correlation achieved in each of those cases, using identical vehicle parameters, indicates that all significant aspects of the collisions and "spin-out" trajectories are adequately represented. The comprehensive output information (kinematics, vehicle damage, tire tracks and acceleration components) will permit both extensive, detailed comparisons with physical evidence and a relatively refined categorization of occupant exposures. The generality included in the program inputs will permit the inclusion of effects of control inputs, vehicle damage and terrain zones with different friction properties (e.g., pavement and roadside surfaces), as well as variation of the effective tire-terrain friction coefficient with speed.

2.1.3 The prototype optical measurement system that was devised within this research program merits consideration for further development and adoption as an investigation aid for general use, as well as for use in an on-scene reconstruction system.

The capabilities of the prototype measurement system (Section 3.4) for speed, accuracy and reduced interference with other activities at the accident scene would be generally advantageous, whether or not an on-scene reconstruction system is in use. While the prototype equipment is somewhat cumbersome and relatively expensive, it is considered to be quite feasible to develop a compact, economical package with equal performance. Paper tapes of dimensional data gathered at the scene

could be processed later on a time-sharing computer terminal, and computer-graphics displays of the scene measurements could also be generated for use in the accident report.

2.1.4 The conventional assumptions that the effects of vehicle deformations and of tire forces can be neglected in analytical reconstructions of collisions can lead to significant errors.

This is particularly true for intersection-type collisions at low to moderate vehicle speeds, in which prolonged or multiple contacts and significant movements of the involved vehicles occur. Such errors tend to be obscured by the general nature of "sample" impulse-momentum type calculations, in which the specific assumptions are frequently adjusted to achieve agreement with known test results (e.g., relative vehicle positions at contact, effective "point" of force application, vehicle dimensions and properties, etc.). In a calculation procedure for general applications, it is essential that the cited effects be included in the collision contact phase of the reconstruction.

2.2 Recommendations

2.2.1 Development of the system for on-site reconstruction should be continued through the planned evaluation phase in which direct applications will be made to staged collisions.

A realistic evaluation of the validity and accuracy of any procedure for accident reconstruction should include applications to staged collisions for which the initial conditions are kept unknown until the reconstruction is completed. While such an evaluation was planned for inclusion within this first-year effort, its performance was precluded by developmental problems and a resulting slip in schedule.

2.2.2 Subsequent to successful trial applications to staged collisions, the overall concept of utilizing a digital computer program to process data transmitted from the accident scene and to thereby provide reconstructions of the event should be field tested, at actual accidents. Data should be transmitted from the scene to a remote terminal and the reconstruction calculations performed on a time-sharing basis. Trained investigators should be utilized for at-scene purposes. Based upon this field experience, the computer program should be modified and expanded as required. A final program should be prepared, oriented toward practical field application, and utilizing state police investigators for at-scene purposes.

In the first year of this research program, every effort has been applied toward making the investigation aid both practical and convenient to use. However, the degree of success cannot be fully evaluated prior to the performance of field trials, at staged collisions and subsequently at actual highway accidents.

The continuing performance, under separate support, of on-scene accident investigations (e. g., CAL research program supported by NHTSA Contract FH-11-7098, AMR. Contract 6903-C-129) will tend to make such field trials reasonably convenient and economical if the field trials can be "piggybacked" on the ongoing investigations.

2.2.3 Effort should be applied toward an expansion of the data processing capabilities of the developed computer program to include indirect supporting evidence.

This recommendation constitutes a logical extension of the overall concept. The first year effort has concentrated on items of direct physical evidence, as required to analytically reconstruct the collision sequence. The intention of the present recommendation is to expand the data processing capabilities to include indirect supporting evidence (e. g., information about the driver, the existence

and timing of traffic control signals, steering column deformation, etc.). Such an expansion of the investigation procedure should make use of the experiences of members of existing interdisciplinary accident investigation teams. In this way, the developed program could be expanded to provide the capability of addressing all phases of the collision sequence (pre-, at-, and post-crash) and all aspects of the man-machine-environmental complex as they may be described by an investigator and as they serve to influence the cause(s) of the accident or resultant injury and the extent of losses incurred.

2.2.4 A continuing analytical effort should be applied to introduce refinements and extensions, as found necessary in field trials, to improve the accuracy and convenience of application of the investigation aid. For example, it may be found necessary to develop special subroutines to treat specific obstacle types and terrain features.

Subsequent to achievement of an operational two-dimensional system, consideration should be given to a long term, parallel development of a three-dimensional version for use by trained investigators at accidents of special interest.

2.2.5 An economical, production version of the optical measurement system described in Section 3.4 should be developed for general application, as well as for use within an on-scene reconstruction system.

A set of performance specifications should be developed and bids sought for the design and fabrication of such a system. Consideration should be given to the inclusion of an automatic leveling system in the transit and a portable paper-tape recorder for those applications in which data will be processed after completion of the investigation.

3.0

DISCUSSION OF RESULTS

3.1 Literature Search and Review

3.1.1 Analytical Approach

A review of literature on the topic of automobile collision analysis, that included References 5 through 12, was performed as the first task in this research program. It was found that, with the exception of Reference 12, the analytical relationships were derived for application in manual calculation procedures. In all cases, extensive simplifications were employed.

Prediction of the nature and extent of damage is not included, except for the case of full-width frontal or rear contacts (e.g., Reference 9). Also, no tests of the validity of reconstructions were found in which analytical results, based purely on physical evidence from staged collisions, were compared with measured test conditions that were kept unknown during the reconstruction process. Thus, physical evidence in the form of vehicle damage is not fully utilized in reconstructions and the accuracy of existing calculation procedures has not been established in a rigorous manner.

It was concluded that the purposes of the present research program would be best served by the development of extensions and refinements in existing calculation procedures to include approximations of the nature and extent of damage incurred by the individual vehicles and to generally increase the prediction accuracy by taking advantage of the planned use of computer solutions. A rigorous investigation of the validity of the developed calculation procedure should then be performed.

The general approach of Marquard (Reference 7) was adopted, whereby an iterative procedure is first applied to the post-collision, or "spin-out", phase of the event to approximate the linear and angular velocities at the end of the collision contact. In this approach, the position and orientation at rest of each vehicle, physical evidence defining the trajectory (e.g., tire marks) and the approximate location of the collision are applied in successive attempts to define compatible velocity components for the start of the spin-out. Marquard (Reference 7) treats this aspect of the reconstruction with a simplified two-wheel, or "bicycle", approximation of the vehicle and uses either all locked or all freely rotating wheels. In the present analytical development, the vehicle representation has been extended to include effects of the vehicle tread dimension on yaw responses and a capability for locking individual wheels and/or steering the individual front wheels and the rear axle (i.e., effects of control inputs and/or damage). Variation of the effective tire-ground friction coefficient with speed and a linear boundary between two different nominal friction coefficients (e.g., between pavement and roadside) have also been incorporated to moderately extend the generality and accuracy of analytical predictions of trajectories.

When an acceptable match of the post-collision trajectory and of the position and orientation at rest of each vehicle has been achieved, Marquard (Reference 7) applies an iterative procedure to the collision phase to approximate the velocities, positions and orientations at initial contact corresponding to the required velocity components at separation. In the present case, an approximate analytical treatment of vehicle damage has been developed and added to the procedure to permit additional correlation with the physical evidence. Thus, collision conditions are sought that will produce a best match of both the post-collision trajectories and the damage to the individual vehicles.

The detailed analytical approach developed within the present research program is presented in Section 3.3.

3.1.2 Measurement Techniques

The literature review revealed significant weaknesses in existing measurement techniques, specifically with regard to the speed and accuracy with which the data can be collected and to the adaptability to a format for efficient data transmission. This aspect of the investigation procedure is of critical importance to the present research, since it determines the extent and the quality of dimensional information that can be made available for remote reconstruction calculations. Because of the transitory nature of physical evidence and the frequently chaotic activity at an accident scene, it is essential to have a rapid, efficient and non-intrusive system for gathering dimensional information regarding the positions and orientations at rest of involved vehicles and any other physical evidence that may be useful in the reconstruction procedure. The important need for an improved measurement system led to an early concentration of effort on this topic.

There are two widely used methods for locating evidence at the accident scene. Triangulation requires that each object be located by two distances, one from each of two known reference points. The other measurement system makes use of a selected base line and fixed origin, and it involves the measurement of rectilinear coordinates of points of interest with respect to the base line. The distances are usually obtained with steel measuring tapes or distance wheels. One of the major objections to the described procedures is the necessity for frequently crossing the roadway and either interfering with or being obstructed by local traffic.

The triangulation procedure can produce ambiguities in the form of mirror images of the points of interest unless a third reference point is included, thus compounding the measurement task. In the rectilinear coordinate technique, the directional accuracy of "perpendicular" measurements is a source of errors. A mutual shortcoming of the two measurement systems is the effect of topographical irregularities which makes it difficult to determine precise distances.

It should also be noted that each of the cited measurement techniques is time consuming and that the data formats are not readily adaptable to efficient transmission for remote processing.

Because of the numerous disadvantages of the described methods of locating points of interest at an accident scene for the purposes of the present research program, it was concluded that alternate methods should be explored. The investigation of measurement techniques and the selected optical system, which makes use of an instrumented builder's transit, are discussed in detail in Section 3.4.

3.2 Selected Analytical Approach

In keeping with general practice in automobile collision analyses, the initially selected approach consisted of separate treatments of the collision and trajectory phases of the event. This division of the analytical task is based on the assumption that the effects of tire forces are negligible during the existence of collision forces. While this assumption appears to be a reasonable one, its application was found to produce significant errors in the case of moderate speed intersection type collisions in which multiple contacts frequently occur (e.g., front-side followed by side-side and/or rear-side contact).

If secondary contacts are neglected, major errors can be produced in predictions of "spin-out" trajectories. On the other hand, if the tire forces are neglected throughout the time during which the collision contacts occur, significant errors can be introduced in the lateral motions of the vehicles between impacts. Thus, it was concluded late in this research program that the collision and tire forces must be treated simultaneously. The two computer routines that had been developed separately for collision and trajectory calculations were, therefore, combined in a single program.

In the following, the two aspects of reconstruction calculations (i.e., collision and trajectory) are first discussed separately. The method of combination of the corresponding, separately developed routines is then described.

3.2.1 Analytical Treatment of Car-to-Car Collisions

Within the research program described in Reference 4, it was found that crush properties of the peripheral structures of automobiles could be approximated with reasonable accuracy by means of the assumption of a layer of isotropic, homogeneous material that exhibits elastic-plastic behavior. Comparisons of deformations and decelerations measured in pole impacts (Reference 14) with those that occur in full frontal contacts (e.g., Reference 9) indicate general agreement in the required properties for such an assumed peripheral layer.

In collision calculations, the specific crush properties that are assumed do not, of course, affect the conservation of momentum. By producing a finite time duration for the application of forces, they do permit relative motions of the colliding bodies to occur during the reconstructed collision. The time-varying values for the magnitudes, positions and orientations of collision forces that are obtained with relatively gross approximations of crush properties tend to yield results more realistic than those of simple impulse-momentum calculations in which the effects of crush are neglected.

In the present case, it was also considered to be highly desirable to devise a means of analytical generation of damage patterns for comparison with corresponding measurements at the scene. Therefore, a strong case was seen for inclusion of an approximate treatment of crush properties in the present reconstruction program.

3.2.1.1 General Approach

The original (undeformed) boundaries of the vehicles are defined in the form of rectangles. Discrete points defining the body outlines in contacted regions are generated and displaced during the impact calculations. These points serve to define the deformed boundaries. The distance between a displaced point and the initial boundary of the deflected surface is used to determine the dynamic pressure at that point during any time increment in which the point is displaced. An iterative procedure is used to achieve equal pressures from the two mutually deformed bodies.

Since a constraint of some kind is required on the directions of deflection of the individual points, to facilitate the determination of contact in previously damaged regions, the points are constrained to move radially toward the coordinate origins of the two deformable bodies (Figure 1).

The effects of partial recovery of deformed peripheral structure are approximated by means of a coefficient of restitution that varies as a function of the magnitude of the deflection (Sections 6.1, 6.5)

3.2.1.2 Specific Analytical Assumptions

(1) The vehicles are treated as rigid bodies surrounded by a layer of isotropic, homogeneous material that exhibits elastic-plastic behavior.

(2) The dynamic pressure in the peripheral layer increases linearly with the depth of penetration relative to the initial boundary of the deflected surface.

(3) The adjustable, nonlinear coefficient of restitution varies as a function of maximum deflection.

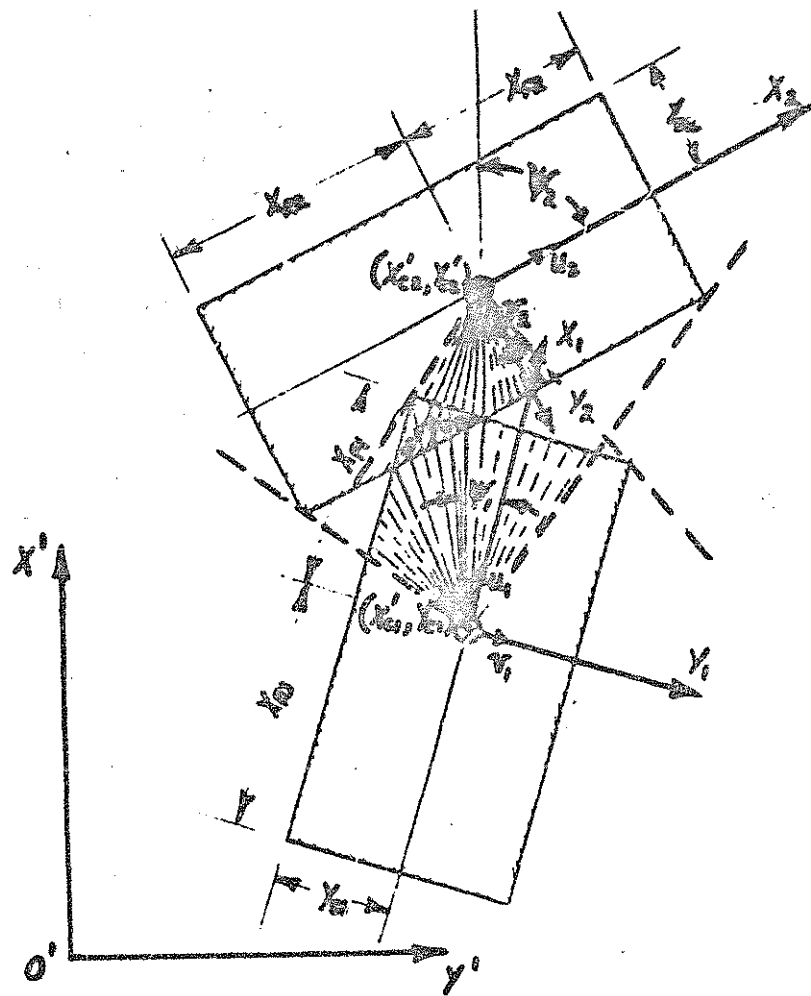


FIGURE 1 ANALYTICAL DEFINITION OF COLLIDING VEHICLES

(4) Tire forces are neglected during the impact.

(5) Plane motion is assumed. The effects of pitch and roll are neglected.

The calculation procedure by which the above assumptions are implemented is outlined in Section 6.1. A post-processing routine for interpretation of damage predictions in terms of a standard collision deformation classification, or "vehicle damage index" (VDI), based on SAE J224a is presented in Section 6.2.

3.2.2 Approximation of Trajectories Preceding and Following Collisions

Existing methods of analytical treatment of the trajectory aspects of accident reconstruction tend to be highly simplified. In the more detailed approaches, the automobile is treated as a two-wheeled vehicle, or "bicycle", in trajectory calculations. It seems obvious that the fidelity of reconstructions obtained with such simplified analytical models is quite limited. Yet, a highly detailed vehicle representation, such as those of Reference 4, would probably require a prohibitive core capacity for most presently available, time-sharing computer terminals. Therefore, an attempt was made to select a compromise approach.

3.2.2.1 General Approach

The vehicle representation is limited to the three degrees of freedom associated with plane motions (Figure 2). The "friction circle" concept (Reference 15) is used to approximate interactions between circumferential forces (i.e., braking or tractive) and side forces of the tires. The cornering stiffnesses of the individual tires are entered separately to permit simulation of damaged tires. A vehicle tread dimension is included to provide realistic effects of individual tire forces



(e.g., locked wheels or flat tires) on yaw behavior and to permit detailed transitions across a boundary defining terrain zones with different friction coefficients.

Tabular inputs, as functions of time, are used for individual wheel torques and for steer angles of the individual front wheels and the rear axle (i.e., control inputs and/or effects of damage). Provision is included for a linear decrement of the effective tire-ground friction coefficient with speed.

Predictions of tire tracks and skid marks are generated as a part of the output information.

3.2.2.2 Specific Analytical Assumptions

(1) Plane motion is assumed. The effects of pitch and roll are neglected.

(2) The terrain surface is assumed to be flat and horizontal.

(3) Effects of camber and of roll-steer of the wheels are neglected.

(4) The "friction circle" concept is used to approximate interactions between circumferential and side forces of the tires.

(5) A step change in the tire-ground friction coefficient is assumed to occur at a linear boundary between two terrain zones.

(6) Provision is included for a linear decrement of the tire-ground friction coefficient with the resultant speeds of the individual wheel centers.

The calculation procedure by which the above assumptions are implemented is outlined in Section 6.3. It should be noted that the output information includes complete definitions of the individual tire tracks and an index to indicate points at which sliding of the individual wheels occurs (Figure 3).

3.2.3 Combination of Collision and Trajectory Calculations

This modification of the computer program retains the existing capability of the trajectory routine for single-vehicle applications (i.e., where the routine is started subsequent to final separation of the colliding vehicles) but it combines the collision and trajectory routines for all collision applications. In the two-vehicle mode of operation, a test for possible collision contact is inserted at the point in the trajectory routine where the summations of tire forces are calculated. If contact is possible, the collision routine is called, the time increment size is reduced and any collision forces are added to the existing force summations prior to integration of the equations of motion.

A detailed definition of the calculation procedure and logic used to combine the two routines is presented in Section 6.4.

3.3 Reconstructions of Staged Collisions in the Literature

In the following, the results of a number of "forward" reconstruction calculations (i.e., the calculations are started with known velocities at impact) are compared with experiments reported by Severy, et al, in Reference 13. While an extensive amount of effort was applied to the formulation and programming of logic to perform "reverse" calculations (i.e, starting with the position and orientation at rest and working

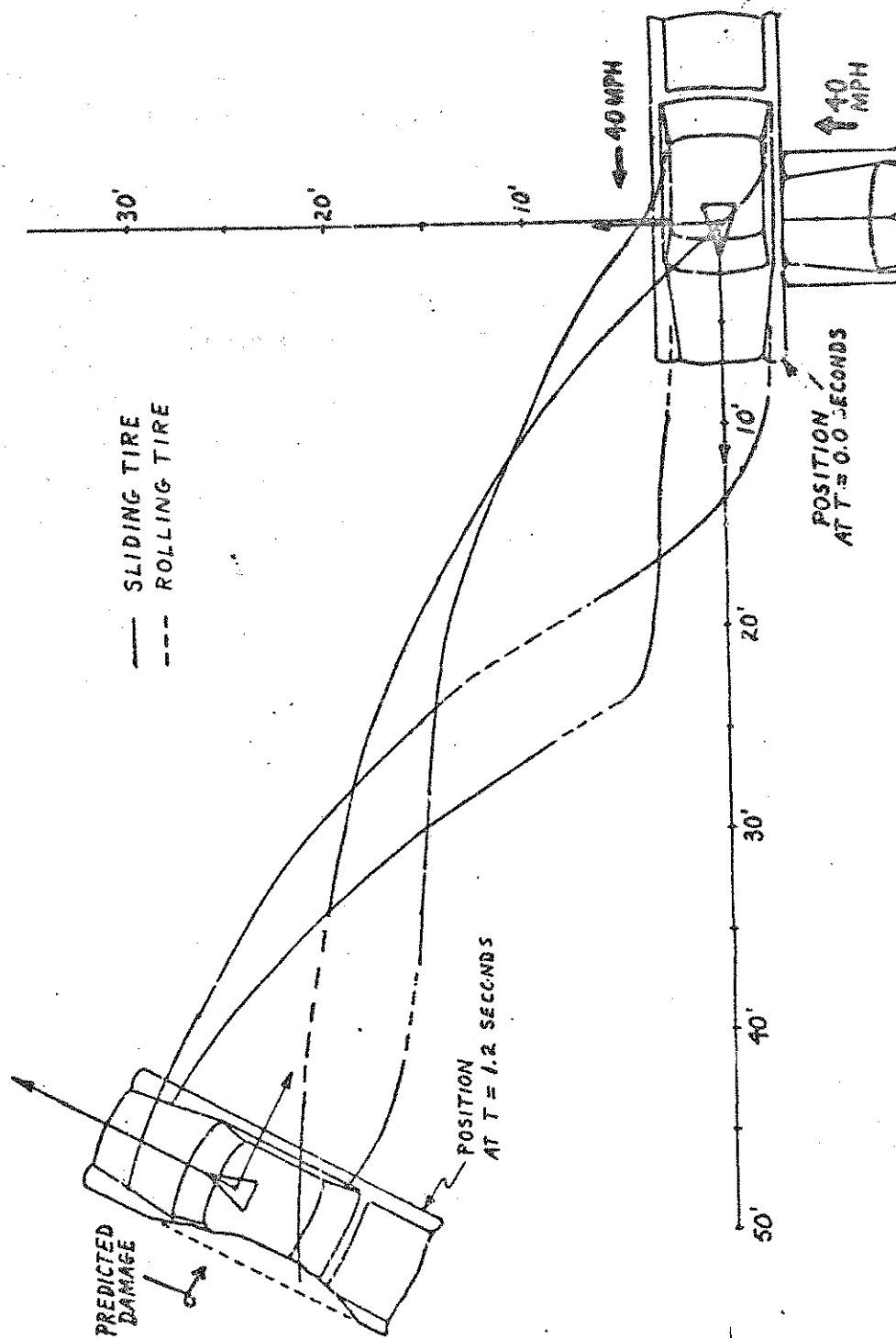


FIGURE 3 PREDICTED TIRE TRACKS OF STRUCK VEHICLE - CENTER-SIDE IMPACT AT 40 MPH

backward through the various items of physical evidence to approximate the velocities at impact), an operational computer program to implement that logic (Section 6.6) was not achieved during the reported research program. Also, a post-processing routine to generate a vehicle damage index (VDI), to supplement the existing output format for vehicle damage, was formulated (Section 6.2) but was not made fully operational within the available performance period.

It is important to note that the analytical predictions that follow have been obtained with identical vehicle parameters in all cases. The only differences in program inputs are those defining the initial positions of the struck vehicles, which have been held constant for each impact configuration.

The experimental results reported in Reference 13 do not include clear definitions of the times of brake applications and of any constraints existing in the steering systems. In view of this lack of information, the analytical predictions do not include inputs for brake applications or for front wheel steer angles, and the trajectories have each been run out for only 2.00 seconds, rather than to the points of rest. In each impact configuration, directly involved wheels have been assumed to be damaged and, thereby, locked by the collision event. It should be noted that the effects of a locked wheel were found to be of secondary importance, but that they were in the general direction of improving agreement with the experimental responses. The specific assumptions of wheel damage were as follows:

Impact Configuration	Wheel Assumed To Be Damaged	
	Striking Vehicle	Struck Vehicle
Front-Side	RF	LF
Center-Side	RF	--
Rear-Side	LF	LR

3.3.1 Kinematics and Damage

In Figures 4 through 9, predicted and experimental kinematics in intersection-type collisions are presented for comparison. While the experimental data do not include direct measures of damage, the sketches from Reference 13 do give an indication of the general locations and extent of damage. Therefore, damage predictions are presented to permit gross comparisons.

In general, the correlation of predicted and experimental kinematics and damage is considered to be very good. The discrepancies in kinematics are considered to be within the probable range of repeatability of the experiments.

It is believed that exploratory changes in the estimated "typical" vehicle parameters, in the relative positions at impact and in the vehicle speeds (e.g., ± 1 MPH about nominal) could produce closer agreement with the experiments. However, in view of the lack of additional details for the specific experiments, such changes could produce a misleading degree of agreement through the mechanism of compensating errors. It should be noted that the occurrence of front wheel steer angle changes, via the mechanism of aligning torques, and the application of brakes could also account for discrepancies in the "spin-out" trajectories. While the computer program includes the capability for simulating such effects, they were not included in the displayed analytical predictions because of the lack of applicable information from the experiments.

3.3.2 Vehicle Accelerations

While vehicle accelerations do not constitute a direct part of the reconstruction process, they do provide a primary means of refining the categorization of occupant exposures. Thus, this additional measure of the validity of predictions must be recognized as being of fundamental importance.

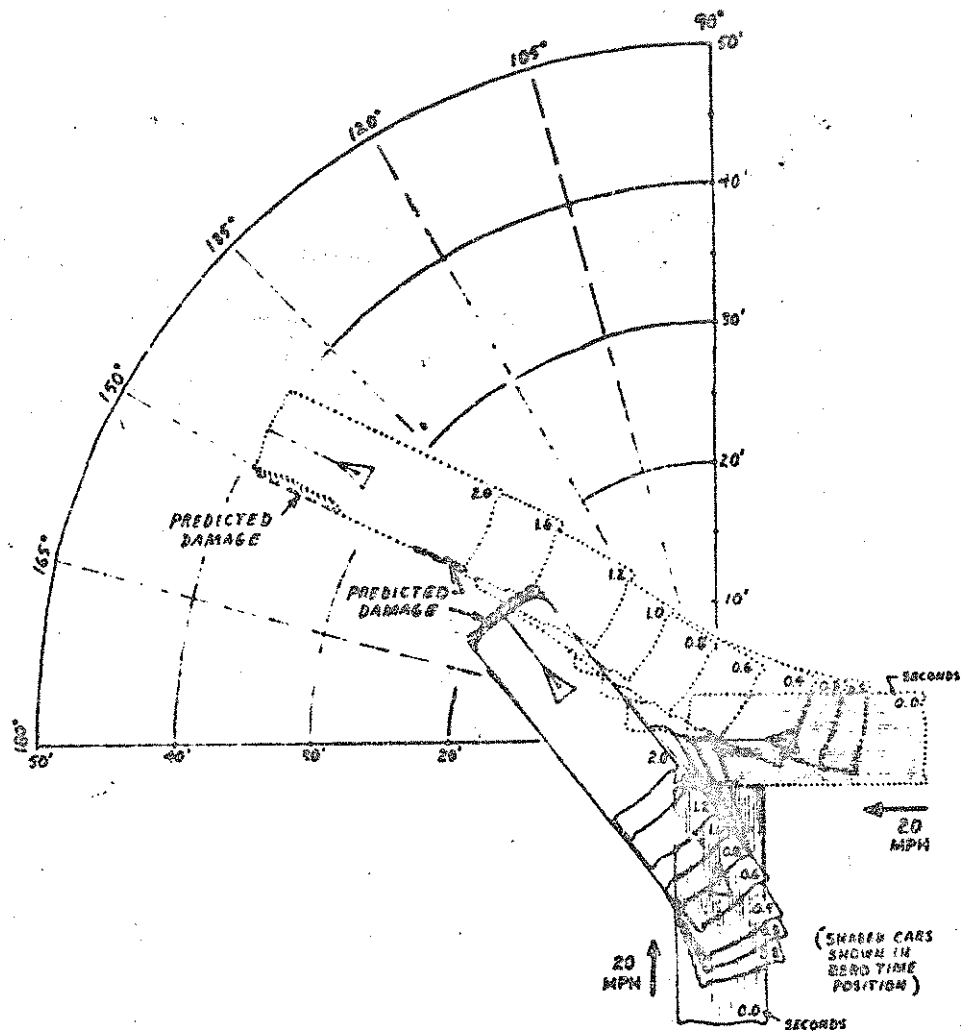


Figure 4A ANALYTICAL PREDICTIONS OF KINEMATICS AND DAMAGE — FRONT-SIDE IMPACT AT 20 MPH

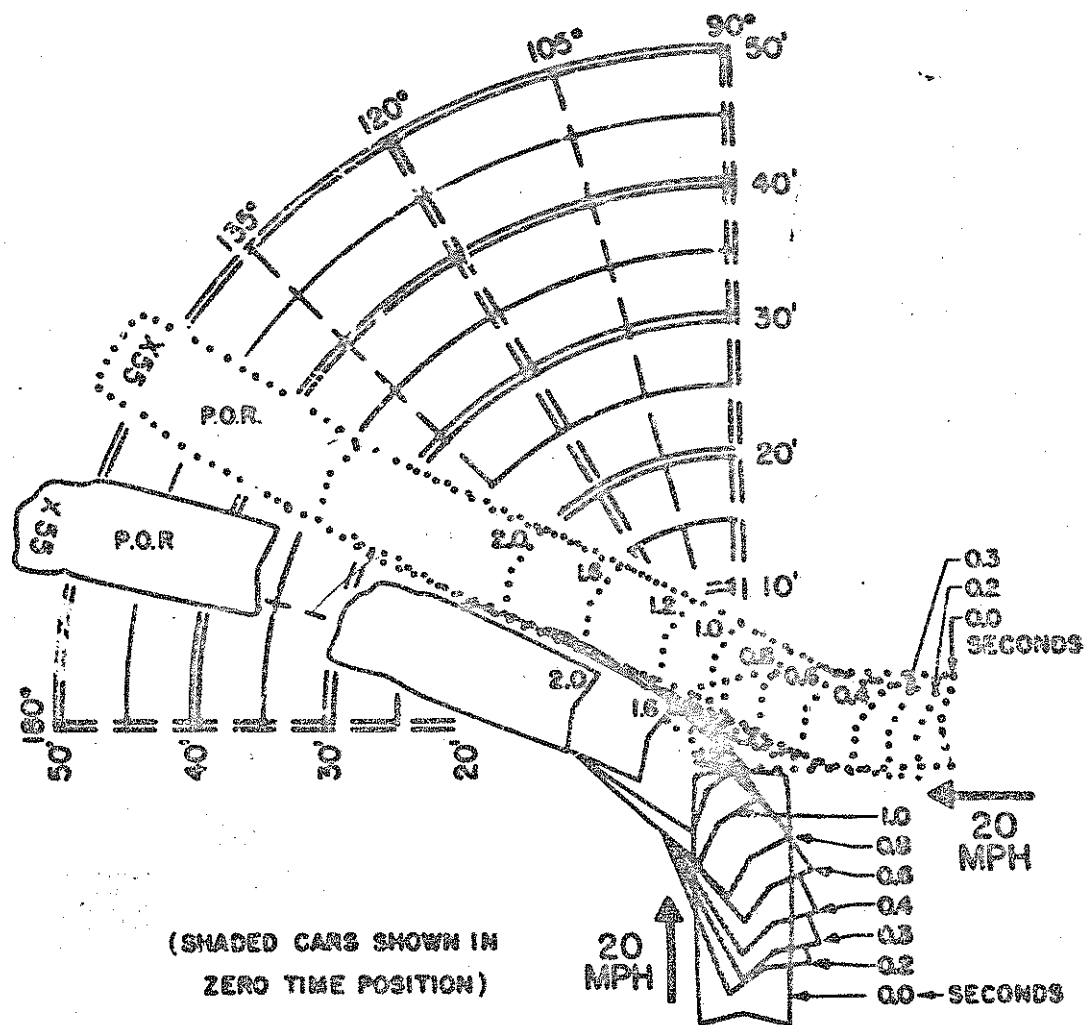


Figure 4B EXPERIMENTAL DATA FROM REFERENCE 13 - FRONT-SIDE IMPACT AT 20 MPH

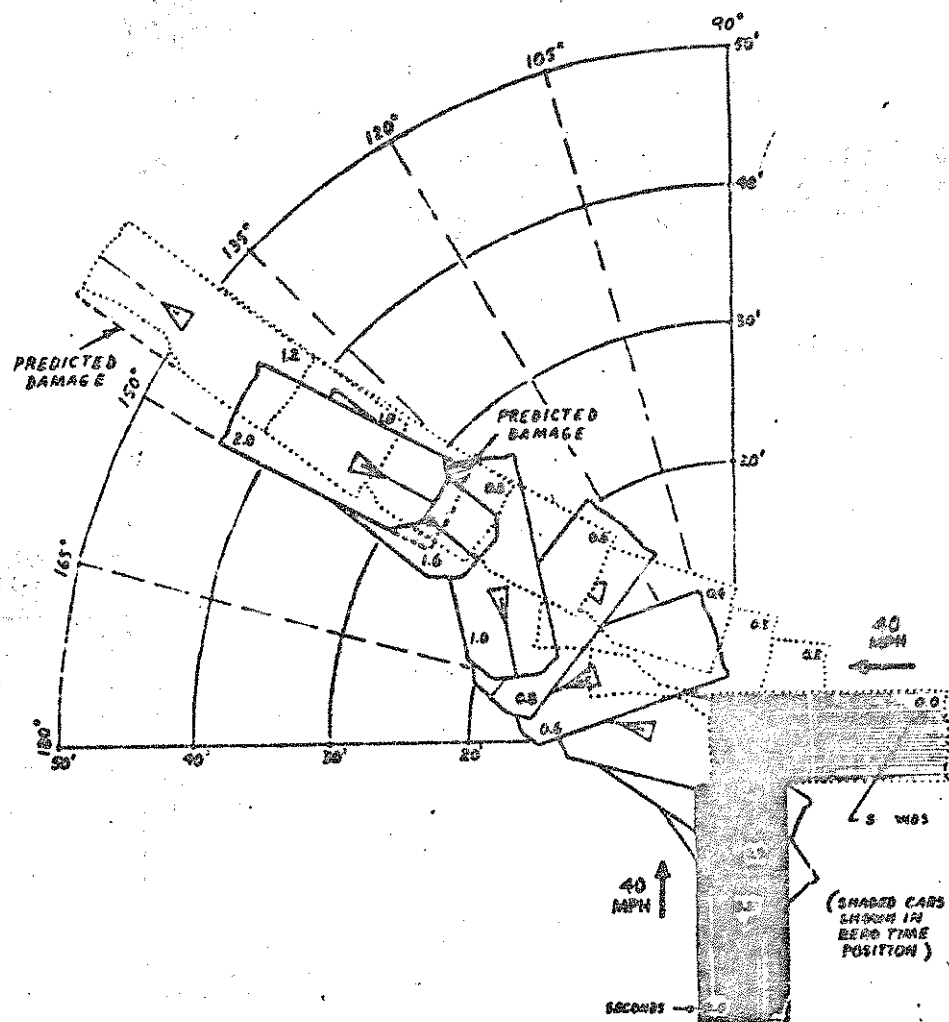


Figure 5A. ANALYTICAL PREDICTIONS OF KINEMATICS AND DAMAGE - FRONT-SIDE IMPACT AT 40 MPH

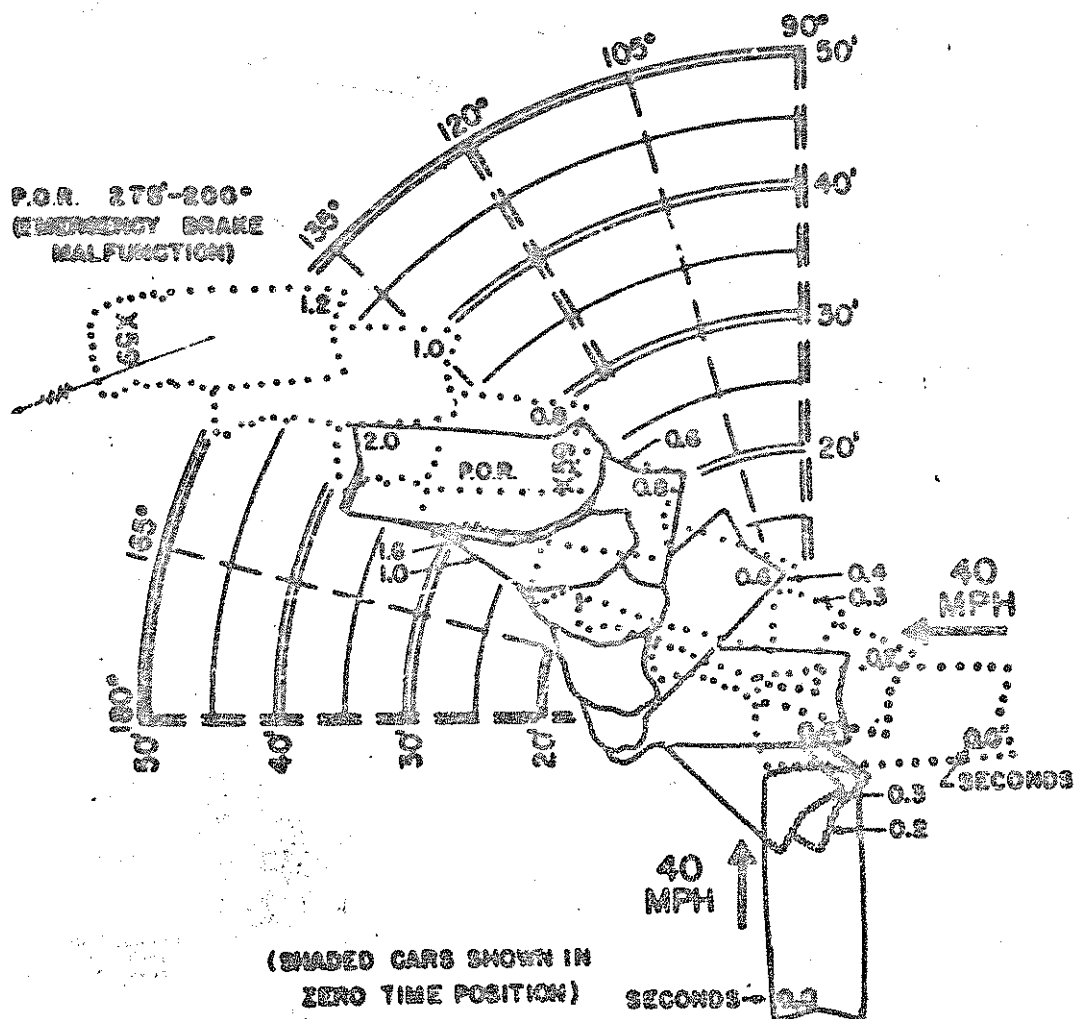


Figure 5B EXPERIMENTAL DATA FROM REFERENCE 13 - FRONT-SIDE IMPACT AT 40 MPH

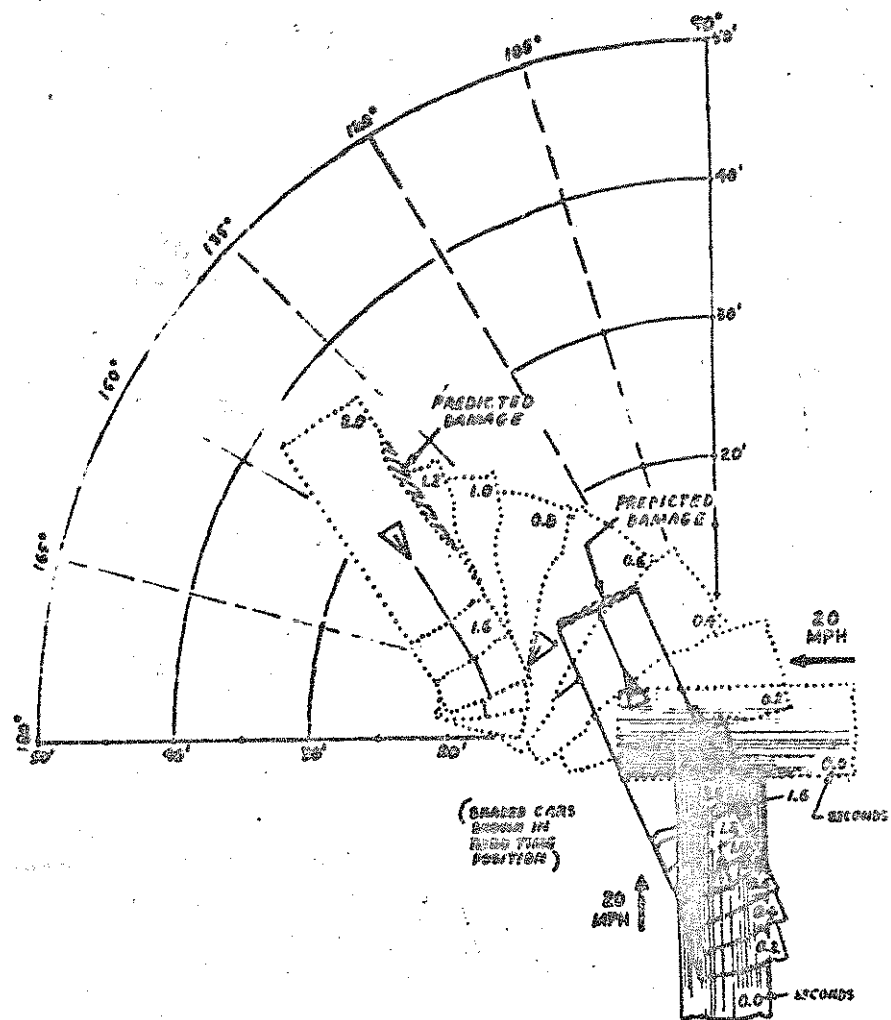


Figure 6A ANALYTICAL PREDICTIONS OF KINEMATICS AND DAMAGE - CENTER-SIDE IMPACT AT 20 MPH

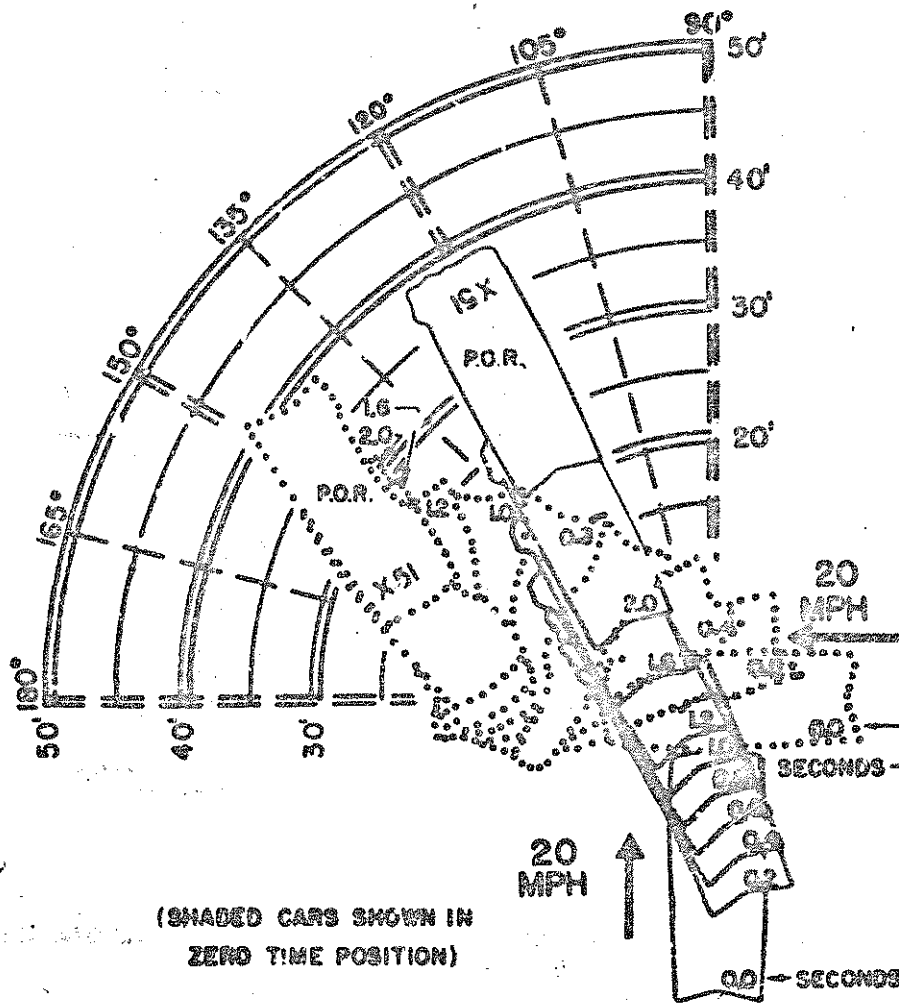


Figure 6B EXPERIMENTAL DATA FROM REFERENCE 13 - CENTER-SIDE IMPACT AT 20 MPH

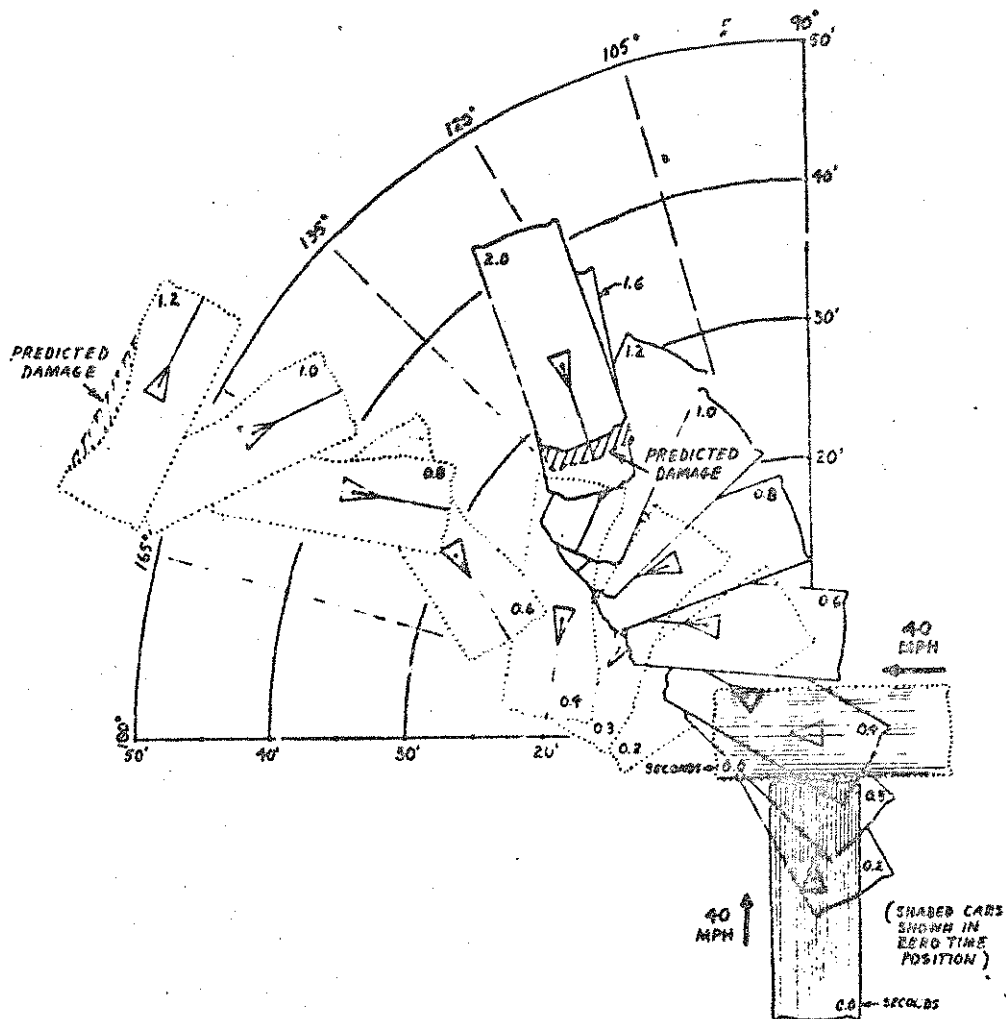


Figure 7A ANALYTICAL PREDICTIONS OF KINEMATICS AND DAMAGE - CENTER-SIDE IMPACT AT 40 MPH

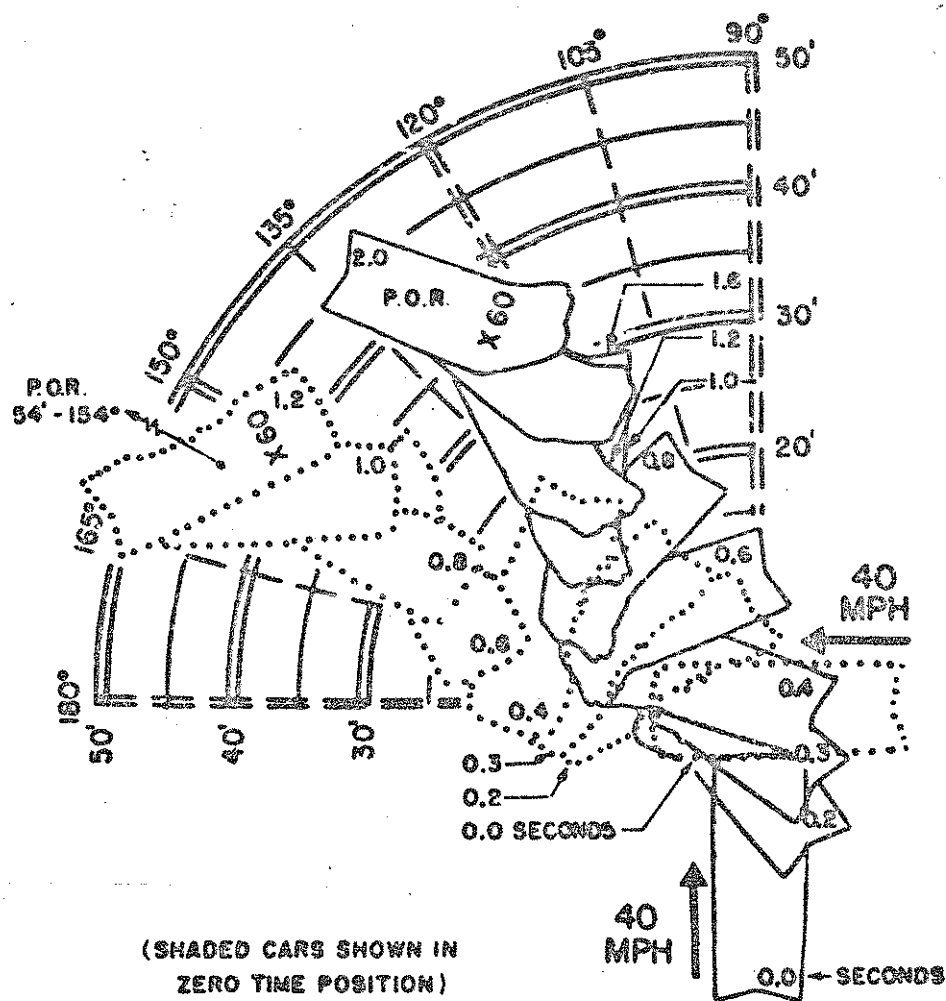


Figure 7B EXPERIMENTAL DATA FROM REFERENCE 13 - CENTER-SIDE IMPACT AT 40 MPH

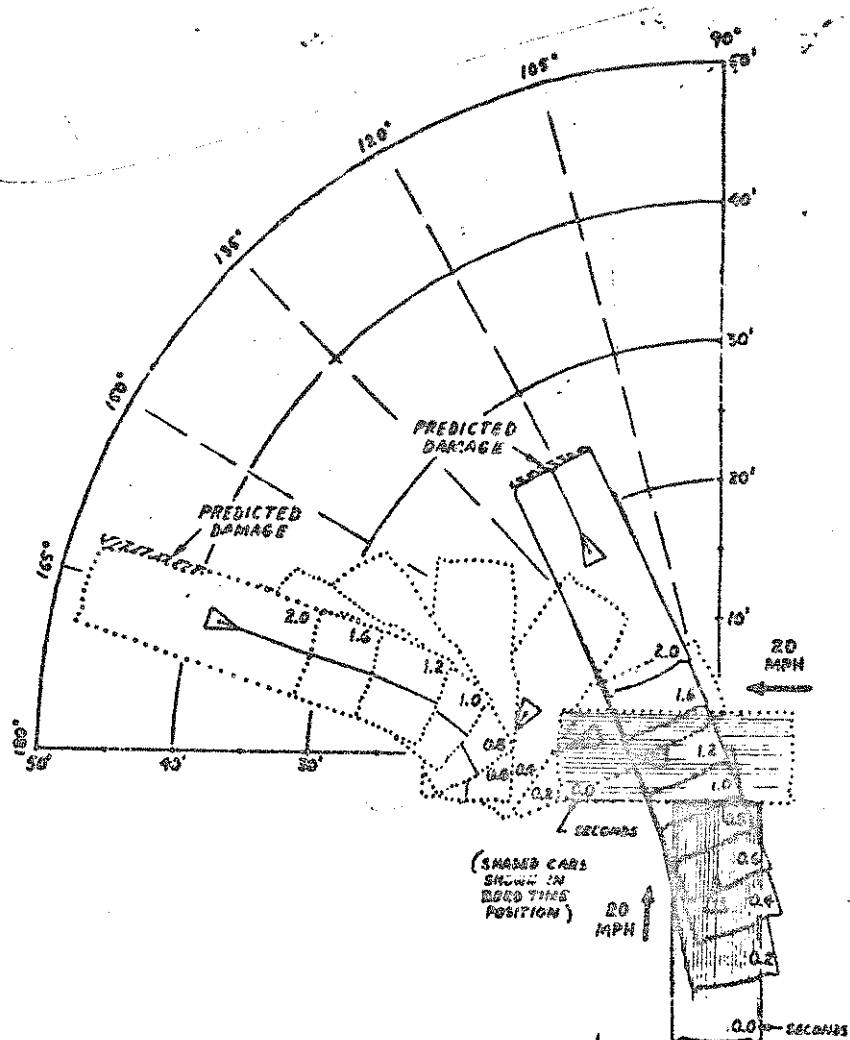


Figure 8A ANALYTICAL PREDICTIONS OF KINEMATICS AND DAMAGE -
REAR-SIDE IMPACT AT 20 MPH

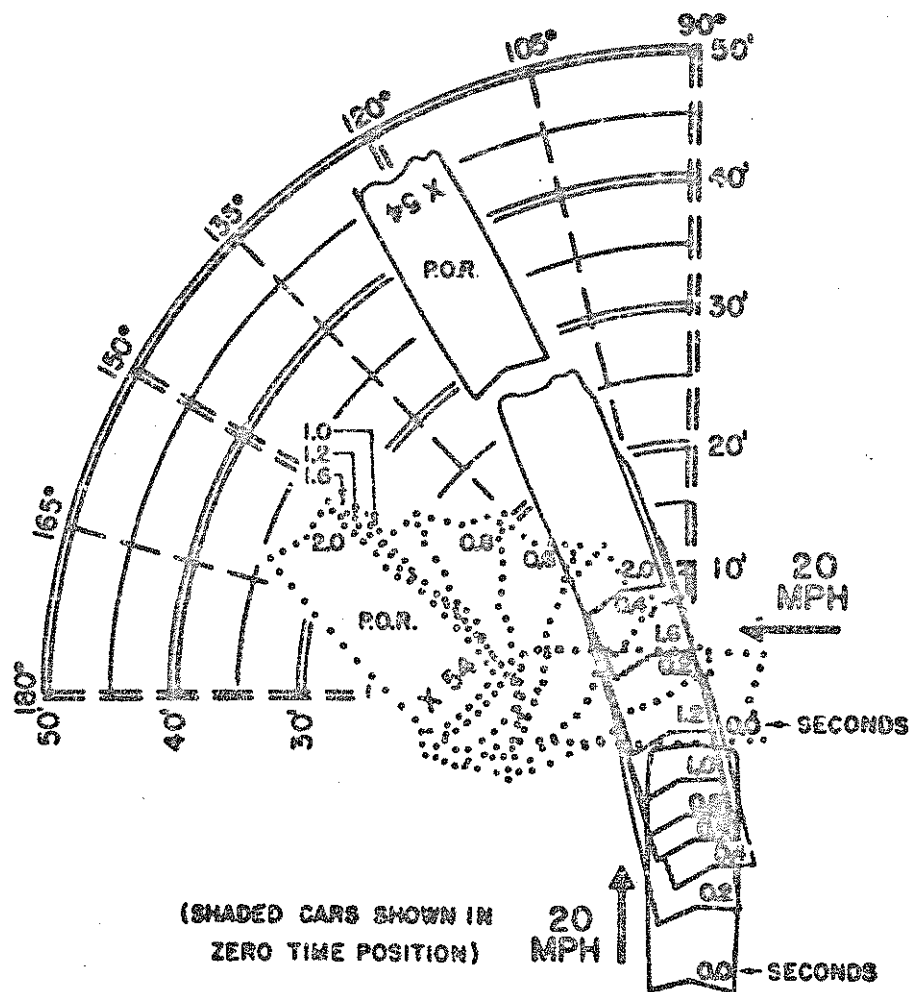


Figure 83 EXPERIMENTAL DATA FROM REFERENCE 13 - REAR-SIDE IMPACT AT 20 MPH

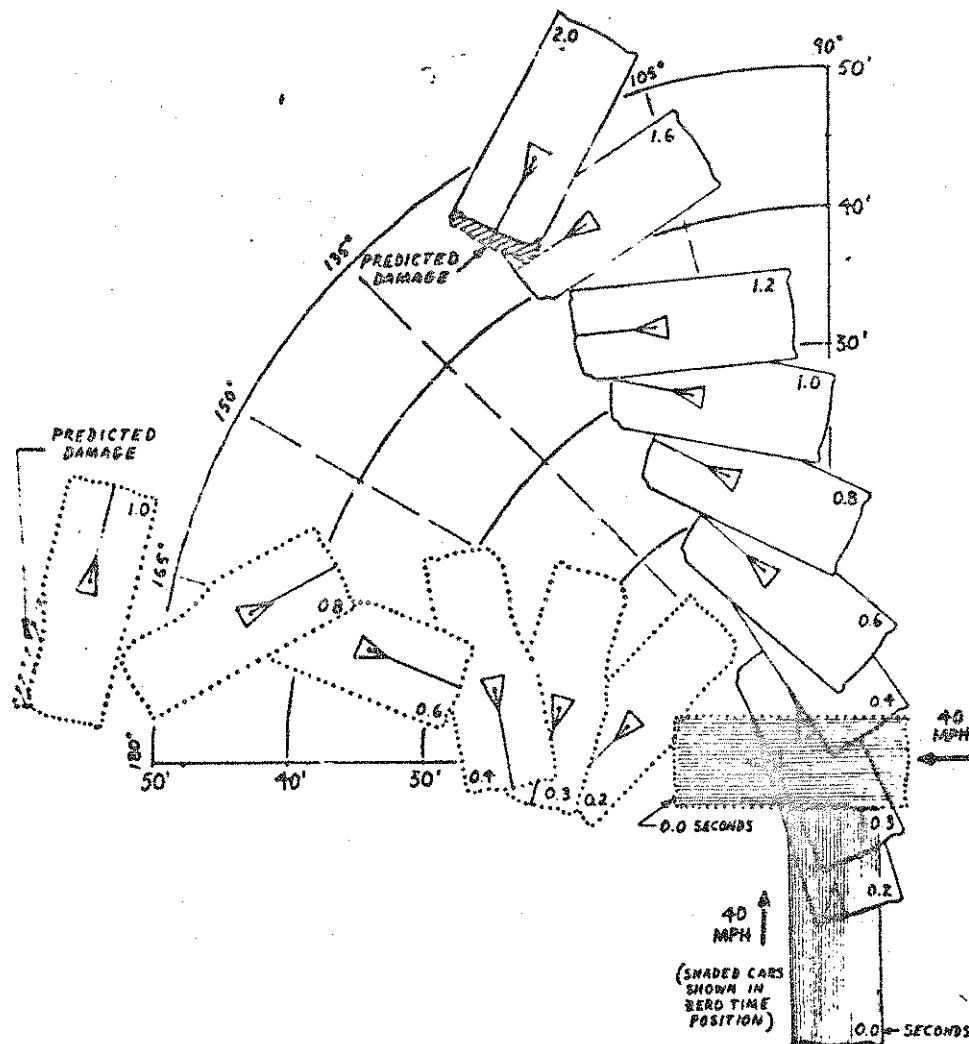


Figure 9A ANALYTICAL PREDICTIONS OF KINEMATICS AND DAMAGE —
REAR-SIDE IMPACT AT 40 MPH

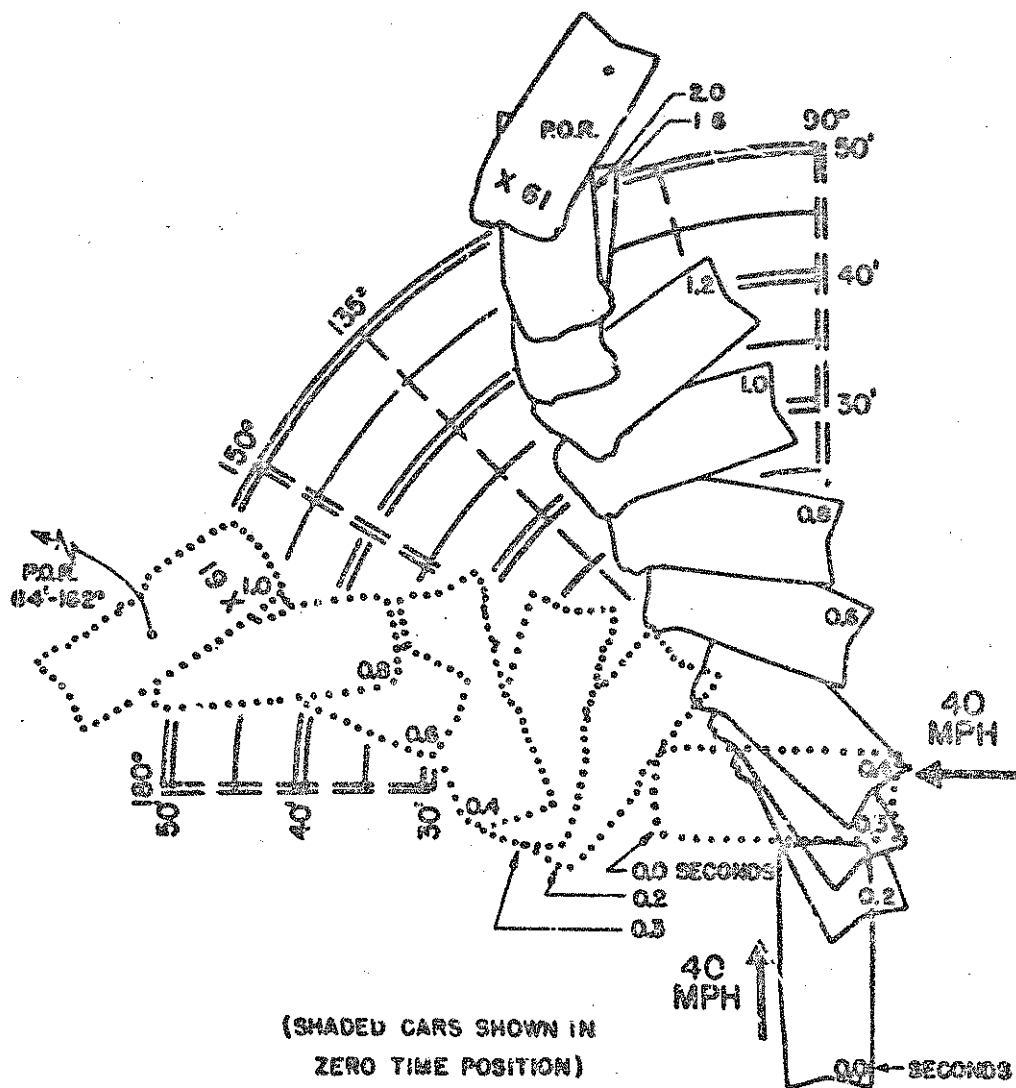


Figure 9B EXPERIMENTAL DATA FROM REFERENCE 13 - REAR-SIDE IMPACT AT 40 MPH

3.3.2.1 Peak Resultant Values

In Figure 10, predicted and measured (Reference 13) values of the resultant peak accelerations are presented for the six collisions of Figures 4 through 9. If the tire forces are assumed to be negligibly small at the moment of occurrence of the peak resultant acceleration, rigid-body behavior would require, on the basis of Newton's Third Law of Motion, that the resultant peak accelerations of the two vehicles be equal and opposite. In the experiments of Reference 13, the measured values of acceleration were obtained by means of tri-axial accelerometers mounted on the vehicle frames near the right center door posts (i.e., in a position that was not directly impacted). Any motions of the accelerometer mounting point relative to the vehicle center of gravity would, of course, produce deviations from equal and opposite readings.

In Figure 10, it may be seen that the averages of the measured peak values of accelerations for the two vehicles agree quite closely with analytical predictions. In the case of the struck vehicle in the 40 MPH center-side impact, an early, short duration, 27 G acceleration spike is followed by a 14 G peak value that matches the timing of the peak acceleration of the striking vehicle (Figure 10 D of Reference 13). It should be noted that this impact configuration would be expected to produce large relative motions of the accelerometer mounting point, since it was opposite the impact point and probably received direct forces through a frame crossmember. In this case, use of the 14 G peak value for the struck vehicle produces close agreement with the corresponding analytical prediction.

COLLISION CONFIGURATION	IMPACT VELOCITY MPH	VEHICLE	MEASURED* RESULTANT PEAK ACCEL. (REF. 13)	AVERAGE OF MEASURED PEAK VALUES OF ACCEL.	PREDICTED RESULTANT PEAK ACCEL.
FRONT-SIDE	20	STRIKING STRUCK	9 G 11 G	10 G	10.1 G
	40	STRIKING STRUCK	17 G 18 G	17.5 G	17.9 G
CENTER-SIDE	20	STRIKING STRUCK	7 G 12 G	9.5 G	10.3 G
	40	STRIKING STRUCK	16 G 27G (14G)**	21.5 (15 G)**	15.7 G
REAR-SIDE	20	STRIKING STRUCK	6 G 9G	7.5 G	7.8 G
	40	STRIKING STRUCK	9 G 17 G	13 G	11.4 G

* TRI-AXIAL ACCELEROMETERS MOUNTED ON VEHICLE FRAMES NEAR RIGHT CENTER DOOR POSTS (REF. 13).

** IN THIS CASE, THE EARLY, SHORT-DURATION, 27G SPIKE IS FOLLOWED BY A 14G PEAK VALUE THAT MATCHES THE TIMING OF THE PEAK ACCELERATION OF THE STRIKING VEHICLE (FIGURE 100 OF REF. 13).

Figure 10 COMPARISON OF PREDICTED AND MEASURED VEHICLE ACCELERATIONS IN 90° INTERSECTION COLLISIONS

3.3.2.2 Time-Histories of Acceleration Components

The developed computer program includes acceleration components at the centers of gravity of the two vehicles. While Reference 13 does not include data for direct comparisons, the durations and wave-forms of Figure 11 are in general agreement with those of resultant frame accelerations in the corresponding experimental data of Reference 13. The magnitudes have, of course, been shown to be in good agreement with experimental data in Section 3.3.2.1.

If desired, the present program outputs could be readily expanded to include acceleration components at individual seating positions (i.e., including effects of angular accelerations about the center of gravity).

3.4 Data Acquisition System

An exploratory investigation of optical systems indicated several potential advantages for purposes of the present research program in the form of increased accuracy and speed, reduced interference with traffic, and adaptability to a standard format for data transmission. It was found, however, that the use of stadia markings for determining range and the reading of vernier scales for azimuth and elevation angles required a significant amount of time and also led to some erroneous data points. Therefore, it was decided to investigate the feasibility of instrumenting a transit with shaft encoders so that the reading process could be reduced to a push-button operation.

The Astrosystems Model AE 30C-2 Shaft Angle Encoder was selected for adaptation to the azimuth and elevation angles of a builder's transit. This device provides a visual digital readout with a resolution of 0.01 degrees. To eliminate the task of manual transcription of the displayed data, and the corresponding possibility of transcription errors, a prototype system was devised to directly punch the transit readings on a

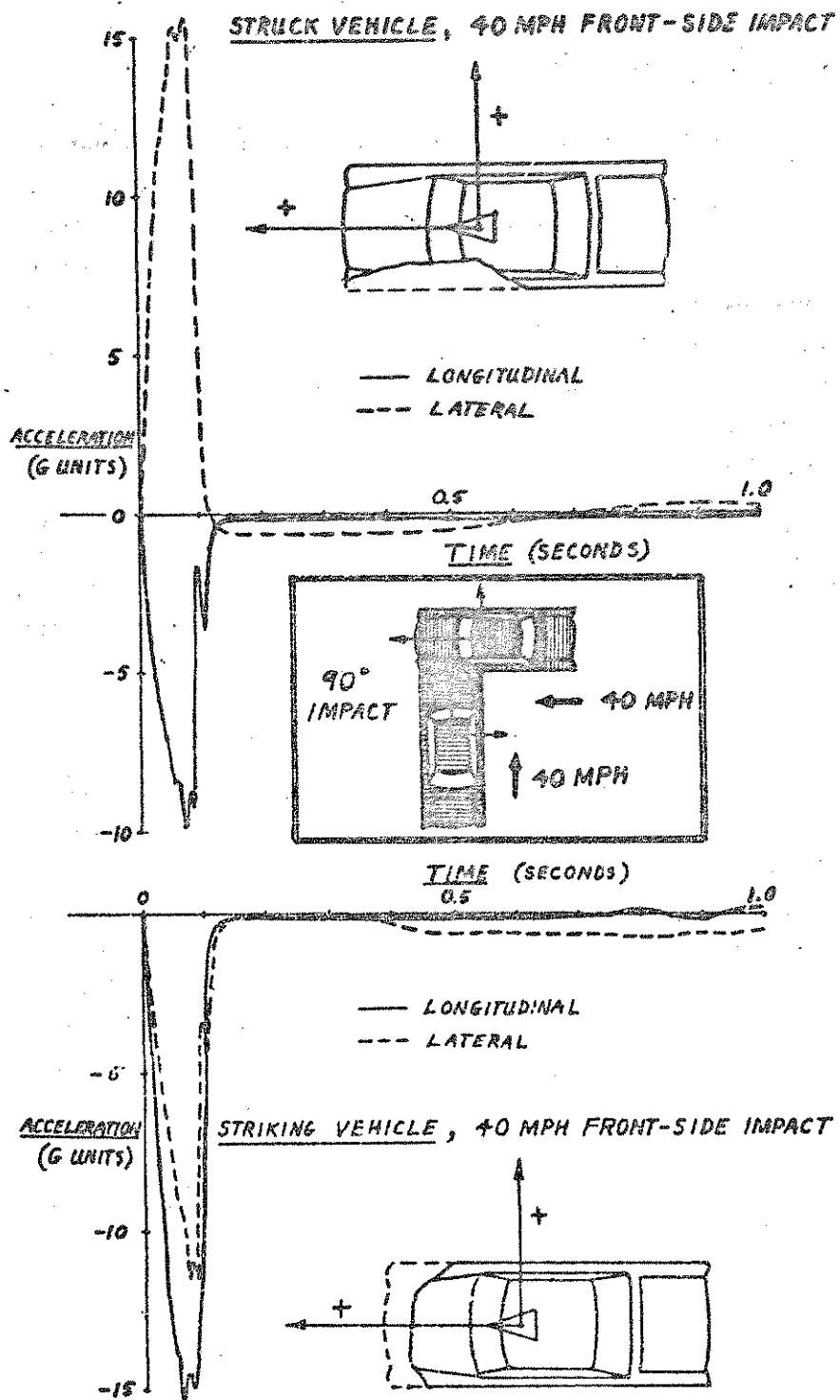


Figure 11 PREDICTED ACCELERATION COMPONENTS AT CENTER OF GRAVITY

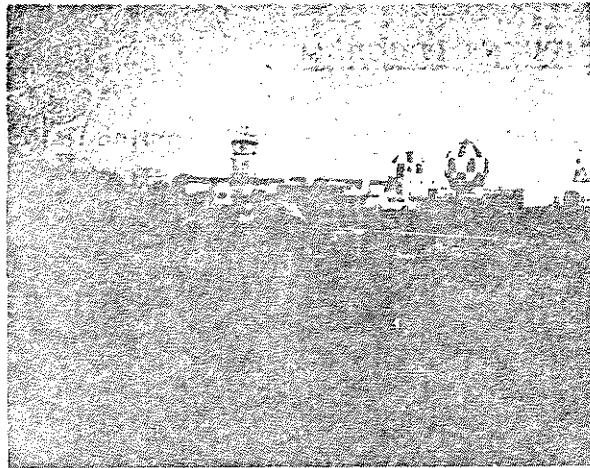
paper tape. While the ultimate form envisioned for this system includes a radio communications link to a paper tape puncher adjacent to a remote time-sharing computer terminal, the prototype system, which was assembled from surplus equipment, makes use of a direct wire to the paper tape puncher.

A standard stadia rod was modified for use in conjunction with the instrumented transit. It has two small lightbulbs mounted five feet apart, a switch and battery pack, and a spirit level. When the lead investigator selects a point to be measured, he places the bottom of the rod on the point, adjusts the rod to stand vertically and turns on the lights to indicate that readings should be taken. Transit readings of the top and bottom lights are taken in sequence for conversion into azimuth angle and range.

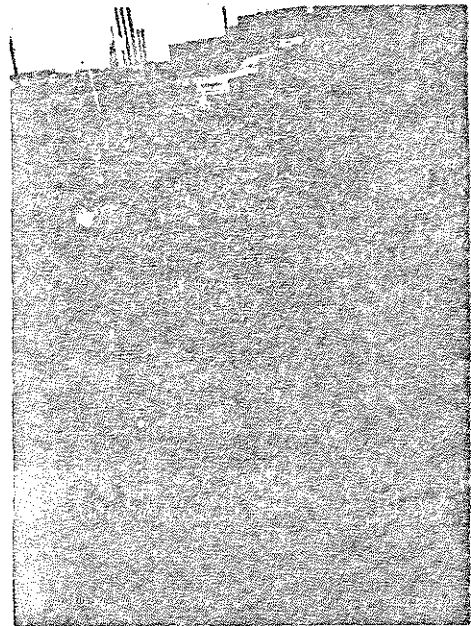
A computer routine has been written to convert the transit data into rectangular coordinates in a selected reference system determined by two permanent reference points located at the accident scene. The initial two readings are used to define the selected reference coordinate system. The first reading locates the origin and the second reading orients the direction of the positive X' axis. In this manner, the measurements are made independent of the specific location of the transit.

A small keyboard with decimal integers is planned for inclusion in the final system so that the operator can code the dimensional information as it is recorded. A five-digit code word is planned which will identify the nature of the data that follows it (e.g., tire skid mark) until a new identification code word is entered.

In Figure 12, photographs taken during a trial application of the described prototype system are shown. An accurate measure of the actual speed capability was not possible in the trial applications because of the experimental use of a very slow paper tape puncher and the manual recording of back-up readings from the digital display. However, even under these conditions, the process was found to be quite rapid. It is obvious that the use of a more efficient recording system will produce a reading speed corresponding to only a momentary pause of the lead investigator at each point of interest. The accuracy and repeatability of the measurement system were demonstrated to be better than one percent.

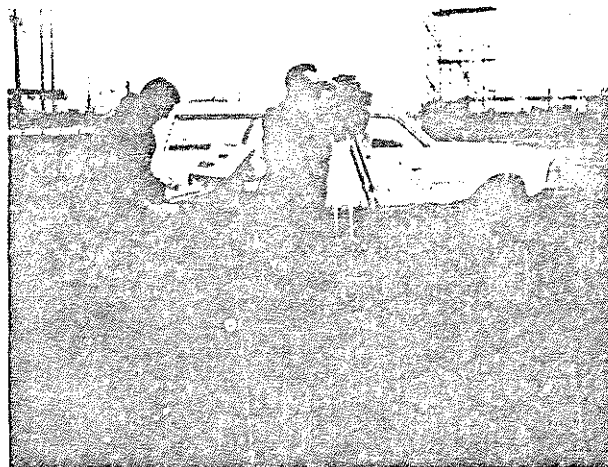


(a) EXPERIMENTAL VEHICLE MANEUVER



(b) SKID MARKS AND REST POSITION

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(c) DATA GATHERING EQUIPMENT

Figure 12 TRIAL APPLICATION OF MEASUREMENT SYSTEM

4.0 DESCRIPTION OF COMPUTER PROGRAM

A major portion of this first-year effort has been applied to the task of development and validation of computer routines to treat the collision and trajectory phases of accident reconstruction. As a result, progress has been hampered on the integration of those routines into an overall computer program for processing accident data. It should be noted, however, that the remaining programming effort is a more straightforward task, consisting of the implementation of relatively simple logic rather than the solution of equations of motion.

As depicted in the simplified flow charts of Figures 13, 14 and 15, applications of the planned overall reconstruction program will consist of three phases: the post-collision trajectories, the collision contact and the precollision trajectories. Each phase will include an iterative procedure aimed at achieving a "best fit" to the overall physical evidence. A library routine will be used to provide system parameters either from storage or by means of estimation procedures (e.g., inertial properties of vehicles, tire characteristics, etc.).

The existing computer programming has been coded in the Fortran IV language to provide ease of adaptation to a variety of computer facilities. While primary attention in this first-year effort has necessarily been focused on the task of achieving operational and valid versions of the major routines, it is recognized that core requirements and operating costs are also items of critical importance. Therefore, those aspects of the program must receive equal attention in future developments.

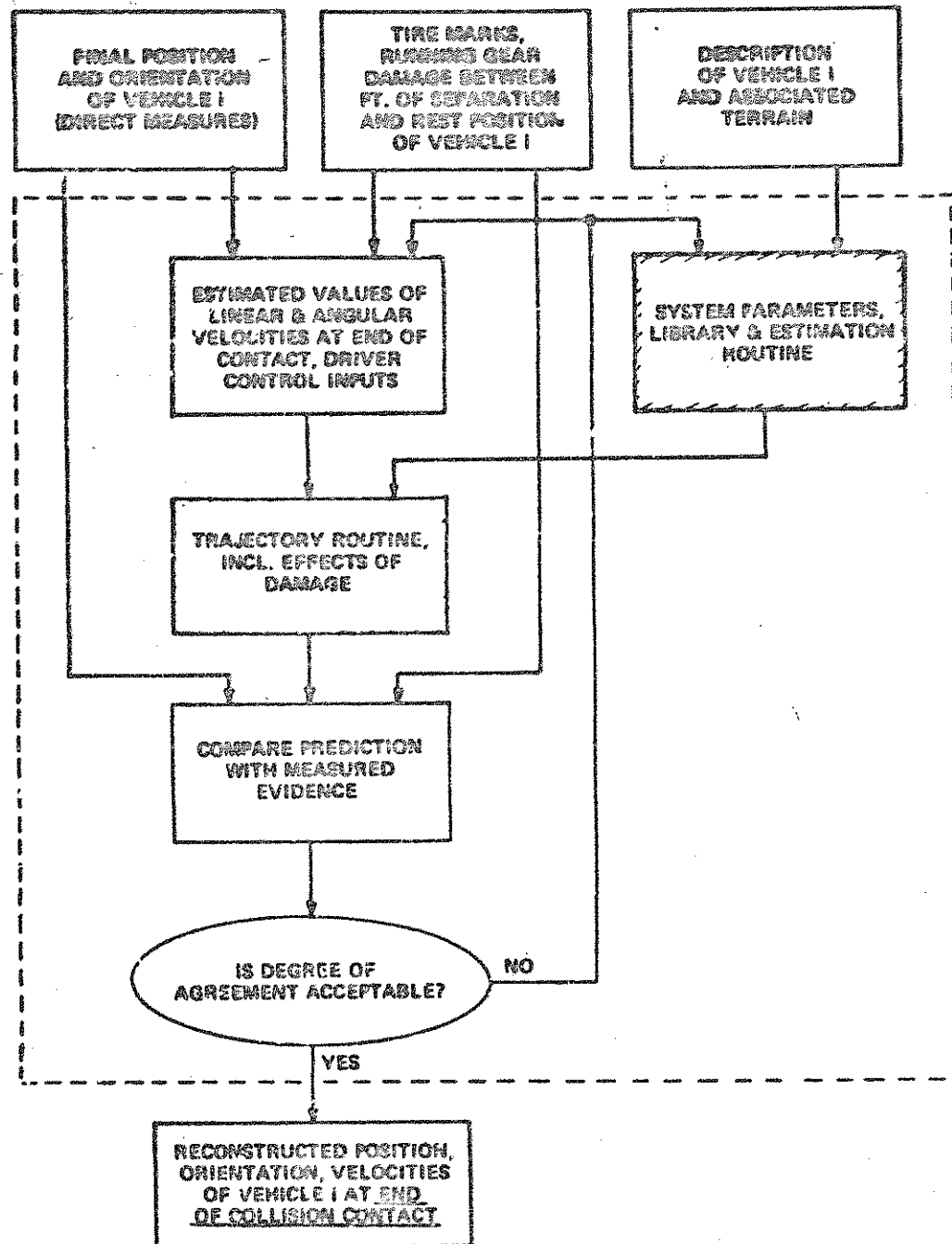


Figure 13 PHASE 1 OF RECONSTRUCTION PROGRAM

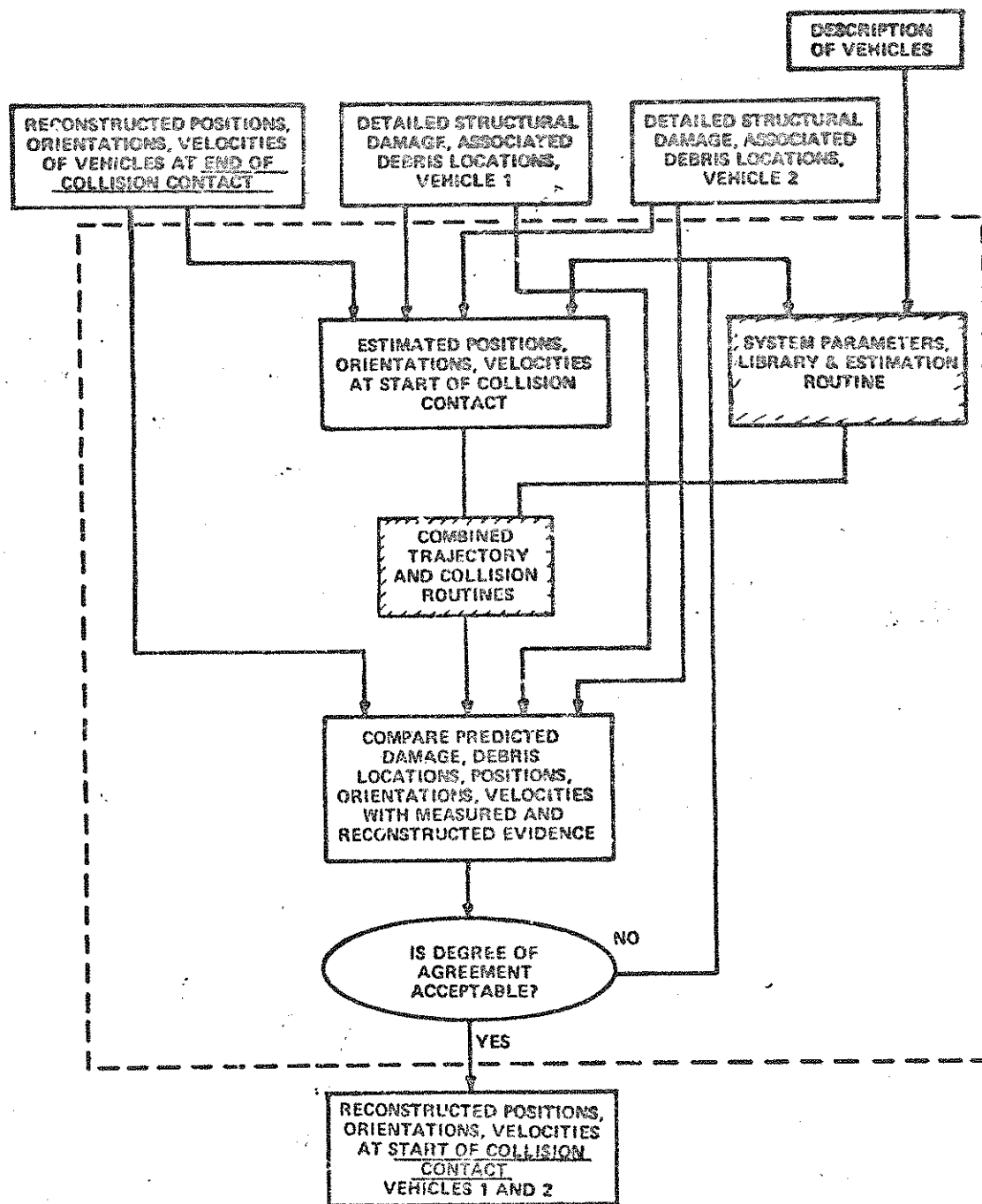


Figure 14 PHASE 2 OF RECONSTRUCTION PROGRAM

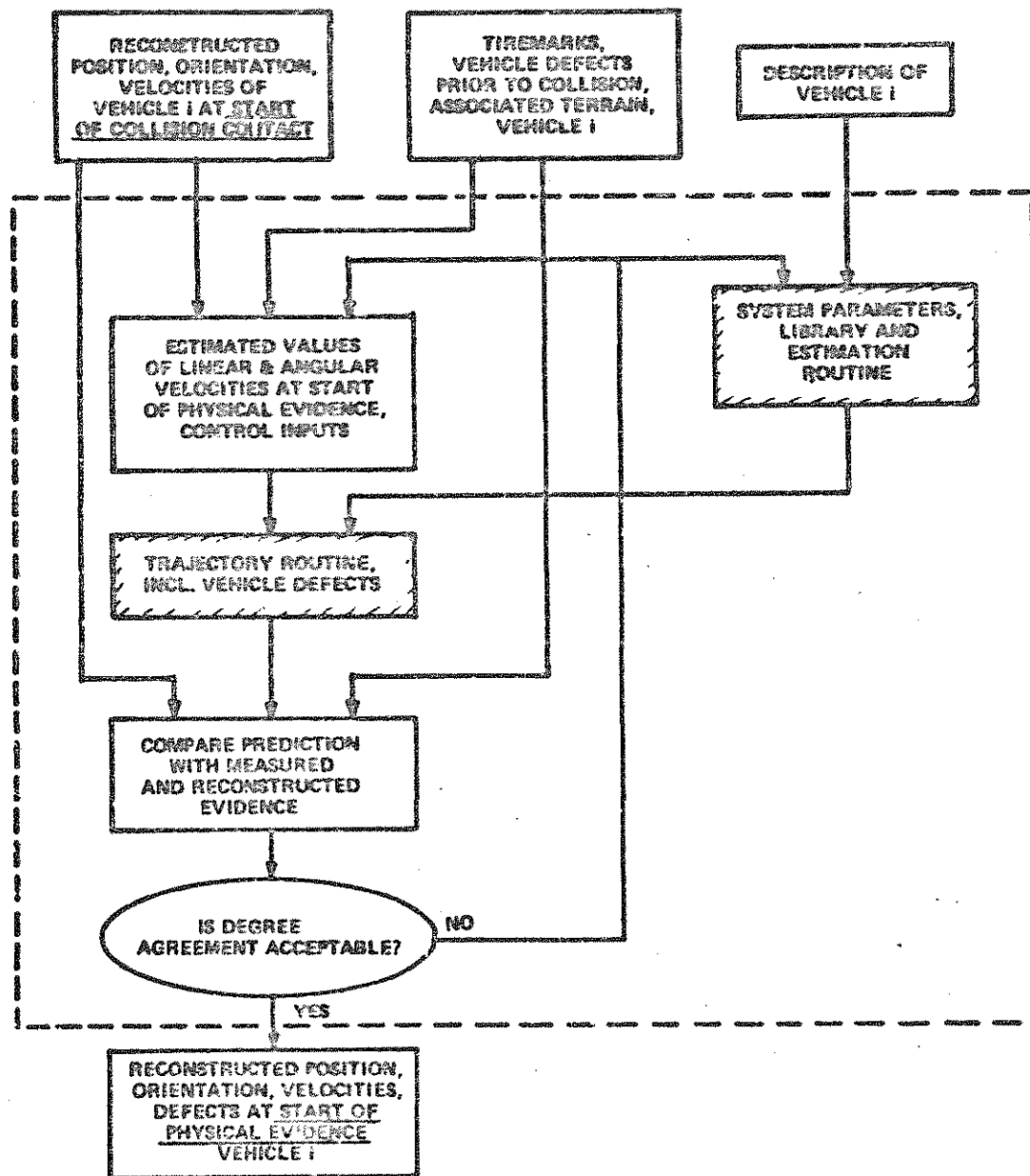


Figure 15 PHASE 3 OF RECONSTRUCTION PROGRAM

The "trajectory" routine (Section 6.3) was initially programmed and checked out on a time-sharing digital terminal (Graphic Controls GC-10 system, utilizing a PDP-10 computer). Because of its greater programming complexity, the "collision" routine (Section 6.1) was developed on the CAL IBM 370 System. As discussed in Section 3.2, it was concluded late in this research program that it was necessary to combine the two routines to achieve acceptable predictions in intersection-type collisions. Therefore, a combined program (Section 6.4) was developed on the IBM 370 system. There has been insufficient time available since completion of that development to establish the minimum core requirements and to explore techniques for reducing operating costs. The range of costs, in this initial form of the combined routines (Section 6.4), has been approximately \$25.00 per application run (i.e., runs presented in Section 3.3).

Also, because of the late decision to combine the two existing routines, it was not possible to achieve an operational version of the partially developed procedure for generating Vehicle Damage Indices (Section 6.2) or to fully implement logic formulated for "reverse" calculations (Section 6.6).

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6.0

APPENDICES

6.1

Collision RoutineInput Requirements1. Dimensional and Inertial Properties

$\left. \begin{array}{l} X_{F1}, X_{R1} \\ X_{F2}, X_{R2} \end{array} \right\}$ = Distances along vehicle fixed X axis from the total vehicle center of gravity to the boundaries of the vehicle at the front and rear, respectively, inches (X_{R1} is entered as a negative quantity).

Y_{S1}, Y_{S2} = Distance along vehicle fixed Y axis from the total vehicle center of gravity to the boundary of the vehicle at the side (i.e., one-half of the total vehicle width), inches.

I_1, I_2 = Moment of inertia of complete vehicle in yaw, lb-sec²-in.

M_1, M_2 = Total vehicle mass, lb-sec²/in.

2. Properties of Deformable Layer of Structure

K_{V1}, K_{V2} = Load-deflection characteristic of peripheral vehicle structure, lb/in.² (corresponding to a given height of contact).

C_0, C_1, C_2 = Constant coefficients in parabolic relationship fitted to approximate variations of the "coefficient of restitution" with deflection.

μ = Friction coefficient for tangential forces between the two interacting bodies.

3. Calculation Constants

$\Delta\psi$ = Angular interval between radial vectors in contact determination, degrees.

λ = Acceptable error in pressure balance between the two bodies, lb/in. (note that a constant height of the contact area is assumed). Also, for solution stability, $(K_{V1}\Delta\rho, K_{V2}\Delta\rho) \leq \lambda$.

$\Delta\rho$ = Increment of change of radial vector length in iterative routine for achieving equilibrium, inches.

S_V = Minimum magnitude of relative velocity for which vehicle-to-vehicle friction forces are calculated, inches/sec.

4. Initial Conditions

$\left. \begin{matrix} X'_{C10}, Y'_{C10} \\ X'_{C20}, Y'_{C20} \end{matrix} \right\}$ = Coordinates of initial position of vehicle center of gravity, inches.

ψ_{10}, ψ_{20} = Initial heading angle, degrees.

$\left. \begin{matrix} u_{10}, v_{10} \\ u_{20}, v_{20} \end{matrix} \right\}$ = Initial velocity components in vehicle coordinate system, inches/sec.

$\dot{\psi}_{10}, \dot{\psi}_{20}$ = Initial angular velocity, degrees/sec.

Output Format

VEHICLE NO. 1

Page 1

Time (sec)	C. G. Position (ft)		Heading Angle (deg)	Velocities (ft/sec)		Angular Velocity (deg/sec)	Acceleration (g units)	
	X'_{c1}	Y'_{c1}		Fwd	Lat		a_{x1}	a_{y1}
t	$X'_{c1/12}$	$Y'_{c1/12}$	$57.296\psi_1$	$U_1/12$	$V_1/12$	$57.296\dot{\psi}_1$	$\frac{\ddot{U}_1 - V_1\dot{\psi}_1}{386.4}$	$\frac{\ddot{V}_1 + U_1\dot{\psi}_1}{386.4}$
0.00	0000.0	0000.0	000.00	000.00	000.00	0000.0	000.00	000.00

VEHICLE NO. 2

Page 2

Time (sec)	C. G. Position (ft)		Heading Angle (deg)	Velocities (ft/sec)		Angular Velocity (deg/sec)	Acceleration (g units)	
	X'_{c2}	Y'_{c2}		Fwd	Lat		a_{x2}	a_{y2}
t	$X'_{c2/12}$	$Y'_{c2/12}$	$57.296\psi_2$	$U_2/12$	$V_2/12$	$57.296\dot{\psi}_2$	$\frac{\ddot{U}_2 - V_2\dot{\psi}_2}{386.4}$	$\frac{\ddot{V}_2 + U_2\dot{\psi}_2}{386.4}$
0.00	0000.0	0000.0	000.00	000.00	000.00	0000.0	000.00	000.00

DAMAGE SUMMARY
(DISPLACED POINTS)

Page 3

VEHICLE 1		VEHICLE 2	
X_1	Y_1	X_2	Y_2
$\rho_{B1} \cos \psi_{B1}$	$\rho_{B1} \sin \psi_{B1}$	$\rho_{B2} \cos \psi_{B2}$	$\rho_{B2} \sin \psi_{B2}$
000.0	000.0	000.0	000.0
(Corresponding to of Table 1)	$\rho_{B1} \cdot \psi_{B1}$	(Corresponding to of Table 2)	$\rho_{B2} \cdot \psi_{B2}$

Car-to-Car Collision Routine

1. Set $i = 1$, $j = 2$.

2. Corner points of vehicle j in space-fixed system:

$$\begin{aligned} X'_{nj} &= X'_{cj} + E_j \cos \psi_j - G_j \sin \psi_j \\ Y'_{nj} &= Y'_{cj} + E_j \sin \psi_j + G_j \cos \psi_j \end{aligned}$$

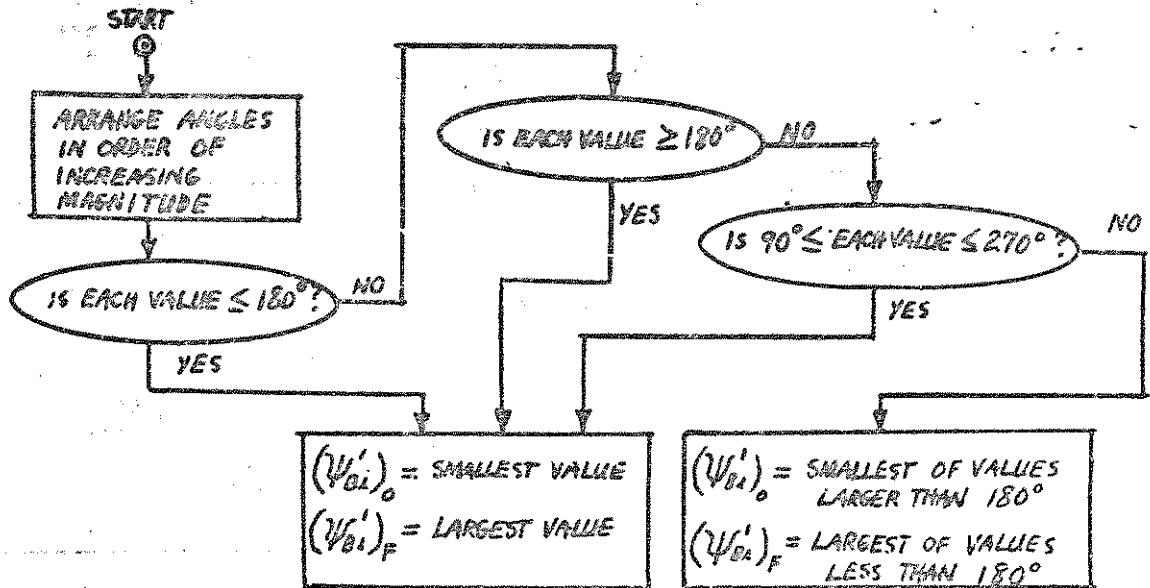
where

n	E_j	G_j	Corner
1	X_{Fj}	$+ Y_{sj}$	LF
2	X_{Fj}	$- Y_{sj}$	RF
3	X_{Rj}	$+ Y_{sj}$	LR
4	X_{Rj}	$- Y_{sj}$	RR

3. Range of clockwise angular sweep for vehicle i :

$$(\psi'_{Bi})_n = \arctan \left[\frac{Y'_{nj} - Y'_{ci}}{X'_{nj} - X'_{ci}} \right] - \psi_i$$

where $n = 1, 2, 3, 4$; $0^\circ \leq (\psi'_{Bi})_n \leq 360^\circ$.



4. Set $(\psi'_{Bi})_o = (n-1)(\Delta\psi)$, where $n = \frac{(\psi'_{Bi})_o}{(\Delta\psi)}$ rounded off to the nearest integer.

5. Start of clockwise sweep of vehicle $\dot{\lambda}$:

$$\text{set } \psi_{Bi} = (\psi_{Bi})_o$$

6. For $360^\circ < \psi_{Bi}$, set $\psi_{Bi} = \psi_{Bi} - 360^\circ$.

7. Is $-180^\circ < [(\psi'_{Bi})_F - \psi_{Bi}] \leq 0$?

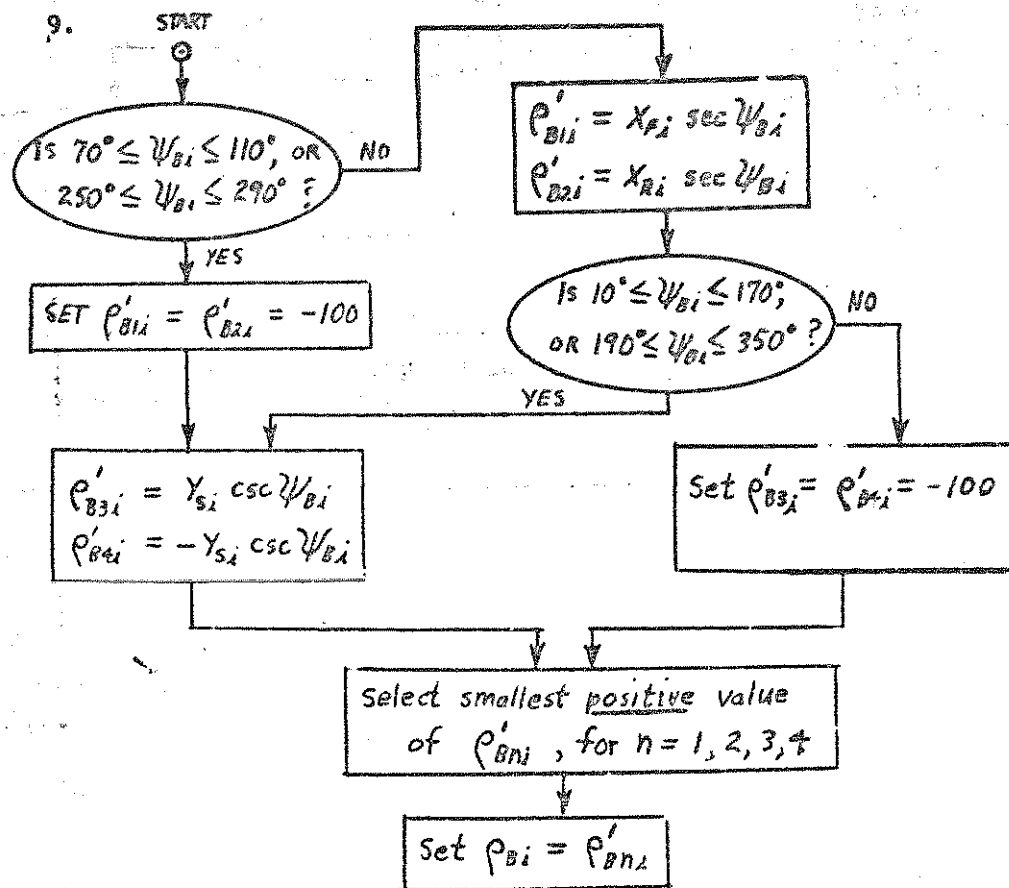
(a) If no, proceed to (8).

(b) If yes, go to (20).

8. For ψ_{Bi} , is there an entry in Table $\dot{\lambda}$?

(a) If yes, set $\rho_{Bi} = \text{tabular value}$, go to (10).

(b) If no, go to (9).



Enter [ψ_{Bi} , $\rho_{Bi} = (\rho_{Bi})_{\max}$, in Table j as temporary entry.

10. Transformation of point (ψ_{Bi} , ρ_{Bi}) to vehicle j coordinate system:

$$(X_{Bi})_j = (Y'_{Ci} - Y'_{Cj}) \sin \psi_j + (X'_{Ci} - X'_{Cj}) \cos \psi_j + \rho_{Bi} \cos (\psi_{Bi} + \psi_i - \psi_j)$$

$$(Y_{Bi})_j = (Y'_{Ci} - Y'_{Cj}) \cos \psi_j - (X'_{Ci} - X'_{Cj}) \sin \psi_j + \rho_{Bi} \sin (\psi_{Bi} + \psi_i - \psi_j)$$

11. Is $X_{Fj} < (X_{Bi})_j < X_{Rj}$ and $-Y_{Sj} < (Y_{Bi})_j < Y_{Sj}$?

(a) If yes, go to (12).

(b) If no, reject (B_i) [remove from Table λ , if a temporary entry, i.e., step (9) entry] increment ψ_{B_i} by $+\Delta\psi$, return to (6).

12. $\psi'_{Bj} = \arctan \frac{(Y_{Bi})_j}{(X_{Bi})_j}$, where $0 \leq \psi'_{Bj} \leq 360^\circ$.

13. $\psi_{Bj} = n(\Delta\psi)$, where $n = \frac{\psi'_{Bj}}{(\Delta\psi)}$ rounded off to the nearest integer.

14. For ψ_{Bj} , is there an entry in Table j ?

(a) If yes, set ρ_{Bj} = tabular value, go to (15).

(b) If no, go to (16).

15. Is $\rho_{Bj} \leq \sqrt{(X_{Bi})_j^2 + (Y_{Bi})_j^2}$?

(a) If yes, reject (B_i) [remove from Table λ , if a temporary entry, i.e., a step (9) entry] increment ψ_{B_i} by $+\Delta\psi$, return to (6).

(b) If no, proceed to (16).

16. Calculate values for the following two sets:

(a) $K_{Vi}(X_{Fi} - X_{Bi}), K_{Vi}(X_{Bi} - X_{Ri}), K_{Vi}(Y_{Si} - |Y_{Bi}|)$

(b) $K_{Vj}[X_{Fj} - (X_{Bi})_j], K_{Vj}[(X_{Bi})_j - X_{Rj}], K_{Vj}[Y_{Sj} - |(Y_{Bi})_j|]$

(Note that all values must be positive.)

17. Select the smallest value from each set of step (16).

P_i = smallest value of set (a).

P_j = smallest value of set (b).

18. Is $|P_i - P_j| < \lambda$ lb/in.?

- (a) If no, increment X_{Bi} , Y_{Bi} , $(X_{Bi})_j$, $(Y_{Bi})_j$ as follows:

$$\begin{aligned}\Delta X_{Bi} &= -(\Delta p) \cos \psi_{Bi} \\ \Delta Y_{Bi} &= -(\Delta p) \sin \psi_{Bi} \\ \Delta (X_{Bi})_j &= -(\Delta p) \cos (\psi_{Bi} + \psi_i - \psi_j) \\ \Delta (Y_{Bi})_j &= -(\Delta p) \sin (\psi_{Bi} + \psi_i - \psi_j)\end{aligned}$$

Return to (16).

- (b) If yes, enter X_{Bi} , Y_{Bi} , or $(X_{Bi})_j$, $(Y_{Bi})_j$, along with P_i , ψ_{Bi} in Table 3.

Table 3 is to contain all points deflected within the current time increment on both vehicles, as defined, or transformed into, the coordinate system of vehicle 1. The force calculation will be based on vehicle 1. Points should be entered in the order of a clockwise sweep of vehicle 1.

19. Recalculate ρ_{Bi} , as follows:

$$\rho_{Bi} = C(\rho_{Bi})_{\text{temp}} + (1-C) \sqrt{X_{Bi}^2 + Y_{Bi}^2}$$

where $(\rho_{Bi})_{\text{temp}}$ = temporary value of ρ_{Bi} entered in Table A in step (9).

$$C = \begin{cases} C_0 - C_1 \delta + C_2 \delta^2, & \text{for } 0 < \delta < \frac{C_1}{2C_2} \\ 0, & \text{for } \frac{C_1}{2C_2} \leq \delta \end{cases}$$

$$\delta = (P_{B1})_{max} - \sqrt{X_{B1}^2 + Y_{B1}^2}$$

Enter new value of P_{B1} in Table 1 as a permanent entry to replace $(P_{B1})_{max}$ of step (9) corresponding to ψ_{B1} . Increment ψ_{B1} by $+\Delta\psi$, return to (6).

20. Is $i = 1$?

(a) If yes, set $i = 2$, $j = 1$, return to (2).

(b) If no, proceed to (21).

21. In Table 3, start with the value of X_{B1} or $(X_{B2})_1$ corresponding to the first point in a clockwise sweep of vehicle 1.

$$F_{n1} = \left[\frac{(P_1)_n + (P_1)_{n+1}}{2} \right] \sqrt{[(X_{B1})_n - (X_{B1})_{n+1}]^2 + [(Y_{B1})_n - (Y_{B1})_{n+1}]^2}$$

$$\psi_{F n1} = \arctan \left[\frac{(Y_{B1})_n - (Y_{B1})_{n+1}}{(X_{B1})_n - (X_{B1})_{n+1}} \right]$$

(Note that the calculation must be able to accommodate a zero denominator (i.e., $\psi_{F n1} = 90^\circ$) since a frontal collision can yield $\psi_{F n1} = 90^\circ$ for the entire set of deflected points.)

$$22. \quad X_1 = \left[\frac{(X_{B1})_n + (X_{B1})_{n+1}}{2} \right], \quad Y_1 = \left[\frac{(Y_{B1})_n + (Y_{B1})_{n+1}}{2} \right]$$

$$\psi_n = \arctan \frac{Y_1}{X_1}$$

$$X_2 = (Y'_{c1} - Y'_{c2}) \sin \psi_2 + (X'_{c1} - X'_{c2}) \cos \psi_2 \\ + \sqrt{X_1^2 + Y_1^2} \cos (\psi_n + \psi_1 - \psi_2)$$

$$Y_2 = (Y'_{c1} - Y'_{c2}) \cos \psi_2 - (X'_{c1} - X'_{c2}) \sin \psi_2 \\ + \sqrt{X_1^2 + Y_1^2} \sin (\psi_n + \psi_1 - \psi_2)$$

$$23. \quad V_{T1} = (U_1 + Y_1 \dot{\psi}_1) \cos \psi_{Fn1} + (V_1 + X_1 \dot{\psi}_1) \sin \psi_{Fn1}$$

$$V_{T2} = (U_2 + Y_2 \dot{\psi}_2) \cos (\psi_2 - \psi_1 - \psi_{Fn1}) \\ - (V_2 + X_2 \dot{\psi}_2) \sin (\psi_2 - \psi_1 - \psi_{Fn1})$$

$$24. \quad \gamma = \begin{cases} 0, & \text{for } |V_{T2} - V_{T1}| < S_v \\ \mu \operatorname{sgn}(V_{T2} - V_{T1}), & \text{for } S_v \leq |V_{T2} - V_{T1}| \end{cases}$$

$$25. \quad F_{nx1} = F_{n1} (\sin \psi_{Fn1} + \gamma \cos \psi_{Fn1})$$

$$F_{ny1} = F_{n1} (\gamma \sin \psi_{Fn1} - \cos \psi_{Fn1})$$

$$N_{n1} = -F_{nx1} Y_1 + F_{ny1} X_1$$

$$26. \quad F_{nx2} = -F_{nx1} \cos (\psi_2 - \psi_1) - F_{ny1} \sin (\psi_2 - \psi_1)$$

$$F_{ny2} = F_{nx1} \sin (\psi_2 - \psi_1) - F_{ny1} \cos (\psi_2 - \psi_1)$$

$$N_{n2} = -F_{nx2} Y_2 + F_{ny2} X_2$$

$$27. \quad F_{x1} = \sum_1^n F_{nx1}, \quad F_{y1} = \sum_1^n F_{ny1}, \quad N_1 = \sum_1^n N_{n1} \\ F_{x2} = \sum_1^n F_{nx2}, \quad F_{y2} = \sum_1^n F_{ny2}, \quad N_2 = \sum_1^n N_{n2}$$

28. Clear Table 3.

29. Equations of motion:

$$\begin{array}{l|l} \ddot{\psi}_1 = \frac{N_1}{I_1} & \ddot{\psi}_2 = \frac{N_2}{I_2} \\ \dot{u}_1 - v_1 \dot{\psi}_1 = \frac{F_{x1}}{M_1} & \dot{u}_2 - v_2 \dot{\psi}_2 = \frac{F_{x2}}{M_2} \\ \dot{v}_1 + u_1 \dot{\psi}_1 = \frac{F_{y1}}{M_1} & \dot{v}_2 + u_2 \dot{\psi}_2 = \frac{F_{y2}}{M_2} \end{array}$$

30. $\dot{\psi}_1 = \dot{\psi}_{10} + \int_0^t \ddot{\psi}_1 dt$
 $\psi_1 = \psi_{10} + \int_0^t \dot{\psi}_1 dt$
 $x'_{c1} = x'_{c10} + \int_0^t (u_1 \cos \psi_1 - v_1 \sin \psi_1) dt$
 $y'_{c1} = y'_{c10} + \int_0^t (u_1 \sin \psi_1 + v_1 \cos \psi_1) dt$

31. $\dot{\psi}_2 = \dot{\psi}_{20} + \int_0^t \ddot{\psi}_2 dt$
 $\psi_2 = \psi_{20} + \int_0^t \dot{\psi}_2 dt$
 $x'_{c2} = x'_{c20} + \int_0^t (u_2 \cos \psi_2 - v_2 \sin \psi_2) dt$
 $y'_{c2} = y'_{c20} + \int_0^t (u_2 \sin \psi_2 + v_2 \cos \psi_2) dt$

32. Return to (1).

Table 1		Table 2		Table 3		
ψ_{01}	ρ_{01}	ψ_{02}	ρ_{02}	x_1	y_1	\bar{p}_1
Deg	Inches	Deg	Inches	Inches	Inches	Lb/in.
(100 Pt. Capacity)		(100 Pt. Capacity)		(100 Pt. Capacity)		
Retained for Print-Out				Cleared Between Time Intervals		

6.2

Post-Processing of Collision Routine Outputs to Generate Vehicle Damage Index (VDI)

(Based on SAE J244a, 9/29/71 Draft)

The classification system consists of seven characters, three numeric and four alphanumeric, arranged in a specific order. In the collision routine output, the classification should appear with each "DAMAGE SUMMARY" table as VDI = 0000000.

1. Direction of Principal Force at Impact

(Columns 1 and 2)

1.1 In outputs for each vehicle, scan AX_j , AY_j to find the maximum value of $[(AX_j)^2 + (AY_j)^2]$.

If more than one impact occurred (i.e., both $|AX_j|$ and $|AY_j|$ less than 1.0 g's subsequent to a maximum, followed by larger values) find the cited maximum quantity for each impact. Order multiple impacts according to the relative magnitudes of the values $[(AX_j)^2 + (AY_j)^2]$ (i.e., starting with the largest value, in order of decreasing magnitudes).

1.2 For each maximum value determined in 1.1, let $\psi_p = \arctan AY_j/AX_j$, where $0 \leq \psi_p \leq 360^\circ$. (Use $\text{sgn } AY_j$, $\text{sgn } AX_j$ to determine quadrant.)

1.3 $N' = \frac{\psi_p}{30}$ rounded off to nearest integer.

1.4 $N = N' + 6$

1.5 For $N \leq 12$, proceed.
For $12 < N$, set $N = N - 12$.

1.6 For each impact, set columns 1 and 2 = classification to N (i.e., 01, 02, 03, , 10, 11, 12).

2. Deformation Location and Classification Code
(Column 3)

2.1 In the damage summary tables, determine the end points of each continuous angular range (i.e., ranges where $\Delta(\psi_{ij}) = \Delta\psi$). Note that a continuous range across 0° will appear at the ends of the table. Designate end points as ψ_{D1} (beginning) and ψ_{D2} (end) of continuous clockwise sweeps, where $0 \leq \psi_{D1} \leq 360^\circ$ and $0 \leq \psi_{D2} \leq 360^\circ$.

2.2 Calculation of midpoints of ranges determined in 2.1.

(a) If range contains zero, go to 2.4.

(b) If range does not contain zero, proceed.

2.3
$$\psi_M = \frac{\psi_{D1} + \psi_{D2}}{2}, \text{ go to 2.5.}$$

2.4
$$\psi_M = \frac{(\psi_{D1} - 360^\circ) + \psi_{D2}}{2}$$

For $\psi_M < 0$, set $\psi_M = \psi_M + 360^\circ$.

2.5 For multiple impacts (i.e., more than one value of N in 1.6, the values of N can be matched with values of ψ_M as follows:

$$\psi_M \approx 30N, \text{ find nearest values.}$$

For several values of N near one ψ_M (i.e., several impacts within one region of damage) select the N corresponding to the largest magnitude of $[(Ax_j)^2 + (Ay_j)^2]$.

2.6 Calculate the following 12 angles. All angles to be between zero and 360°.

$$\begin{array}{ll}
 \psi'_1 = \arctan 0.333 \frac{1}{2} & \psi'_7 = \arctan -0.333 \frac{1}{2} \\
 \psi'_2 = \arctan \frac{1}{2} & \psi'_8 = \arctan -\frac{1}{2} \\
 \psi'_3 = \arctan 2.702 \frac{1}{2} & \psi'_9 = \arctan -1.754 \frac{1}{2} \\
 \psi'_4 = \arctan 1.754 \frac{1}{2} & \psi'_{10} = \arctan -2.702 \frac{1}{2} \\
 \psi'_5 = \arctan \frac{1}{2} & \psi'_{11} = \arctan -\frac{1}{2} \\
 \psi'_6 = \arctan 0.333 \frac{1}{2} & \psi'_{12} = \arctan -0.333 \frac{1}{2}
 \end{array}$$

Note that the above angles lie in the following quadrants:

$$\begin{array}{lll}
 \text{1st Quadrant:} & \psi'_1 & \psi'_2 & \psi'_3 \\
 \text{2nd Quadrant:} & \psi'_4 & \psi'_5 & \psi'_6 \\
 \text{3rd Quadrant:} & \psi'_7 & \psi'_8 & \psi'_9 \\
 \text{4th Quadrant:} & \psi'_{10} & \psi'_{11} & \psi'_{12}
 \end{array}$$

2.7 For $\psi'_{11} \leq \psi_M \leq 360^\circ$, or $0 \leq \psi_M \leq \psi'_2$, set column 3 = F, go to 3.1.

For $\psi'_2 < \psi_M < \psi'_5$, set column 3 = R, go to 3.2.

For $\psi'_5 \leq \psi_M \leq \psi'_8$, set column 3 = B, go to 3.3.

For $\psi'_8 < \psi_M < \psi'_{11}$, set column 3 = L, go to 3.4.

3.

Specific Horizontal Location of Deformation

(Column 4)

FRONT

3.1 For $\psi'_1 \leq \psi_{D1} < \psi'_{10}$, set column 4 = R, go to 4.1.

For $\psi'_{10} \leq \psi_{D1} < \psi'_{12}$, go to 3.1.1.

For $\psi'_{12} \leq \psi_{D1} \leq 360^\circ$ or $0 \leq \psi_{D1} < \psi'_1$, go to 3.1.2.

3.1.1 For $\psi'_{12} < \psi_{D2} \leq 360^\circ$ or $0 \leq \psi_{D2} < \psi'_1$, set column 4 = Y, go to 4.2.

For $\psi'_1 < \psi_{D2} \leq \psi'_3$, set column 4 = D, go to 4.2.

For $\psi'_{11} < \psi_{D2} \leq \psi'_{12}$, set column 4 = L, go to 4.1.

3.1.2 For $\psi'_{12} < \psi_{D2} \leq 360^\circ$ or $0 \leq \psi_{D2} \leq \psi'_1$, set column 4 = C, go to 4.2.

For $\psi'_1 < \psi_{D2} \leq \psi'_3$, set column 4 = Z, go to 4.2.

RIGHT SIDE

3.2 For $\psi'_1 \leq \psi_{D1} < \psi'_3$, go to 3.2.1.

For $\psi'_3 \leq \psi_{D1} < \psi'_4$, go to 3.2.2.

For $\psi'_4 \leq \psi_{D1}$, set column 4 = B, go to 4.1.

3.2.1 For $\psi'_2 < \psi_{D2} \leq \psi'_3$, set column 4 = F, go to 4.1.

For $\psi'_3 < \psi_{D2} \leq \psi'_4$, set column 4 = Y, go to 4.2.

For $\psi'_4 < \psi_{D2} \leq \psi'_6$, set column 4 = D, go to 4.2.

3.2.2 For $\psi'_3 < \psi_{D2} \leq \psi'_4$, set column 4 = P, go to 4.2.

For $\psi'_4 < \psi_{D2} \leq \psi'_6$, set column 4 = Z, go to 4.2.

REAR (BACK)

3.3 For $\psi'_4 \leq \psi_{D1} < \psi'_6$, go to 3.3.1.

For $\psi'_6 \leq \psi_{D1} < \psi'_7$, go to 3.3.2.

For $\psi'_7 \leq \psi_{D1}$, set column 4 = L, go to 4.1.

3.3.1 For $\psi'_5 < \psi_{D2} \leq \psi'_6$, set column 4 = R, go to 4.1.

For $\psi'_6 < \psi_{D2} \leq \psi'_7$, set column 4 = Z, go to 4.2.

For $\psi'_7 < \psi_{D2} \leq \psi'_9$, set column 4 = D, go to 4.2.

3.3.2 For $\psi'_6 < \psi_{D2} \leq \psi'_7$, set column 4 = C, go to 4.2.

For $\psi'_7 < \psi_{D2} \leq \psi'_9$, set column 4 = Y, go to 4.2.

LEFT SIDE

3.4 For $\psi'_7 \leq \psi_{D1} < \psi'_9$, go to 3.4.1.

For $\psi'_9 \leq \psi_{D1} < \psi'_{10}$, go to 3.4.2.

For $\psi'_{10} \leq \psi_{D1}$, set column 4 = F, go to 4.1.

3.4.1 For $\psi'_8 < \psi_{D2} \leq \psi'_9$, set column 4 = B, go to 4.1.

For $\psi'_9 < \psi_{D2} \leq \psi'_{10}$, set column 4 = Z, go to 4.2.

For $\psi'_{10} < \psi_{D2} \leq \psi'_{12}$, set column 4 = D, go to 4.2.

3.4.2 For $\psi'_9 < \psi_{D2} \leq \psi'_{10}$, set column 4 = P,
go to 4.2.

For $\psi'_{10} < \psi_{D2} \leq \psi'_{12}$, set column 4 = Y,
go to 4.2.

4. General Type of Damage Distribution

(Column 6)

{ Note: Set Column 5 = 0
Vertical Location, Not Applicable }

4.1 Corner Damage

Column 4	Scan Damage Range For	(DIS)
F	$(X)_{\min}$	$X_F - X_{\min}$
R	$(Y)_{\min}$	$Y_S - Y_{\min}$
B	$(X)_{\max}$	$X_{\max} - X_R$
L	$(Y)_{\max}$	$Y_S + Y_{\max}$

	Column 6
$0 < (DIS) \leq 4.5$	S
$4.5 < (DIS) \leq 16.5$	E
$16.5 < (DIS)$	W

4.2

Side or End Damage

Column 3	(DIS)
F	(Y) ψ D2 - (Y) ψ D1
R	(X) ψ D1 - (X) ψ D2
B	(Y) ψ D1 - (Y) ψ D2
L	(X) ψ D2 - (X) ψ D1

	Column 6
$0 < (DIS) \leq 16.0$	N
$16.0 < (DIS)$	W

5.0

Extent of Deformation

Column 3	Scan Damage Range For	(EXT)	Go To
F	(X) _{min}	$X_F - X_{min}$	5.1
R	(Y) _{min}	$Y_S - Y_{min}$	5.2
B	(X) _{max}	$X_{max} - X_R$	5.3
L	(Y) _{max}	$Y_S + Y_{max}$	5.2

5.1 For Column 1 = F

	Column 7
$0 < (EXT) \leq 0.125 X_F$	1
$0.125 X_F < (EXT) \leq 0.250 X_F$	2
$0.250 X_F < (EXT) \leq 0.375 X_F$	3
$0.375 X_F < (EXT) \leq 0.500 X_F$	4
$0.500 X_F < (EXT) \leq 0.625 X_F$	5
$0.625 X_F < (EXT) \leq 0.846 X_F$	6
$0.846 X_F < (EXT) \leq 0.946 X_F$	7
$0.946 X_F < (EXT) \leq 1.046 X_F$	8
$1.046 X_F < (EXT)$	9

5.2 For Column 3 = R, L

	Column 7
$0 < (EXT) \leq 0.165 Y_S$	1
$0.165 Y_S < (EXT) \leq 0.253 Y_S$	2
$0.253 Y_S < (EXT) \leq 0.502 Y_S$	3
$0.502 Y_S < (EXT) \leq 0.751 Y_S$	4
$0.751 Y_S < (EXT) \leq 1.000 Y_S$	5
$1.000 Y_S < (EXT) \leq 1.249 Y_S$	6
$1.249 Y_S < (EXT) \leq 1.498 Y_S$	7
$1.498 Y_S < (EXT) \leq 1.747 Y_S$	8
$1.747 Y_S < (EXT)$	9

5.3

For Column 3 = B

	Column 7
$0 < (EXT) \leq -0.084 X_R$	1
$-0.084 X_R < (EXT) \leq -0.168 X_R$	2
$-0.168 X_R < (EXT) \leq -0.252 X_R$	3
$-0.252 X_R < (EXT) \leq -0.336 X_R$	4
$-0.336 X_R < (EXT) \leq -0.421 X_R$	5
$-0.421 X_R < (EXT) \leq -0.588 X_R$	6
$-0.588 X_R < (EXT) \leq -0.769 X_R$	7
$-0.769 X_R < (EXT) \leq -0.950 X_R$	8
$-0.950 X_R < (EXT)$	9

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6.3 Trajectory Routine

1. INPUT REQUIREMENTS

1.1 Dimensional and Inertial Properties

a, b = Distances along vehicle fixed X axis from the total vehicle center of gravity to the center lines of the front and rear wheels, respectively, inches (both entered as positive quantities).

T = Tread at front and rear wheels (average), inches.

M = Total vehicle mass, lb-sec²/in.

I_z = Moment of inertia of complete vehicle in yaw, lb-sec²-in.

1.2 Tire Properties

C_1, C_2, C_3, C_4 = Cornering stiffnesses of the tires at wheels 1, 2, 3, 4 for small slip angles, pounds/radian (entered separately to permit simulation of damaged tires).

μ_1, μ_2 = Tire-terrain friction coefficients at zero speed.

C_μ = Coefficient of linear decrement of friction with tire speed.

T_i = Tabular inputs of tractive or braking forces at wheel i (positive for traction, negative for braking), pounds (0.10 sec intervals).

x'_{Bj}, y'_{Bj} = Points defining boundary between terrain zones ($j = 1, 2$), inches.

1.3 Steer Angles

ψ_R = Rear axle angle produced by damage, deg.

ψ_1, ψ_2 = Tabular inputs of steer angles as functions of time to simulate steering or damage, degrees (entered as degrees, converted to radians, 0.10 sec intervals).

1.4 Initial Conditions

x'_{co}, y'_{co} = Coordinates of initial position of vehicle center of gravity, inches.

ψ_o = Initial heading angle, degrees.

u_o, v_o = Initial velocity components in vehicle coordinate system, inches/sec.

$\dot{\psi}_o$ = Initial angular velocity, deg/sec.

2. OUTPUT FORMAT

Time (Sec)	C. G. Position (Ft)		Heading Angle (Deg)	Velocities (Ft/Sec)		Angular Velocity (Deg/Sec)	Velocity Vector (Deg)
	x'	y'		Fwd	Lat		
t	$x'_c/12$	$y'_c/12$	57.296 ψ	$u/12$	$v/12$	57.296 ψ	δ
0.00	0000.0	0000.0	000.00	000.00	000.00	000.00	000.00

Time (Sec)	Accel. at C. G. (G Units)		Tire Tracks (Ft) *							
			RF		LF		RR		LR	
	Long	Lat	x'_1	y'_1	x'_2	y'_2	x'_3	y'_3	x'_4	y'_4
t	a_x	a_y	$x'_1/12$	$y'_1/12$	$x'_2/12$	$y'_2/12$	$x'_3/12$	$y'_3/12$	$x'_4/12$	$y'_4/12$
	386.4	386.4								
0.00	0.000	0.000	0000.0	0000.0	0000.0	0000.0	0000.0	0000.0	0000.0	0000.0

* If $3 < |\bar{\beta}_i|$ and/or $(W_i - 1.00 \text{ lb}) < F_{ci}$, this information should be made available, along with the wheel locations, in order that the tracks can be displayed as skids (i.e., solid lines). Otherwise, the tracks should be dotted lines in the graphic display. Mark with asterisk in print-out.

3. NORMAL FORCES AT WHEELS (ASSUMED CONSTANT)

$$W_1 = W_2 = \frac{b}{2(a+b)} Mg$$

$$W_3 = W_4 = \frac{a}{2(a+b)} Mg$$

Input Calculations (1)
Only
($g = 386.4 \text{ in/sec}^2$) (2)

4. EFFECTIVE FRICTION COEFFICIENTS

4.1 Positions of Individual Wheels

$$\text{RF} \begin{cases} X_1' = X_c' + a \cos \psi - \frac{T}{2} \sin \psi \\ Y_1' = Y_c' + a \sin \psi + \frac{T}{2} \cos \psi \end{cases} \quad (3)$$

$$\text{LF} \begin{cases} X_2' = Y_c' + a \cos \psi + \frac{T}{2} \sin \psi \\ Y_2' = Y_c' + a \sin \psi - \frac{T}{2} \cos \psi \end{cases} \quad (4)$$

$$\text{RR} \begin{cases} X_3' = X_c' - b \cos \psi - \frac{T}{2} \sin \psi \\ Y_3' = Y_c' - b \sin \psi + \frac{T}{2} \cos \psi \end{cases} \quad (5)$$

$$\text{LR} \begin{cases} X_4' = X_c' - b \cos \psi + \frac{T}{2} \sin \psi \\ Y_4' = Y_c' - b \sin \psi - \frac{T}{2} \cos \psi \end{cases} \quad (6)$$

4.2 Determination of Terrain Zones for Individual Wheels

$$\rho_i = \sqrt{(X_i')^2 + (Y_i')^2} \quad (7)$$

$$\rho_{P_i} = \sqrt{(X_{P_i}')^2 + (Y_{P_i}')^2}, \quad i = 1, 2, 3, 4 \quad (8)$$

where

$$X_{P_i}' = \left(\frac{Y_{B2}' - \xi X_{B1}'}{Y_i' - \xi X_i'} \right) X_i' \quad (9)$$

$$\xi = \left(\frac{Y_{B2}' - Y_{B1}'}{X_{B2}' - X_{B1}'} \right) \left\{ \begin{array}{l} \text{Test } (X_{B2}' - X_{B1}'), \\ \text{if } X_{B2}' - X_{B1}' = 0, \\ \text{set } X_{P_i}' = X_{B1}' \end{array} \right\} \quad (10)$$

$$y_{p_i}' = \left(\frac{y_i'}{x_i'} \right) x_{p_i}' \quad (11)$$

TEST: If $p_i \leq p_{p_i}$, $\mu_{s_i} = \mu_1$
 If $p_{p_i} < p_i$, $\mu_{s_i} = \mu_2$

4.3 Wheel Speeds

$$S_1 = \sqrt{(u - \frac{I}{2}\dot{\psi})^2 + (v + a\dot{\psi})^2} \quad \text{in/sec} \quad (12)$$

$$S_2 = \sqrt{(u + \frac{I}{2}\dot{\psi})^2 + (v + a\dot{\psi})^2} \quad \text{in/sec} \quad (13)$$

$$S_3 = \sqrt{(u - \frac{I}{2}\dot{\psi})^2 + (v - b\dot{\psi})^2} \quad \text{in/sec} \quad (14)$$

$$S_4 = \sqrt{(u + \frac{I}{2}\dot{\psi})^2 + (v - b\dot{\psi})^2} \quad \text{in/sec} \quad (15)$$

$$\mu_i = \mu_{s_i} (1 - C_{\mu} S_i), \quad i = 1, 2, 3, 4 \quad (16)$$

5. SLIP ANGLES (SATURATION AT APPROXIMATELY 20°, $\tan \alpha \approx \alpha$)

Test for zero denominator.
 If zero, test numerator.

Set $\frac{0}{0} = 0$

$\frac{n}{0} = 1.57$

$$\alpha_1 = \frac{v + a\dot{\psi}}{|u - \frac{I}{2}\dot{\psi}|} - \psi_{F1} \operatorname{sgn} u \quad \text{radians} \quad (17)$$

$$\alpha_2 = \frac{v + a\dot{\psi}}{|u + \frac{I}{2}\dot{\psi}|} - \psi_{F2} \operatorname{sgn} u \quad \text{radians} \quad (18)$$

$$\alpha_3 = \frac{v - b\dot{\psi}}{|u - \frac{I}{2}\dot{\psi}|} - \psi_R \operatorname{sgn} u \quad \text{radians} \quad (19)$$

$$\alpha_4 = \frac{v - b\dot{\psi}}{|u + \frac{I}{2}\dot{\psi}|} - \psi_R \operatorname{sgn} u \quad \text{radians} \quad (20)$$

6. CIRCUMFERENTIAL TIRE FORCES ($i = 1, 2, 3, 4$)

6.1 If $T_i = 0$ (input table), bypass calculation of F_{ci} , set $F_{ci} = 0$.

6.2 If $0 < T_i$ (Traction),

$$F_{ci} = \begin{cases} T_i, & \text{for } T_i \leq \mu_i W_i \\ \mu_i W_i, & \text{for } \mu_i W_i < T_i \end{cases} \quad (21)$$

6.3 If $T_i < 0$ (Braking),

$$F_{ci} = \begin{cases} T_i (\text{sgn } U), & \text{for } |T_i| \leq \mu_i W_i \cos \alpha_i \\ -\mu_i W_i (\text{sgn } U) \cos \alpha_i, & \text{for } \mu_i W_i \cos \alpha_i < |T_i| \end{cases} \quad (22)$$

For $|U| < 2$ inches/sec, multiply Equation (22) by

$$\frac{|U|}{2.00}$$

7. NONDIMENSIONAL SLIP ANGLE VARIABLE

7.1 If $(\mu_i W_i - 1.00 \text{ lb}) \leq F_{ci}$, bypass side force calculation, set $F_{si} = 0$,* go to 9.

$$\bar{\beta}_i = \frac{C_i \alpha_i}{\sqrt{\mu_i^2 W_i^2 - F_{ci}^2}} \quad (23)$$

8. SIDE FORCE CALCULATION ($i = 1, 2, 3, 4$)

8.1 If both $|V|$ and $|U| < 0.5$ inches/sec, bypass side force calculation, set $F_{s_i} = 0$, go to 9.

8.2 If either $0.5 \text{ in/sec} \leq |V|$ or $0.5 \text{ in/sec} \leq |U|$, and $|\bar{\beta}_i| < 3$,

$$F_{s_i} = \sqrt{\mu_i^2 W_i^2 - F_{c_i}^2} \left[\bar{\beta}_i - \frac{1}{3} \bar{\beta}_i |\bar{\beta}_i| + \frac{1}{27} \bar{\beta}_i^3 \right] \quad (24)$$

For $3 \leq |\bar{\beta}_i|$, *

$$F_{s_i} = \sqrt{\mu_i^2 W_i^2 - F_{c_i}^2}, \text{ where } \text{sgn } F_{s_i} = \text{sgn } \bar{\beta}_i \quad (25)$$

9. SUMMATIONS OF FORCES AND MOMENTS ($i = 1, 2, 3, 4$)

$$F_{y_i} = F_{s_i} \cos \psi_i + F_{c_i} \sin \psi_i \quad (26)$$

$$F_{x_i} = -F_{s_i} \sin \psi_i + F_{c_i} \cos \psi_i \quad (27)$$

* Sliding Wheel. Retain this information for print-out and graphics.

where ψ_1, ψ_2 = separate tabular inputs

$\psi_3 = \psi_4 = \psi_R$ = constant.

$$\Sigma F_y = \sum_{i=1}^{i=4} F_{yi} \quad (28)$$

$$\Sigma F_x = \sum_{i=1}^{i=4} F_{xi} \quad (29)$$

$$\Sigma N_\psi = (F_{x2} - F_{x1} + F_{x4} - F_{x3}) \frac{I}{2} + (F_{y1} + F_{y2}) a - (F_{y3} + F_{y4}) b \quad (30)$$

10. EQUATIONS OF MOTION

$$\ddot{\psi} = \frac{\Sigma N_\psi}{I_z} \quad (31)$$

$$\dot{\psi} = \dot{\psi}_0 + \int_0^t \ddot{\psi} dt \quad (32)$$

$$\psi = \psi_0 + \int_0^t \dot{\psi} dt \quad (33)$$

$$\dot{u} - v\dot{\psi} = \Sigma F_x / M \quad (34)$$

$$\dot{v} + u\dot{\psi} = \Sigma F_y / M \quad (35)$$

$$x'_c = x'_{c0} + \int_0^t (u \cos \psi - v \sin \psi) dt \quad (36)$$

$$y'_c = y'_{c0} + \int_0^t (u \sin \psi + v \cos \psi) dt \quad (37)$$

11. OUTPUT CALCULATIONS

11.1 Direction of Velocity Vector

$$\gamma = \arctan \left(\frac{U \sin \psi + V \cos \psi}{U \cos \psi - V \sin \psi} \right) \quad (38)$$

11.2 Acceleration at C. G. in Body Coordinate System

$$a_x = \dot{U} - V\dot{\psi} \quad \text{inches/sec}^2 \quad (39)$$

$$a_y = \dot{V} + U\dot{\psi} \quad \text{inches/sec}^2 \quad (40)$$

6.4

Combination of Collision and Trajectory Routines

1.

General Plan

(1) Retain existing separate mode of operation of trajectory routine as an option. Add index to inputs to indicate single or two-vehicle application.

(2) In two-vehicle mode of operation, perform existing trajectory calculations for each vehicle up to point of force summations. Add test for possible collision contact. If no contact, bypass collision routine and proceed with integrations. If contact is possible, reduce time increment size, call collision routine, add collision forces to force summations.

(3) Combined outputs:

Pages 1 and 2 Vehicle #1

Pages 3 and 4 Vehicle #2

Page 5 Damage Summary (at end of run only).

Note that with the exception of VELOCITY VECTOR and TIRE TRACKS (trajectory outputs), the outputs are the same items.

(4) Combined inputs:

Trajectory - Cards 1 and 5 common to two vehicles.

Cards 2, 3, 4, 6, 7 - two sets required.

Collision - Omit Card Nos. 1, 2 and 3.

Cards 4 and 5, omit fields 4 and 5.

Cards 6 and 7, unchanged.

(5) Print out every time increment. The time increment size will be changed within the program.

(6) A counter for time increments subsequent to separation permits the use of a reduced time increment during the time that further contacts are likely and a large time increment for the "spin-out".

2. Specific Program Modifications

Following the force summations of the trajectory routine, insert the following tests.

1. If index = 0 (single vehicle application), proceed to equation 31. Set $\Delta t = 0.025$ sec.

If index = 1.0 (two vehicle application), set $g = 0$, perform the following steps:

Note that above index test should be applied only once in a given run. The index must be added to the input data. g = counter for time increments subsequent to separation, or prior to initial contact.

2. Proceed with second vehicle calculations through force summations.

3. Test for possible collision contact:

3.1 Set $i = 1, j = 2$ } Vehicle Identification

3.2 Calculate the following coordinates of vehicle i corner points in vehicle j system:

$$\begin{aligned}(X_{Ni})_j &= (Y'_{ci} - Y'_{cj}) \sin \psi_j + (X'_{ci} - X'_{cj}) \cos \psi_j \\ &+ X_{Ni} \cos (\psi_i - \psi_j) - Y_{Ni} \sin (\psi_i - \psi_j) \\ (Y_{Ni})_j &= (Y'_{ci} - Y'_{cj}) \cos \psi_j - (X'_{ci} - X'_{cj}) \sin \psi_j \\ &+ X_{Ni} \sin (\psi_i - \psi_j) + Y_{Ni} \cos (\psi_i - \psi_j)\end{aligned}$$

where

N	X_{Ni}	Y_{Ni}	Corner
1	X_{Fi}	Y_{Si}	RF
2	X_{Fi}	$-Y_{Si}$	LF
3	X_{Ri}	Y_{Si}	RR
4	X_{Ri}	$-Y_{Si}$	LR

3.3 Set $N = 1$.

3.4 Is $X_{Rj} < (X_{Ni})_j < X_{Fj}$ and
 $-Y_{Sj} < (Y_{Ni})_j < Y_{Sj}$?

(a) If yes, set $\Delta t = 0.001$, $q = 0$, call Collision Routine, determine collision forces, add to force summations and proceed with integration of equations of motion for each vehicle.

(b) If no, go to 3.5.

3.5 Is $N < 4$?

(a) If yes, set $N = N + 1$, return to 3.4.

(b) If no, go to 3.6.

3.6 Is $\dot{A} = 1$?

(a) If yes, set $\dot{A} = 2$, $\dot{j} = 1$, return to 3.2.

(b) If no, bypass Collision Routine, go to 4.

4. Set time increment counter, q , to $q + 1$.

5. If $q < 100$, set $\Delta t = 0.005$, proceed with integration of equations of motion for both vehicles.

If $100 \leq q$, set $\Delta t = 0.025$, proceed with integration of equations of motion for both vehicles.

6.5

Coefficient of Restitution

Program inputs control the extent of recovery of vehicle structural deflections, and the load-deflection rate is assumed to be identical for loading and unloading. For compatibility with this form of analytical treatment, the following relationship must be maintained:

$$C = 1.00 - \sqrt{1 - \epsilon^2} \quad (\text{see Figure})$$

where C = program variable
 ϵ = coefficient of restitution

The program variable, C , is defined

$$C = 1 - \frac{\delta_f}{\delta_{max}}$$

where δ_f = final deflection, inches
 δ_{max} = maximum deflection, inches

With the assumption of identical load-deflection rates, K , for loading and unloading, the returned energy, E_R , may be expressed

$$\begin{aligned} E_R &= \frac{1}{2} K (\delta_{max}^2 - \delta_f^2) \\ &= \frac{1}{2} K \delta_{max}^2 (2C - C^2) \end{aligned}$$

Thus, the ratio of returned to absorbed energy

$$\epsilon^2 = 2C - C^2$$

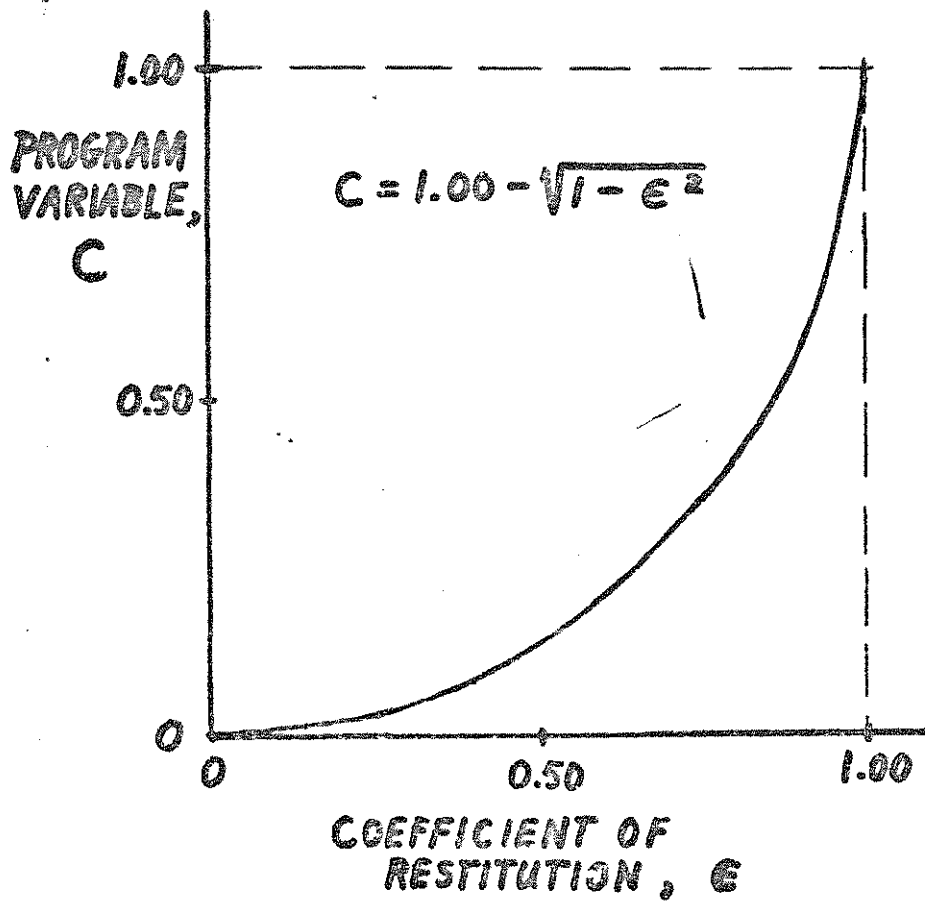


Figure RELATIONSHIP BETWEEN PROGRAM VARIABLE C AND COEFFICIENT OF RESTITUTION

Solving for C ,

$$C = 1.00 - \sqrt{1 - e^2}$$

An approximation of the properties indicated in Reference 8 has been obtained with the following program inputs:

$$\left. \begin{array}{l} C_0 = 0.1150 \\ C_1 = 5.100 \times 10^{-3} \\ C_2 = 5.679 \times 10^{-5} \end{array} \right\}$$

Approximation of
"typical" automobile
frontal properties

6.6 Logic for "Reverse" Calculations

RECON

1. Set $j = 0$ (j = vehicle identification)
2. $j = j + 1$
 $g = 0$ (g = counter for passes through 5-12)
3. Read vehicle type $\left\{ \begin{array}{l} \text{Type 1} = \text{Subcompact} \\ \text{Type 2} = \text{Compact} \\ \text{Type 3} = \text{Full Size} \end{array} \right.$
4. Call PARAM, return with $a, b, T, M, I_2, C_1, C_2, C_3, C_4, (\psi_1)_{\max}$.
5. Any damaged tires?
 - 5.1 For $g = 1$, transmit request for recheck.
For $2 \leq g$, go to 6.
 - 5.2 If no, go to 6.
 - 5.3 If yes, identify involved wheel(s), call PARAM for revised C_i .
6. More than one surface type?
 - 6.1 For $g = 1$, transmit request for recheck.
For $2 \leq g$, go to 7.

6.2 If no, set X'_{ej} , Y'_{ej} out of range,
 set $M_1 = M_2 =$ measured value. Go to 7.
 In absence of measured value, call PARAM
 for "typical" value for M_1 .

6.3 If yes, enter X'_{ej} , Y'_{ej} as measured,
 enter M_1 , M_2 as measured.
 In absence of measured values, call PARAM
 for "typical" values for M_1 , M_2 .

7. Measurement available for C_μ ?

7.1 For $0 < q$, go to 8.

7.2 If yes, enter measured value. Go to 8.

7.3 If no, call PARAM for "typical" value for C_μ .

8. Position and orientation at separation.

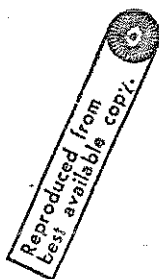
8.1 For $q < 3$, transmit request for recheck.
 For $3 \leq q$, go to 9.

8.2 Set $X'_{eo} = X'_{es}$, $Y'_{eo} = Y'_{es}$,
 $\psi_o = \psi_s$.

9. Fixed steer angles produced by damage?

9.1 For $q = 1$, transmit request for recheck.
 For $2 \leq q$, go to 10.

9.2 If no, go to 10.



- 9.3 If yes, generate appropriate tables for ψ_1 and/or ψ_2 and/or ψ_R (i.e., enter three equal values in appropriate table(s), based on measurements at scene). (Use ψ_1 max for "full" steer.)

10. Evidence of driver inputs of steering, subsequent to separation?

- 10.1 For $q < 3$, transmit request for recheck.

For $3 \leq q$, go to 11.

- 10.2 If no, set ψ tables, other than those generated in 9.3, to zero. Go to 11.

- 10.3 If yes, generate appropriate tables for ψ_1 and ψ_2 . (Use ψ_1 max for "full" steer). Start with ramp input (1.0 sec. duration) to indicated value in indicated direction. For $1 \leq q < 3$, transmit request for X' , Y' coordinates at which steer changes occur, adjust (time) tables on basis of predicted trajectory coordinates.

11. Any wheels locked by damage?

- 11.1 For $q = 1$, transmit request for recheck.

For $2 \leq q$, go to 12.

- 11.2 If no, go to 12.

- 11.3 If yes, generate appropriate table(s) for T_i . (i.e., enter three equal values, $T_i = -2 \mu_i w_i$ for each locked wheel.)

12. Evidence of driver inputs of braking, subsequent to separation?

12.1 For $g < 3$, transmit request for recheck.
For $3 \leq g$, go to 13.

12.2 If no, set T_i tables, other than those generated in 11.3, to zero. Go to 13.

12.3 If yes, generate appropriate tables for T_i , other than those generated in 11.3.

Start with ramp inputs (1.0 sec. duration) to indicated braking level. e.g., "Locked wheel" braking, $T_i = -\mu_i W_i$; "Hard" but non-locking braking, $T_i = -0.8 \mu_i W_i$; "Moderate" braking, $T_i = -0.5 \mu_i W_i$.

For $1 \leq g < 3$, transmit request for x' , y' coordinates at which wheel lock or evidence of braking occurred. Adjust (time) tables on basis of predicted trajectory coordinates.

13. For $g = 0$, go to 14.
For $1 \leq g$, go to 15.

14. Call STARTT, return with u_0 , v_0 , ψ_s .

15. Call TRAJ, return with x_{CR}'' , y_{CR}'' , ψ_R' and track definitions.

16. Call ADJTT.

17. If $\dot{j} = 1$, return to 2.
 If $\dot{j} = 2$, go to 18.

18. [Iteration of collision phase of event.
 Formulation not yet completed.]

PARAM

Vehicle Type

		1	2	3
		Sub compact	Compact	Full Size
Tire Cond. {	a			
	b			
	T			
	M			
	I_z			
	1. Normal { C_1, C_2			
		C_3, C_4		
	2. Low Inflation Pressure { C_1, C_2			
		C_3, C_4		
	3. Flat { C_1, C_2			
		C_3, C_4		
Max. Steer Angle $(\psi)_{max.}$				

Surface Type

1	2	3	4	5	6	7
Ice					Dry Asphalt	Dry Concrete
μ						
$C\mu$						

STARTT

{ Used only to generate approximate
starting values for TRAJ }

INPUTS: X'_{CR} , Y'_{CR} , ψ_R = Rest position and orientation (inches and degrees)

X'_{CS} , Y'_{CS} , ψ_S = Position and orientation at separation (inches and degrees).

$a+b$ = Wheelbase, inches.

I_z = Moment of inertia in yaw, lb-sec²-in.

M = Mass, lb-sec²/in.

μ_1 = Nominal tire-ground friction coefficient.

$$1. \quad S = \sqrt{(X'_{CR} - X'_{CS})^2 + (Y'_{CR} - Y'_{CS})^2} \quad \text{inches}$$

$$2. \quad \Delta\psi = \frac{(\psi_R - \psi_S)}{57.3} \quad \text{radians}$$

3. $\gamma_s = \arctan \left(\frac{Y'_{CR} - Y'_{CS}}{X'_{CR} - X'_{CS}} \right)$
4. $PR = \frac{(\Delta\psi)(a+b)}{2S}$ (Path Ratio)
5. $\phi_\psi = 0.78 (PR) - 0.16 (PR)^2$
6. $\phi_v = 1.00 - 0.10 (PR) - 0.28 (PR)^2$
7. $\psi_s = \left\{ \sqrt{\phi_\psi \frac{(a+b)\mu_1 M g}{I_z}} |\Delta\psi| \right\} \text{sgn}(\Delta\psi)$
8. $\Sigma V_s = + \sqrt{2\phi_v \mu_1 g S}$
9. $U_s = (\Sigma V_s) \cos(\gamma_s - \psi_s)$
10. $V_s = (\Sigma V_s) \sin(\gamma_s - \psi_s)$
11. Return to RECON, Step 15, with

$$\begin{aligned} U_o &= U_s \\ V_o &= V_s \\ \psi_o &= \psi_s \\ q &= 1 \\ p &= 1 \end{aligned} \quad \left\{ \begin{array}{l} q = \text{counter for passes through 5-12} \\ p = \text{counter for passes thorough ADJTT.} \end{array} \right.$$

ADJTT

Used for iterative adjustments of
 $\{ u_0, v_0, \psi_0 \}$ in TRAJ

INPUTS: Measured $x'_{CR}, y'_{CR}, \psi'_R$ and Track.
 Definitions, if available (i.e., x'_A, y'_A coordinates;
 wheel identification, if available; and definition of
 skid portions).

Predicted $x''_{CR}, y''_{CR}, \psi'_R$ and Track.
 Definitions from TRAJ (i.e., x'_A, y'_A coordinates,
 wheel identification and skid portions (asterisks)).

Starting values $u_0, v_0, \psi_0 (\equiv u_s, v_s, \psi_s)$

- (1) If no track data are available from scene, go to (20).
- (2) STOP (Temporary, until development of logic is completed for the processing of track data).
- (3)-(9) TO BE DEVELOPED.

$$(20) \quad K_1 = \frac{x'_{CR} - x'_{CS}}{x''_{CR} - x'_{CS}}, \quad K_2 = \frac{y'_{CR} - y'_{CS}}{y''_{CR} - y'_{CS}},$$

$$K_3 = \frac{\psi'_R - \psi_s}{\psi'_R - \psi_s}$$

- (21) If $0.98 \leq |K_1| \leq 1.02$ and $0.98 \leq |K_2| \leq 1.02$
 and $0.98 \leq |K_3| \leq 1.02$, return to RECON, step (17).

- (22) For $p \leq 5$, go to (24).
For $5 < p$, set $q = q + 1$, proceed.
- (23) For $q \leq 5$, return to RECON, step (5).
For $5 < q$, STOP. Print the following message:
"Cannot match rest position of vehicle j . Please seek cause and resubmit case."
- (24) $\dot{x}'_s = K_1 (u_s \cos \psi_s - v_s \sin \psi_s)$
- (25) $\dot{y}'_s = K_2 (u_s \sin \psi_s + v_s \cos \psi_s)$
- (26) $u_s = \dot{x}'_s \cos \psi_s + \dot{y}'_s \sin \psi_s$
- (27) $v_s = \dot{y}'_s \cos \psi_s - \dot{x}'_s \sin \psi_s$
- (28) $\dot{\psi}_s = K_3 \dot{\psi}_s$
- (29) Set $u_0 = u_s$, $v_0 = v_s$, $\dot{\psi}_0 = \dot{\psi}_s$,
 $p = p + 1$, return to RECON, Step (15).