Commercial Vehicle Tyre Adhesion
By Brian J. Robinson
TRL Ltd.
TRL Report 237

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TRL REPORT 237

COMMERCIAL VEHICLE TYRE ADHESION

by Brian J Robinson

Prepared for: **Vehicle Standards & Engineering Division, DETR**
Project: **Commercial vehicle tyre adhesion (S340A/VD)**

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EXECUTIVE SUMMARY

The Department of the Environment, Transport and the Regions has a target of reducing the number of road casualties by one third, from the levels of the early 1980s, by the year 2000. Tyre adhesion is a major primary safety factor and it is therefore very important that current levels of tyre adhesion are maintained (or improved) to help meet the above target.

The Vehicle Standards and Engineering (VSE) Division of the DETR contracted the Transport Research Laboratory (TRL) to look at the adhesion levels of current generation commercial vehicle tyres and to consider the feasibility of an adhesion test procedure. A further aim of this project was to compare current levels of tyre adhesion with those from similar tests conducted in the late seventies.

Twenty types of commercial vehicle tyres have been tested, involving several different makes, models and sizes. The adhesion levels have been measured using braking tests and cornering tests. All the tests made use of a specially modified test vehicle. In each test the vehicle's longitudinal or lateral acceleration was measured and converted into a dimensionless Braking or Side Force coefficient (BFC or SFC).

There was found to be a large difference between peak and locked wheel values on all the surfaces, indicating the potential for improved braking performance through the use of systems designed to prevent wheel lock and make full use of the grip available.

The peak Side Force Coefficients, measured on a wet asphalt surface and a wet Bridport gravel surface, did not vary much between tyres, though the wide trailer tyres gave slightly better results than the other, narrower tyres on the asphalt surface.

Where tentative comparisons were possible, there was some evidence that the adhesion levels of current generation tyres on a dry surface, as indicated by straight line braking tests, are significantly better than those from CV tyres tested previously. This trend was not evident when comparing results on a wetted surface but it is possible that other improvements in tyre performance, such as increased durability and reduction in noise, have been made over the years without any sacrifice in tyre adhesion.

The results indicate that it would be very difficult to measure tyre adhesion by using one test procedure. The tyres generally gave different rank orderings with the braking tests on the various surfaces tested. Another different rank ordering was produced from the cornering tests. It would therefore be difficult to conclude too much from any one test, as a tyre that performs well in that test might perform badly in another. There were found to be statistically significant correlations between the stopping distances calculated on the wet concrete and wet motorway, wet Bridport and wet Mastic, wet Bridport and wet FTA, dry FTA and wet concrete and dry FTA and wet motorway surfaces, but not between any other surface pairs. Although some patterns emerged on some surfaces, for example retreaded tyres were found to give significantly shorter stopping distances (typically 8-13% from 90 km/h) than new tyres on the wet motorway, wet concrete and dry FTA surfaces, there were no consistent variations across all the surfaces tested, or even across all the high-grip surfaces typical of normal roads.

When one considers the results of all the studies into truck tyre performance, it is clear that the factors which influence tyre and vehicle adhesion levels are many and complex. The trend to reduce tyre noise levels could possibly lead to an increase in commercial vehicle emergency stopping distances, but other trends, such as fitting anti-lock brakes and electronic braking systems ("brake by wire"), may well have the opposite effect. The crucial question is whether legislation (or international standards) should be used to ensure that tyre adhesion levels are maintained or improved, despite all the other influencing factors. It is important that any legislative test is simple to perform and representative of the conditions tyres encounter in normal use. It is also important that the design of tyres which easily meet any adhesion requirements are not changed (to give better noise or tread wear characteristics for example) in such a way that the adhesion performance is reduced to nearer the minimum standard required. It is also of vital importance that improvements in tyre adhesion levels translate into improvements in commercial vehicle braking and cornering performance. This can only be done through further legislation or standards governing all the various vehicle components and systems (brakes, suspension etc.) which influence this performance.

In summary there are various factors which influence tyre adhesion. The substantial international pressure to reduce tyre noise levels is likely to result in legislation in the very near future. Such legislation might well adversely affect tyre adhesion levels. It is therefore the author's opinion that there is a need to develop a legislative test for commercial vehicle tyre adhesion with some urgency. It is likely that whatever form of test may be proposed or developed, the main problem will be in specifying what road surfaces the adhesion levels are to be measured on. The research described in this report has found that any given tyre can perform quite differently, relative to another tyre, from one surface to another. Any standard test must therefore cover a variety of surfaces. Further research into the effect of road surface on tyre adhesion needs to be carried out in preparation for the development of a standard test. One surface which ought to be considered is porous asphalt, which is becoming increasingly common on British and European roads.

In future, the development of computer models or more useful and accurate drum rigs may make the testing of real tyres on real vehicles on real road surfaces unnecessary. At present, however, it is the author's opinion that the straight line front-wheel braking method, as described in this report, offers the best combination of cost, simplicity, accuracy, reliability and repeatability.

A standard cornering test would be much more difficult to justify, as the ultimate cornering grip of commercial vehicle tyres may be rarely needed in real situations. It is considered unlikely that the difficulties in developing a simple, safe and realistic cornering test for commercial vehicle tyres will be justified by the improvements in vehicle safety that might result. There was some evidence from the recent tests that peak BFC can be used to predict maximum SFC, thus avoiding the need for any separate test to measure cornering adhesion.

COMMERCIAL VEHICLE TYRE ADHESION

ABSTRACT

Tyre adhesion is critical to primary safety and it is therefore very important that current levels of tyre adhesion are maintained or improved. The UK Department of the Environment, Transport and the Regions contracted the Transport Research Laboratory (TRL) to look at the adhesion levels of current generation commercial vehicle tyres and to consider the feasibility of an adhesion test procedure. The results indicate that it would inappropriate to measure tyre adhesion by using any one test procedure. The tyres generally gave different rank orderings with the braking tests on the various surfaces tested. Another different rank ordering was produced from the cornering tests. There are various factors which influence tyre adhesion. The substantial international pressure to reduce tyre noise levels is likely to result in legislation in the very near future. Such legislation might well adversely affect tyre adhesion levels. It is therefore the author's opinion that there is a need to develop a legislative test for commercial vehicle tyre adhesion with some urgency.

1. INTRODUCTION

The Department of the Environment, Transport and the Regions (DETR) has a target to reduce the number of road casualties by one third of the levels of the early 1980s, by the year 2000. Tyre adhesion is critical to primary safety and it is therefore very important that current levels of tyre adhesion are maintained (or improved) to help meet the above target.

Pressures from the environmental lobby to reduce tyre noise may result in international legislation to limit the peak sound pressure produced by car and commercial vehicle (CV) tyres. The DETR therefore needs to be aware of the current tyre adhesion coefficients to ensure that proposals to reduce tyre noise do not result in reduced adhesion levels and a consequent potential increase in vehicle accidents. It is likely that a standard for tyre adhesion will need to be legislated to ensure that this does not happen.

The Vehicle Standards and Engineering (VSE) division of the DETR has contracted the Transport Research Laboratory (TRL) to measure the adhesion coefficients of current commercial vehicle tyres and to consider the feasibility of an adhesion test procedure. TRL were also requested to compare current values of tyre adhesion with those obtained from similar tests in the late seventies. In a separate project, TRL were commissioned by VSE to study the relationship between tyre noise and tyre safety; reference 1 gives the results of the noise and safety measurements and describes the relationship between the two.

This report describes the method and the results of measuring the adhesion coefficients of a range of CV tyres including those used in the tyre noise and safety project. The adhesion was assessed by determining braking force and cornering force on different surfaces with the tyres fitted to a vehicle specially designed for this purpose. The tyres tested are described in section 2 below and the vehicle, instrumentation, test surfaces and test methods are described in sections 3 to 6. The results are presented and discussed in section 7 and this is followed by a comparison of these results with those obtained from similar tests in the late seventies (1977/78) and reported by Wilkins and Riley (ref 2). The final part of the report summarises the findings and discusses the need for, and feasibility of, a legislative test procedure.

2. TYRES TESTED

The size, typical usage and type of the twenty commercial vehicle tyres tested are shown in Table 1. The tyres were selected to be representative of those in use in the UK.

The type of each tyre refers to the process used in its manufacture. New tyres are first use, newly manufactured tyres. Retread (hot) covers two forms of retreading; the first is a bead to bead retreading process where the casing is buffed all the way across the section, including the sidewalls. A thick layer of uncured rubber is applied to the tread area with a thin sheet to the sidewalls. The tyre is then placed in a mould which forms the tread pattern and heats up to cure the rubber. The second hot retreading process is known as re-capping, where new rubber covers the shoulder of the tyre and the tread is again formed in a mould. Retread (cold) is a pre-cure process, also known as top-capping, where only the tread area is buffed. A thick strip of pre-cured rubber, complete with tread pattern, is bonded round the circumference, at a much lower temperature than that required for bead to bead or re-cap retreading.

Photographs of all the tyres, showing the tread patterns, are presented in Figure 1. All the tyres were obtained in as new condition with maximum tread depths. Prior to testing, all the tyres were run in by being driven gently (without harsh cornering or braking) for approximately 200 miles. The tyres were inflated to pressures appropriate to the load being carried (see next section), typically about 65 psi (4.5 bar) for the wide trailer tyres and 80-90 psi (5.5 - 6.2 bar) for the other, smaller sizes.

3. TEST VEHICLE

The vehicle used for the tests was a two axle Daimler bus chassis specially modified for the purpose of tyre adhesion testing. The front brakes can be applied separately from the rears and are sufficiently powerful to obtain quick wheel locking, even on dry surfaces. This is the same vehicle as was used for the tests conducted in the late seventies, as referred to elsewhere in this report.

The static axle weights were:

Front axle	4,320 kg
Rear axle	6,670 kg
	10,990 kg

The test tyres were fitted to the front axle, the loading of which was chosen to be typical for the range of tyres to be tested (Reference 3). This reference relates to some work on HGV tyre debris left on the M4 motorway and involved a survey of vehicles using the motorway. That survey found that, on average, HGV axles were loaded to about 71 per cent of their plated values. Though not reported in Reference 3, the survey also found that, on average, tyres fitted to 22.5 inch diameter wheels had individual wheel loads of approximately 2000 kg, implying an axle load on the test vehicle of 4000 kg. The load was positioned such that each tyre was loaded, as near as possible, to the same value. The wheelbase was 5.65m. The centre of gravity was 2.22m in front of the rear axle and 1.00m above the ground, as measured for, and quoted in, Reference 2.

The single formation Load Indices of the tyres ranged from 146 (for one of the 11 X 22.5 tyres) to 160 (for one of the 385/65 X 22.5 tyres). These equate to maximum possible axle loads of between 6,000 kg and 9,000 kg, so the constant axle load used of 4,320 kg represents between 48% and 72% of the tyres' maximum permissible static loading.

In order to fully repeat the late seventies tests described in Reference 2, a small series of braking tests was also conducted with different front axle weights. The lightly laden tests were carried out with an axle weight of 3150 kg and the heavily laden tests with an axle weight of 4920 kg.

4. INSTRUMENTATION

The vehicle was fitted with a contactless optical speed and distance measuring device. This was connected to data logging equipment within the vehicle, set to store test data on magnetic disk. An accelerometer, with a measuring range from -3g to +3g, was also connected to the data logger and its signal was stored on disk. The test files were then transferred to a PC for analysis using a dedicated software package.

TABLE 1.

Commercial vehicle tyres used for adhesion tests

Tyre	Size	Typical usage	Type
A	11 X 22.5	Drive axle	New
B	11 X 22.5	Steer axle	New
C	11 X 22.5	On/off highway	New
D	11 X 22.5	General purpose	New
E	11 X 22.5	Trailer	New
F	11 X 22.5	Drive axle	Retread (hot)
G	295/80 X 22.5	Drive axle	New
H	295/80 X 22.5	Steer axle	New
I	295/80 X 22.5	Drive axle	New
J	295/80 X 22.5	General purpose	Retread (hot)
K	295/80 X 22.5	Steer axle	New
L	315/80 X 22.5	Drive axle	New
M	315/80 X 22.5	Steer axle	New
N	315/80 X 22.5	On/off highway	New
O	315/80 X 22.5	Drive axle	Retread (hot)
P	12 X 22.5	General purpose	New
Q	12 X 22.5	Trailer	Retread (cold)
R	385/65 X 19.5	Trailer	New
S	385/65 X 22.5	Trailer	New
T	385/65 X 22.5	Trailer	Retread (cold)

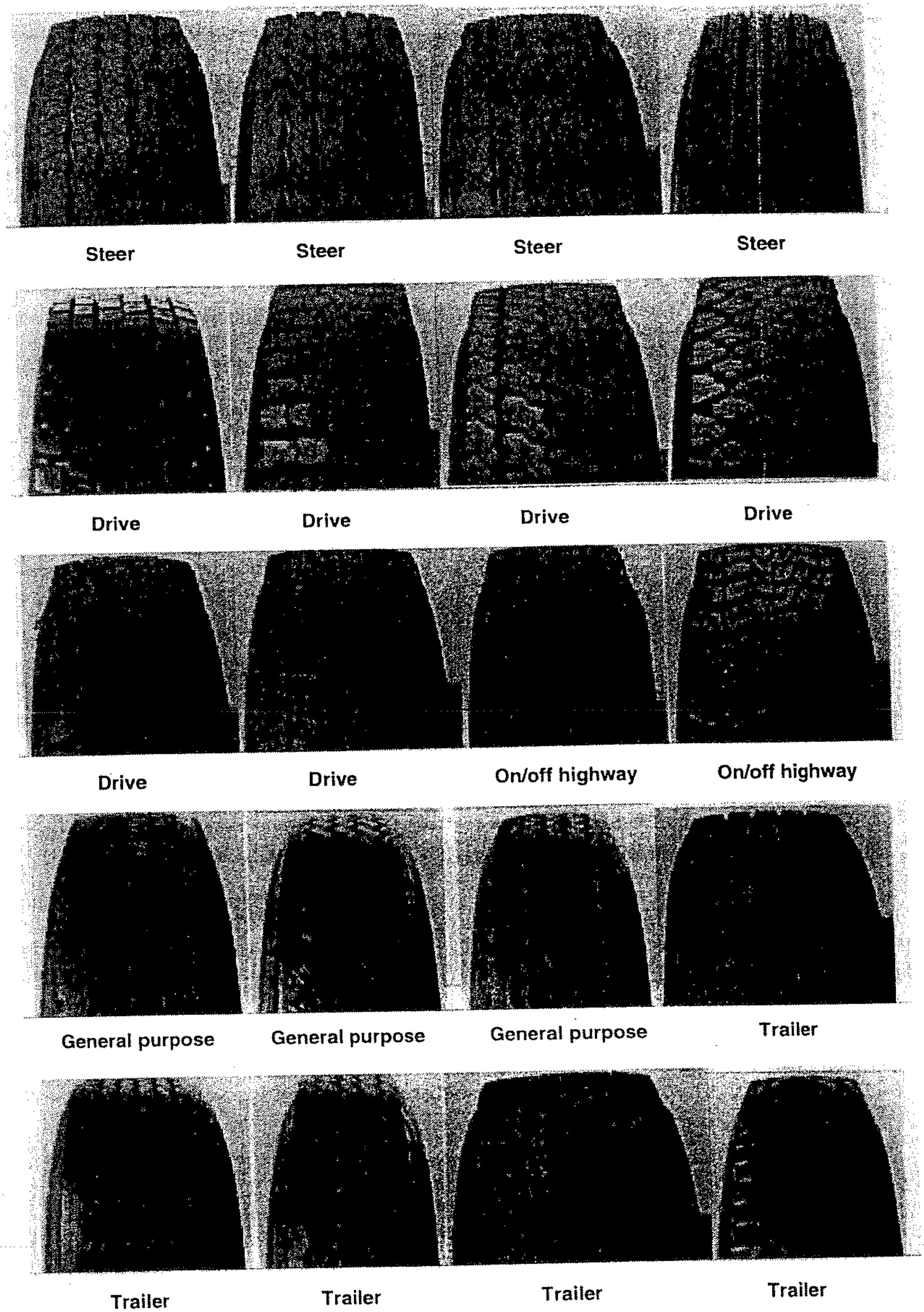

Figure 1. Tyres used for adhesion tests

5. TEST SURFACES

Five wet surfaces and one dry surface were used for braking tests and two (both wet) were used for cornering tests (see below). The surfaces and their macro and micro textures are shown in Table 2, together with the SCRIM coefficients as measured in April 1993. When wet, the surfaces were wetted to a depth of 1-2 mm of water (measured using an electronic probe), which represents typical heavy rainfall conditions.

6. TEST METHODS

6.1 BRAKING TESTS

The test procedure used broadly conformed to ASTM standard F403 - 86 (Reference 4). The vehicle was driven in a straight line onto the test surface and the front brakes were applied as hard and as fast as possible. The initial speeds ranged from approximately 20 km/h to 100 km/h and in roughly 20 km/h increments. During each test the vehicle speed and deceleration were recorded by the instrumentation described above.

From each test run the data was analysed to give peak vehicle deceleration and a locked wheel value at the appropriate vehicle speed. Using the centre of gravity position as quoted in Reference 2, the peak and locked wheel decelerations were then converted into Brake Force Coefficients (BFCs), using the procedure described in the Appendix.

The pitch of the vehicle during braking was very small and insufficient to necessitate correction of the accelerometer measurements.

To minimise tyre damage, the front brakes were released within about a second of wheel locking. Tyres were tested two to four times at each test speed and on each surface, to produce a regression line of peak and locked wheel BFC against vehicle velocity.

6.2 CORNERING TESTS

The vehicle was driven at a constant speed (40 km/h on the cold rolled asphalt and 30 km/h on the Bridport) and the steering wheel was turned at a steady rate to gradually reduce the radius of the turn while maintaining the vehicle's speed. The lateral acceleration was measured using the accelerometer described above and the maximum value, at which the front tyres lost adhesion, was recorded. The test was repeated approximately six times and an average peak lateral acceleration derived. When corrected for vehicle roll angle, the peak acceleration (in units of g) is equivalent to the peak Side Force Coefficient (SFC). The front (test) tyres always lost adhesion before the rears with this procedure. The test speeds were chosen to ensure the limit of adhesion was reached before the limit of turning radius imposed by the vehicle's steering geometry.

7. RESULTS

7.1 BRAKING TESTS

7.1.1 Peak and locked wheel BFCs

The locked wheel and peak BFC results are given in Figures 2 - 7, together with the "best fit" locked wheel regression lines. The strength of the linear relationship between locked wheel BFC and vehicle speed over the range tested is indicated by the values of R^2 shown on the graphs. The nearer the values to 1.0, the greater the evidence for a strong linear relationship.

The peak BFCs on the wet FTA surface (Figures 2(a) and 2(b)) show a slight reduction as speed increases. The tyres generally gave values ranging from around 0.7 to 0.8 at 20 km/h down to approximately 0.6 at 100 km/h. The locked

TABLE 2.

Test surfaces used

Surface	Macro-texture	Micro-texture	Test	SCRIM value
Wet Motorway (hot rolled) asphalt	Rough	Harsh	Braking	0.59
Wet concrete	Smooth	Harsh	Braking	0.43
Wet fine cold rolled asphalt	Smooth	Harsh	Cornering	N/K
Wet Fine Textured Asphalt (FTA)	Smooth	Harsh	Braking	0.61
Wet Bridport Gravel	Rough	Polished	Braking & Cornering	0.22
Wet Mastic asphalt	Smooth	Polished	Braking	0.12
Dry Fine Textured Asphalt (FTA)	Smooth	Harsh	Braking	n/a

wheel BFCs are all highly linearly related to vehicle speed. Values of 0.55 - 0.6 at 20 km/h down to 0.25 at 90 km/h are typical. The graphs show very little difference in BFC performance (peak or locked wheel) on the wet FTA surface between any of the tyres tested.

The peak BFCs on the wet motorway surface (Figures 3(a) and 3(b)) are only slightly speed dependent. The tyres generally gave values of around 0.75 at low speed (20-30 km/h) reducing to 0.65 at 100 km/h. The locked wheel BFCs are all highly linearly related to vehicle speed. Values of 0.5 - 0.6 at 20 km/h down to 0.2 - 0.25 at 90 km/h are typical. The graphs show little difference in BFC behaviour (peak or locked wheel) on the wet motorway surface between the tyres tested.

The peak BFCs on the wet concrete surface (Figures 4(a) and 4(b)) do generally show a slight reduction as speed increases, typically from around 0.65 - 0.7 at 20 or 30 km/h down to 0.5 - 0.6 at 100 km/h. There is again a clear linear speed dependence apparent with the locked wheel BFCs. Values of 0.4 - 0.5 at 20 km/h and 0.15 - 0.25 at 90 km/h are typical. As was the case on the motorway and wet FTA surfaces, there was found to be little difference between any of the tyres tested on the wet smooth concrete surface.

The peak BFCs on the Bridport surface (Figures 5(a) and 5(b)) are only slightly speed dependent. The tyres generally gave values of around 0.5 at low speed (20-30 km/h) reducing to 0.4 at 80 km/h (the size of the available area of Bridport gravel limited the maximum test speeds to around 80 km/h, rather than 100 km/h on the other surfaces). The locked wheel BFCs are all linearly dependent on vehicle speed, but the dependency is not as great as was found on the wet FTA, motorway and concrete surfaces, as indicated by the values of R^2 and the gradient of the regression lines. Values of 0.3 - 0.4 at 20 km/h down to 0.2 at 80 km/h are typical. The graphs show very little difference in peak or locked wheel performance on the wet Bridport surface between any of the tyres tested.

The peak BFCs on the wet Mastic asphalt surface (Figures 6(a) and 6(b)) do generally show a reduction as speed increases, typically from around 0.2 - 0.3 at 20 km/h down to 0.1 at 100 km/h. There is evidence for a linear speed dependence with the locked wheel BFCs, but the slopes of the regression lines and values of R^2 are much lower than on any of the other surfaces tested. BFC values of 0.1 - 0.15 at 20 km/h and 0.05 at 90 km/h are typical. There was found to be little difference in locked wheel BFC behaviour between any of the tyres tested on the wet Mastic surface, though there were quite wide variations in peak BFCs. This is probably attributable to the fact that this surface has a very high ratio of peak BFC to locked wheel BFC, making it particularly difficult to ensure that both test tyres reach their peak adhesion at the same time. If the tyres do not peak at the same time, then the accelerometer will inevitably give a falsely low reading of peak deceleration. The variations in peak BFC on the Mastic may therefore be attributable to limitations in the test procedure rather than differences amongst the tyres under test. There is no such problem with the locked wheel readings, once both wheels have locked.

The peak BFCs on the dry FTA surface (Figures 7(a) and 7(b)) remain fairly constant as speed increases. The tyres generally gave values ranging from around 0.7 to 0.9 at all speeds. The locked wheel BFCs are all linearly related to vehicle speed, but the strength of the relationship is lower than was found on the wet FTA surface. Values of 0.6 at 20 km/h down to 0.4 at 90 km/h are typical. The graphs show little difference in BFC performance (peak or locked wheel) on the dry FTA surface between any of the tyres tested.

All the graphs show large differences between locked wheel BFCs and peak BFCs, which generally increase with vehicle speed. This indicates the potential for improvement in braking performance if greater use was made of the adhesion available at peak by means of systems designed to prevent wheel lock.

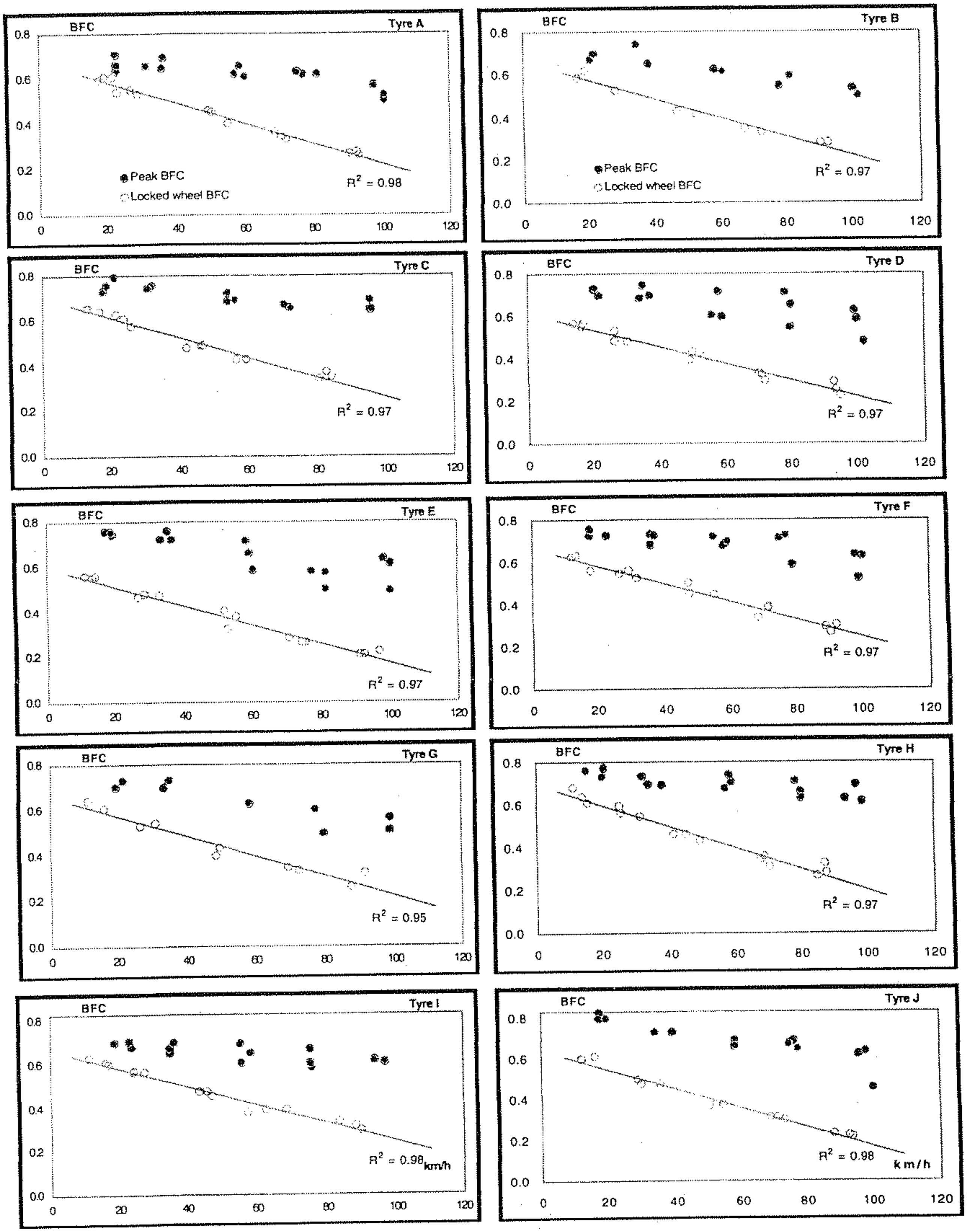

Figure 2(a) - Peak and locked wheel braking force coefficients (BFCs) on wet Fine Textured Asphalt

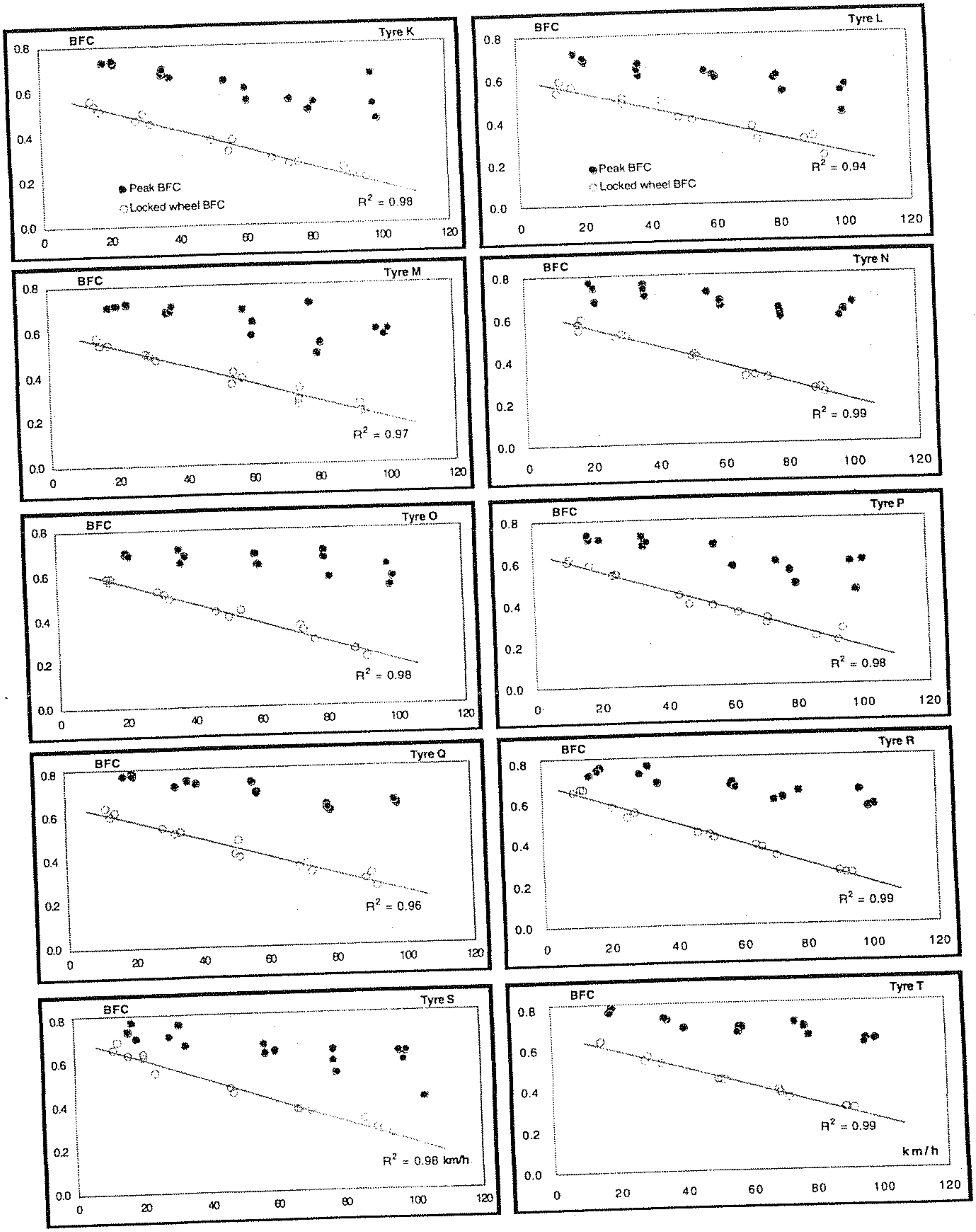

Figure 2(b) - Peak and locked wheel braking force coefficients (BFCs) on wet Fine Textured Asphalt

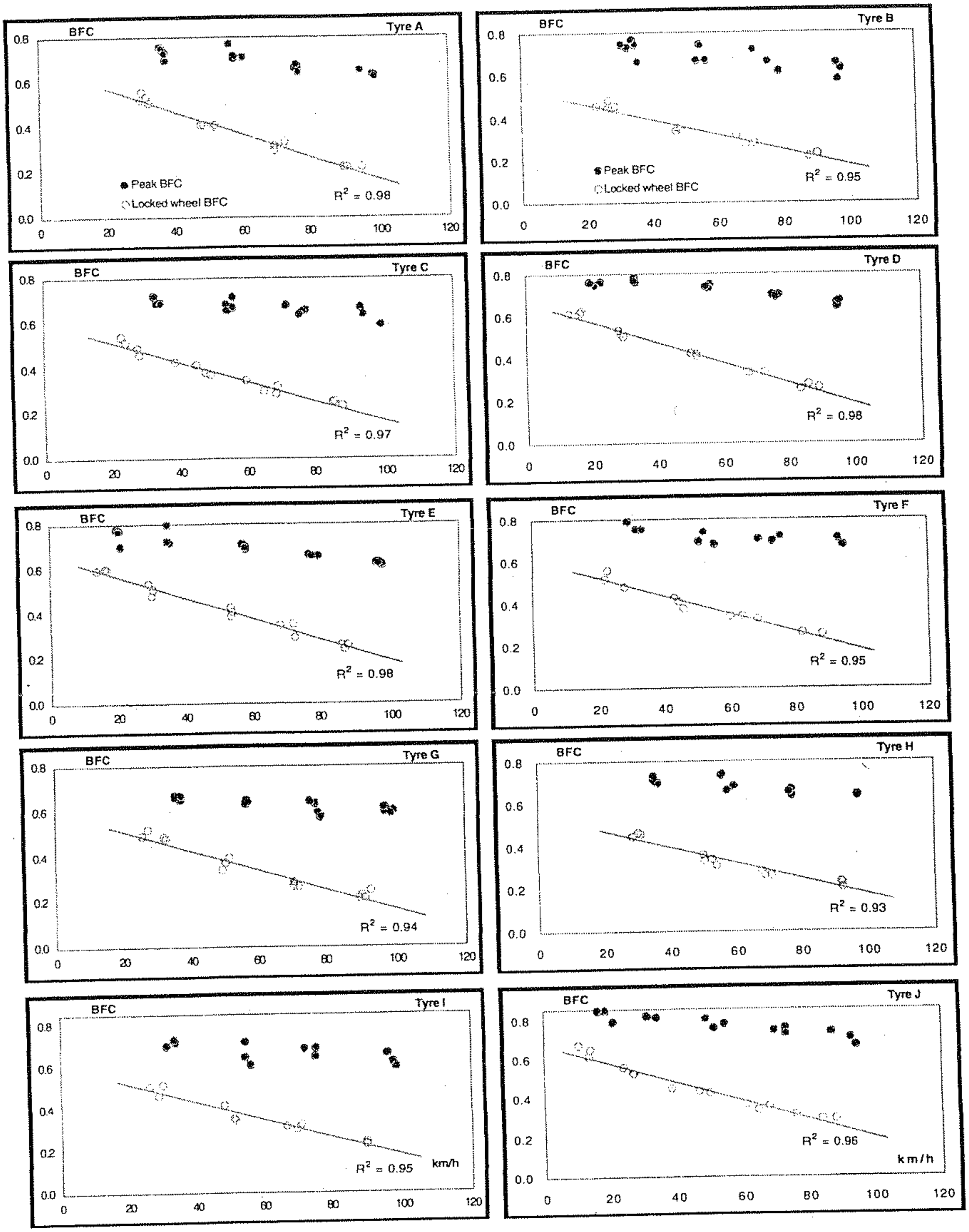

Figure 3(a) - Peak and locked wheel braking force coefficients (BFCs) on wet Motorway Asphalt

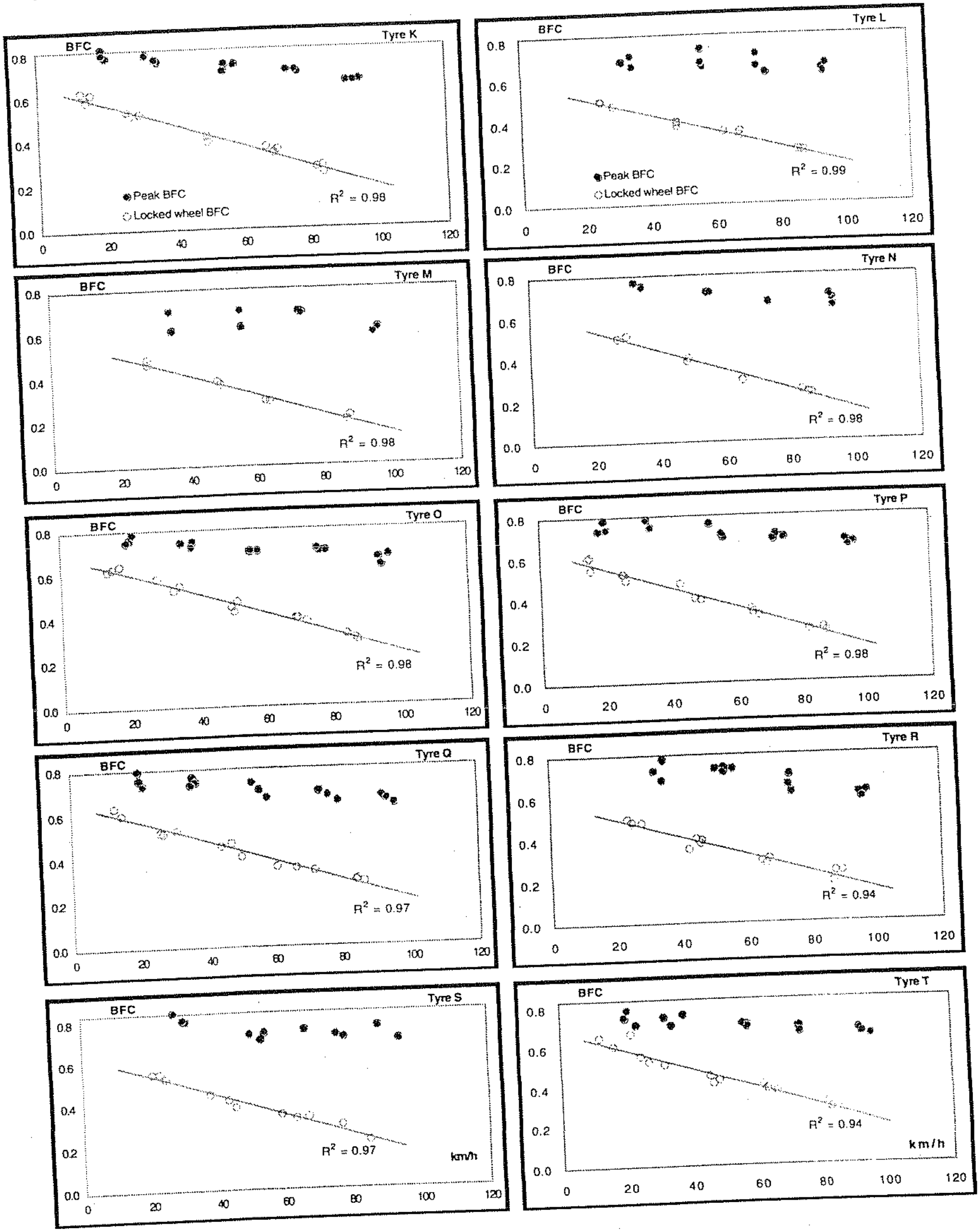

Figure 3(b) - Peak and locked wheel braking force coefficients (BFCs) on wet Motorway Asphalt

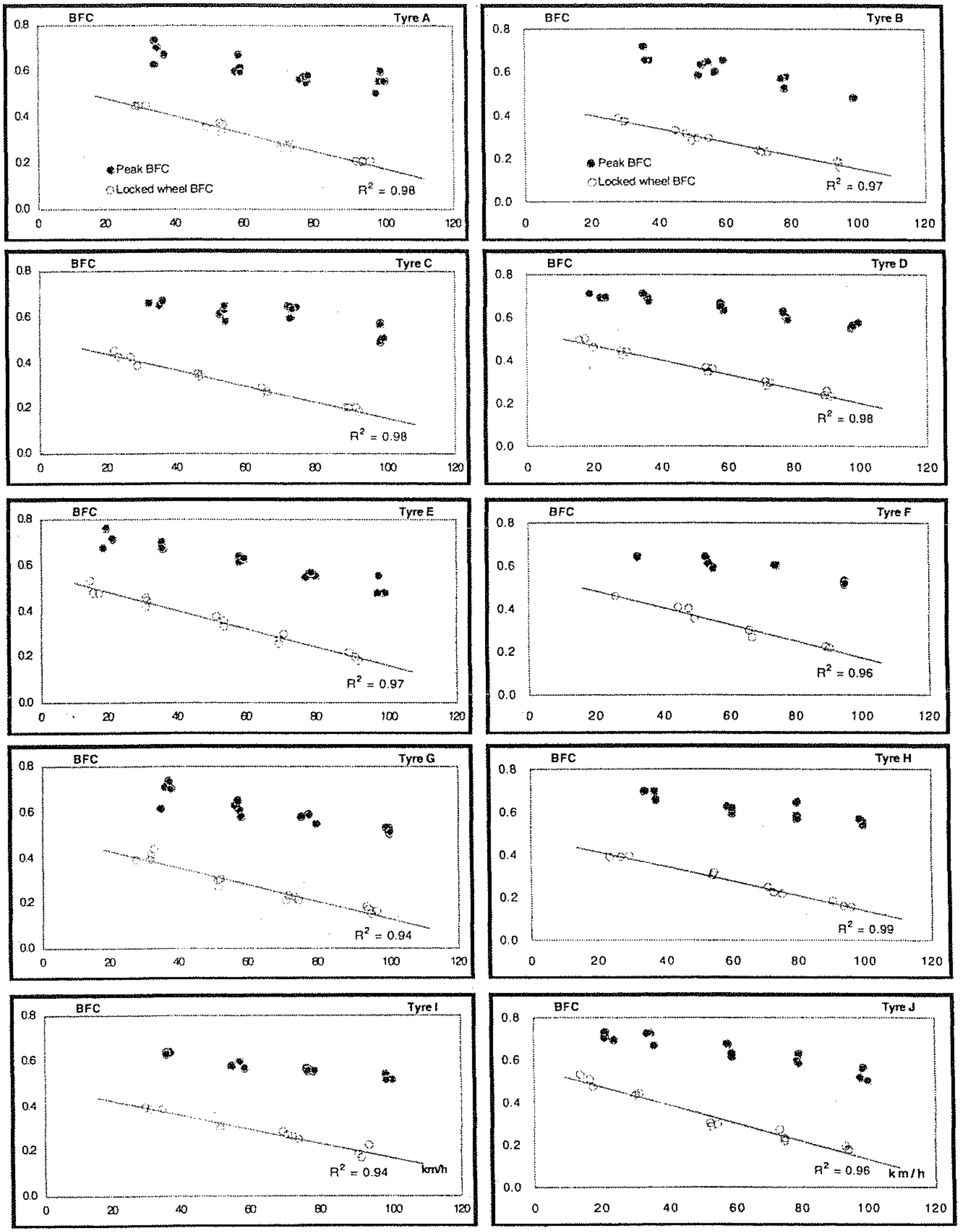

Figure 4(a) - Peak and locked wheel braking force coefficients (BFCs) on wet Concrete

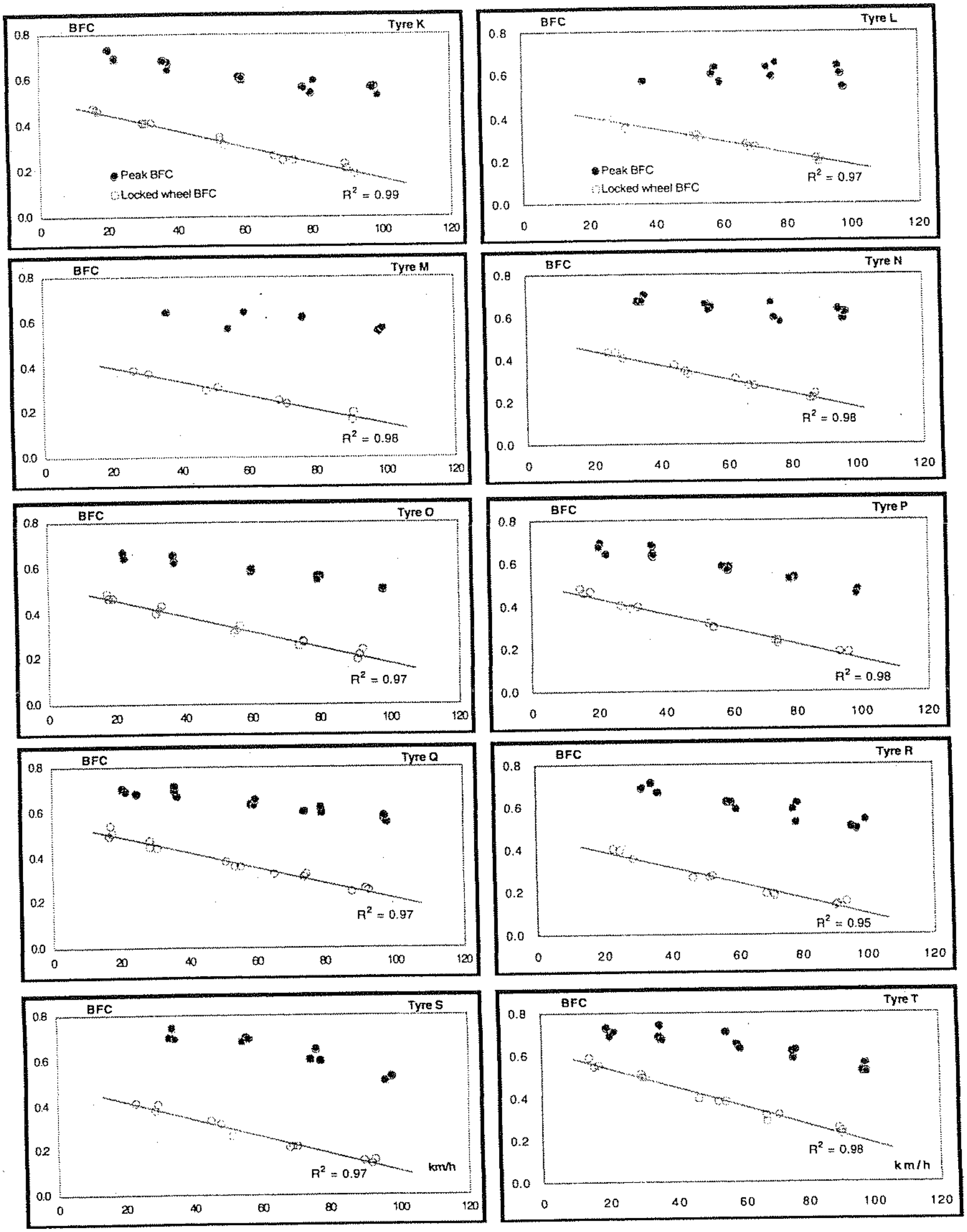

Figure 4(b) - Peak and locked wheel braking force coefficients (BFCs) on wet Concrete

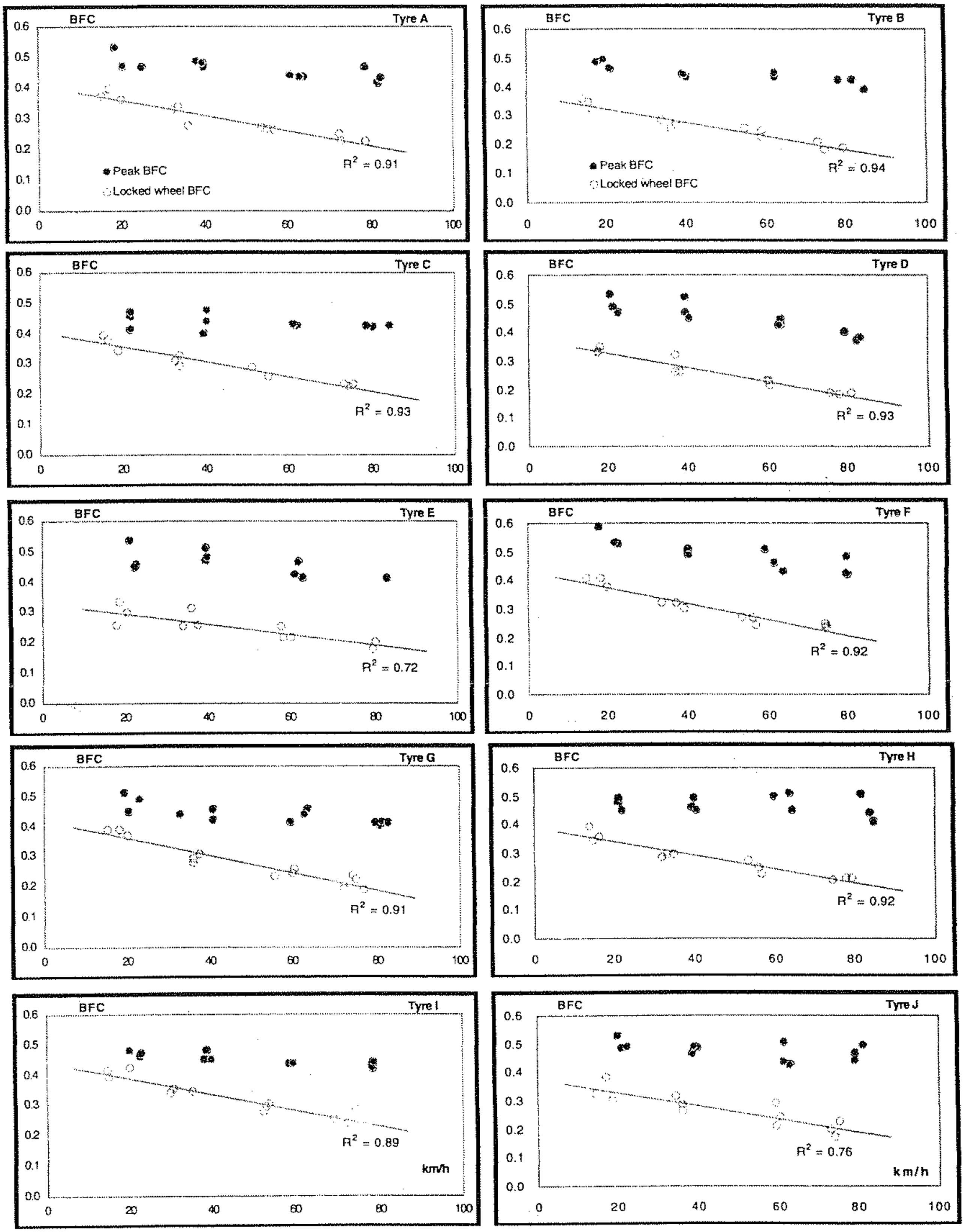

Figure 5(a) - Peak and locked wheel braking force coefficients (BFCs) on wet Bridport Gravel

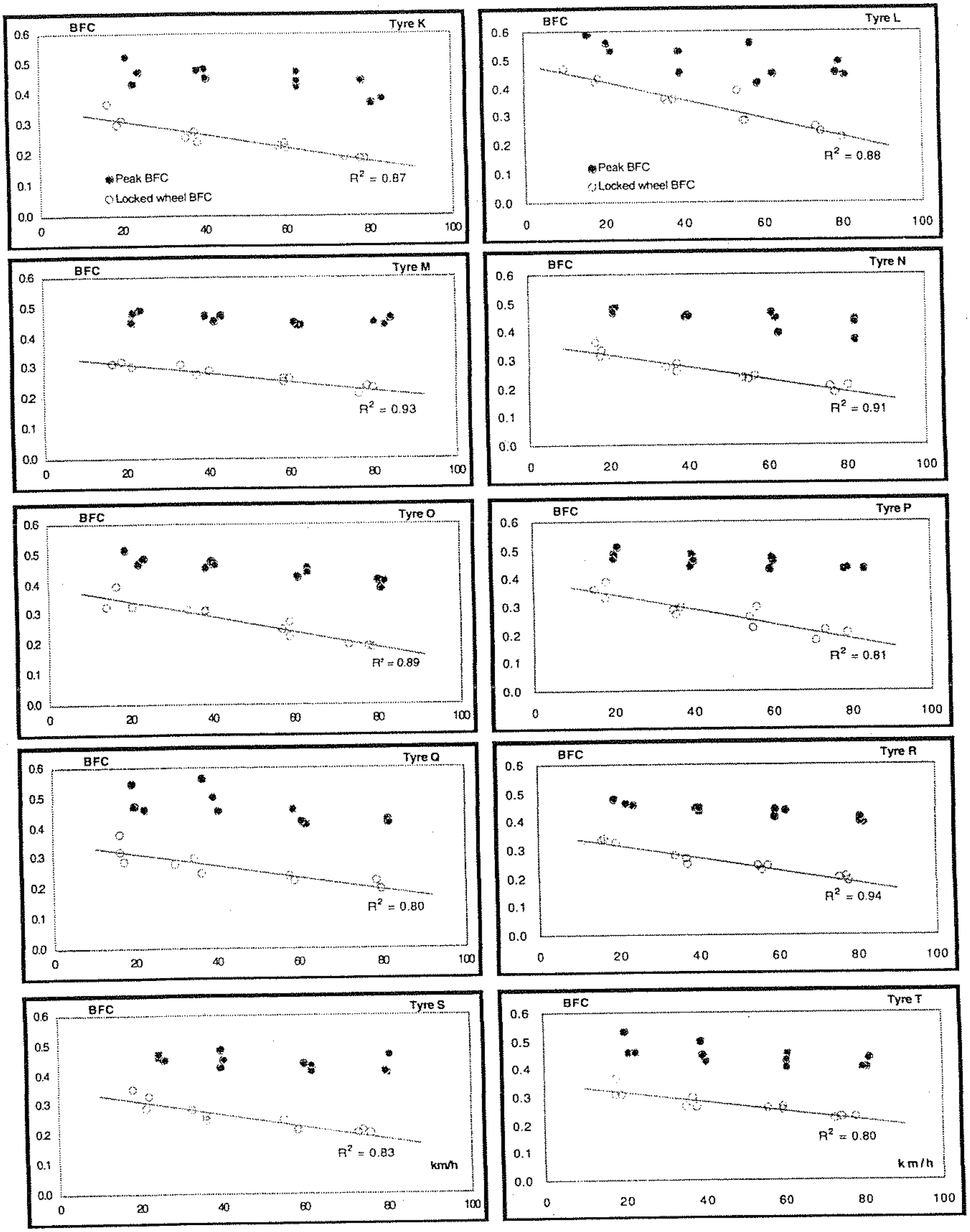

Figure 5(b) - Peak and locked wheel braking force coefficients (BFCs) on wet Bridport Gravel

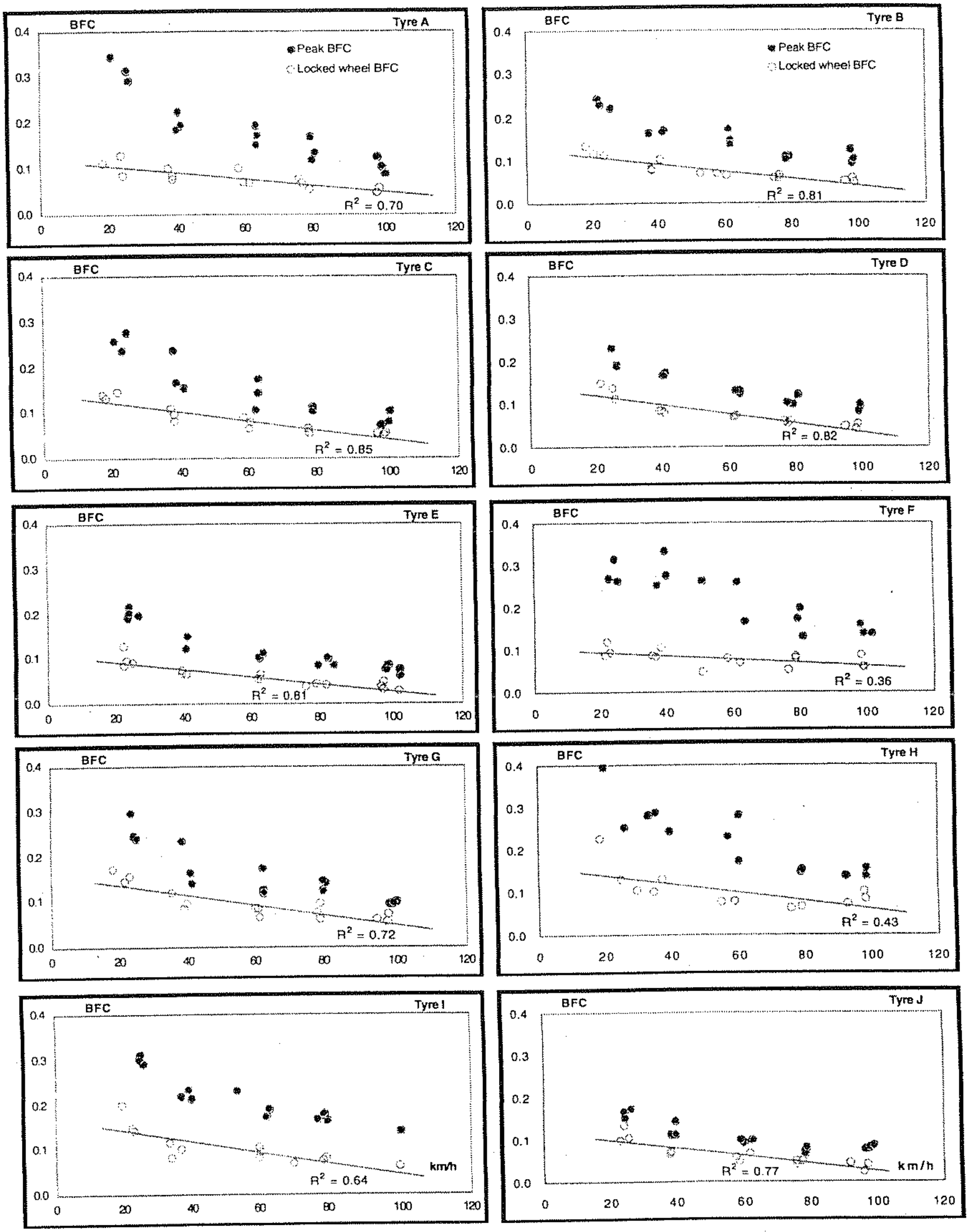

Figure 6(a) - Peak and locked wheel braking force coefficients (BFCs) on wet Mastic Asphalt

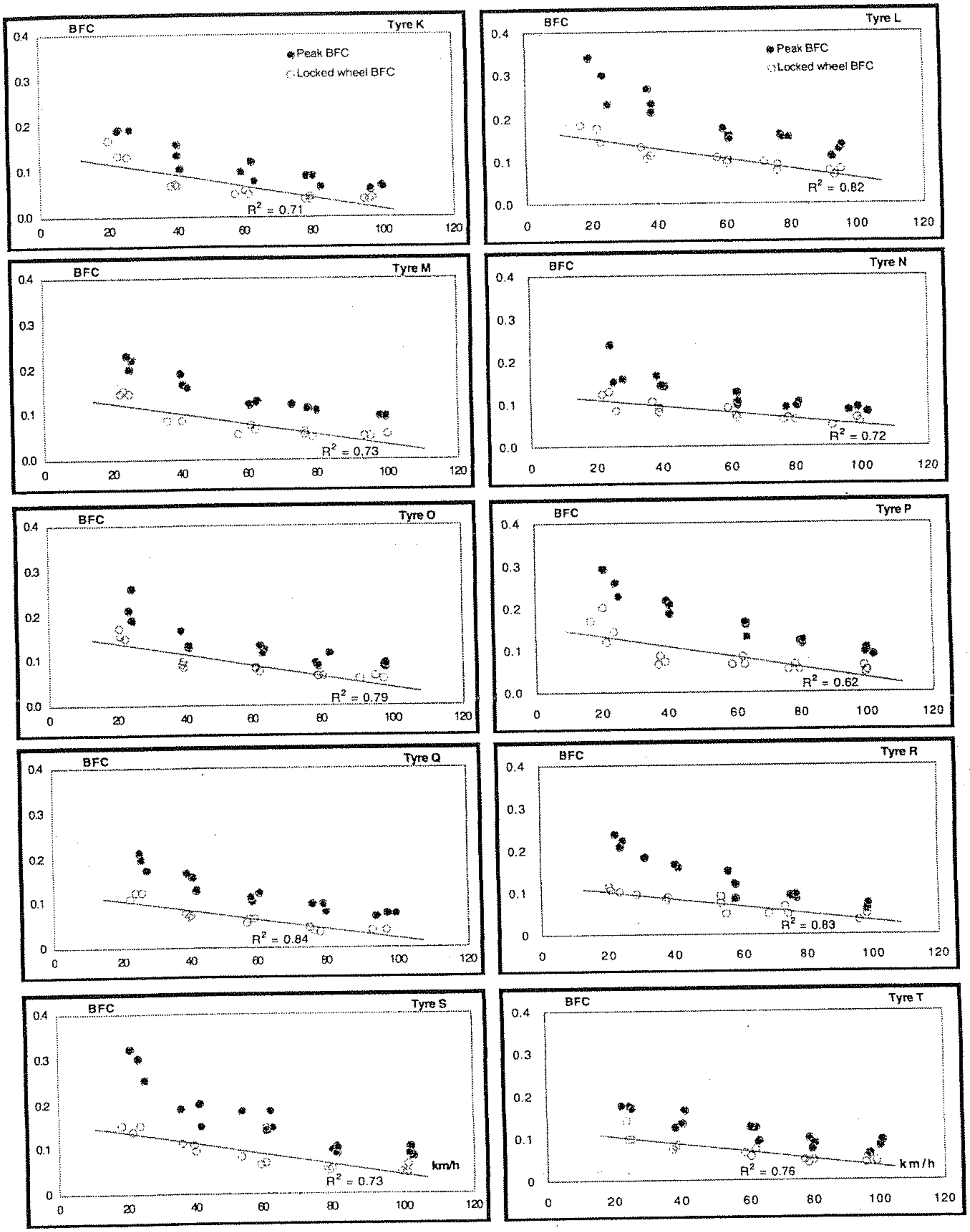

Figure 6(b) - Peak and locked wheel braking force coefficients (BFCs) on wet Mastic Asphalt

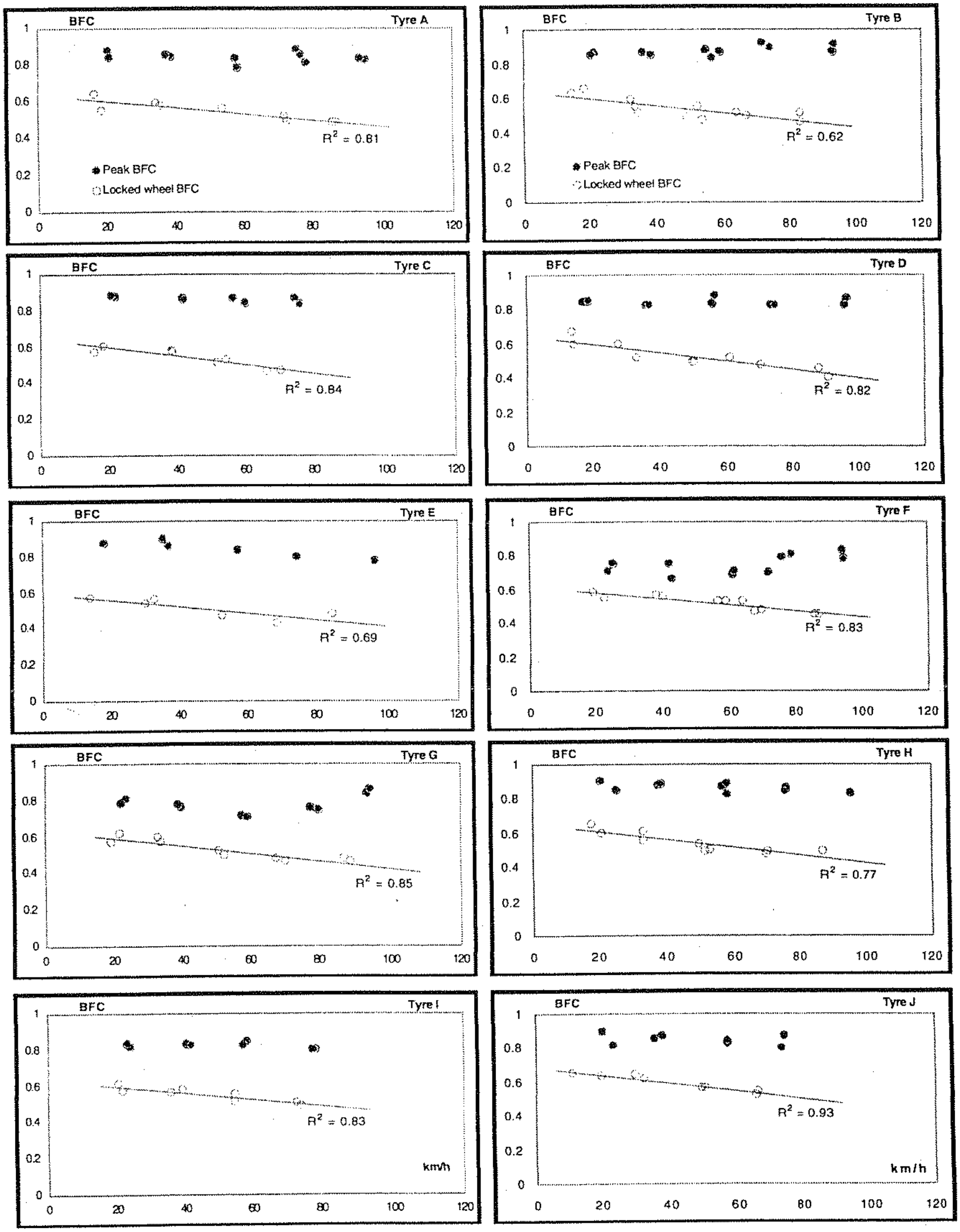

Figure 7(a) - Peak and locked wheel braking force coefficients (BFCs) on dry Fine Textured Asphalt

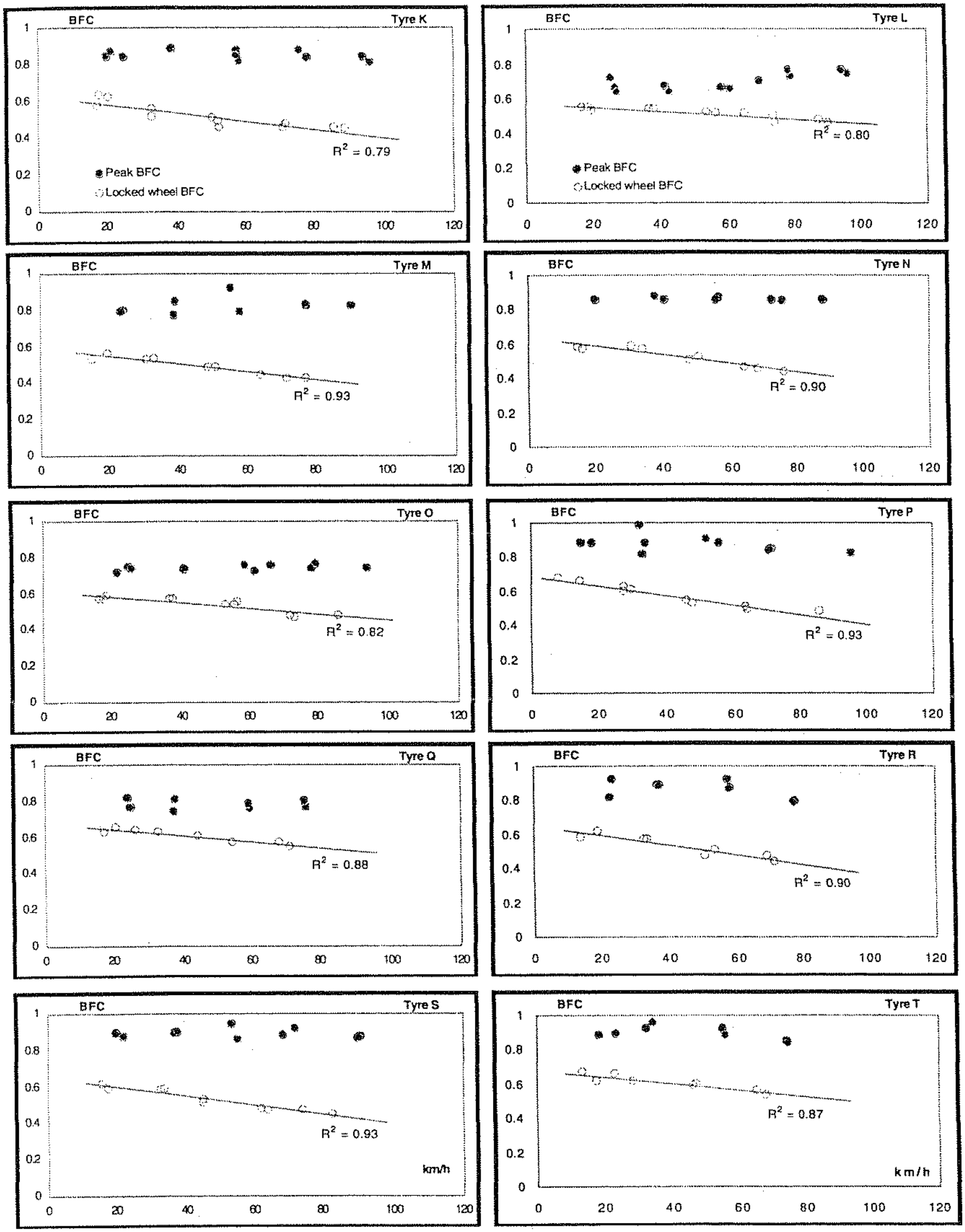

Figure 7(b) - Peak and locked wheel braking force coefficients (BFCs) on dry Fine Textured Asphalt

7.1.2 Stopping distances

Any differences in locked wheel performance of the various tyres would not necessarily be apparent from the graphs of BFC against speed, as low BFCs at high speed are potentially of much greater significance to vehicle safety than low BFCs at low speed. This is because a vehicle's overall stopping distance is determined mainly by the deceleration when the brake is first applied, ie. when the vehicle speed is greatest. To aid comparison between tyres, Figures 8 and 9 show the calculated locked wheel stopping distances from 90 km/h or 80 km/h for each tyre on each surface.

The stopping distances have been calculated from the locked wheel regression lines shown in Figures 2-7. By taking account of the change of BFC with speed, they are representative of the distance it would take a vehicle to travel, with all wheels locked, from a given speed to rest.

Table 3 summarises the stopping distance calculations, showing the minimum, maximum and average distances on each surface. The last column in the Table shows the percentage variation from the average for 95% of the data, assuming a normal population distribution, that is 95% of the results would be expected to be within ±x% of the sample average.

The variation of stopping distance is quite large on all the wet surfaces, especially the concrete and mastic, but smaller on the dry surface. This means that on any given surface, it is likely that some tyres will give substantially shorter stopping distances than others. Since normal roads can be constructed with a variety of different surfaces, this fact will only be significant if any tyres give consistently short or long stopping distances on a wide variety of surfaces. To establish if this is the case, it is sensible to combine the calculated stopping distances described above into a single measure of overall braking performance. The most important consideration is that this measure should be representative of the likely performance of the tyres on real-world road surfaces. Bridport Gravel and Mastic Asphalt are not found on normal roads and have therefore been excluded from the following analysis of the stopping distance results:

To be properly representative of real road surfaces, the measure of stopping distance performance should take into account the relative proportions of each of the road surfaces in common use, including what proportion of their use is when the road surface is wet. This information is not readily available and hence it is necessary to adopt a slightly subjective measure. For simplicity the following analysis assumes that all of the common road surfaces tested (wet FTA, wet motorway asphalt, wet smooth concrete and dry FTA) are equally prevalent on normal roads. By dividing all the calculated stopping distances on any given surface by the average distance on that surface, a series of stopping distance index rankings can be obtained for each tyre. If a tyre has a ranking of more than 1.0 on all the surfaces, then it consistently produces longer than average stopping distances, whereas a tyre with an index consistently less than 1.0 produces shorter than average stopping distances. Table 4 shows the indices for all the tyres studied. The variation of average stopping distance index between tyres is a little lower than was found on individual wetted surfaces (95 percentile range is ± 11%). It is noticeable that very few tyres have indices consistently less than or greater than 1.0 (for clarity, all indices < 1.0 are shown with a shaded background in Table 4). Tyres A, Q and T all gave consistently better than average stopping distances, whereas only tyre M gave longer than average distances on all the surfaces. It is interesting to note that the three best tyres, in terms of low average stopping distance indices, are all retreads of one form or another.

Further analyses of the stopping distance results are presented in the section of this report discussing the feasibility of a standard test procedure.

7.1.3 Effect of axle load and inflation pressure

A short series of tests were carried out to assess the effects of varying the axle load and inflation pressure on braking

TABLE 3.

Summary of locked wheel stopping distances

Surface	Stopping distances (metres) Minimum	Maximum	Average	95 percentile variation from average
Wet FTA (90 km/h)	76.3	100.0	86.9	±14%
Motorway asphalt (90 km/h)	79.6	106.3	95.6	±15%
Concrete (90 km/h)	94.4	146.5	113.6	±22%
Dry FTA (90 km/h)	55.7	69.6	63.0	±11%
Bridport (80 km/h)	83.0	109.9	100.4	±14%
Mastic asphalt (80 km/h)	233	398	315	±30%

TABLE 4.

Stopping Distance Indices on high-grip surfaces

Stopping Distance Indices (SDI, values < 1.00 indicate shorter than average stopping distances)

Tyre / Surface	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
Wet FTA	0.96	0.99	0.88	1.03	1.15	0.93	0.98	0.99	0.92	1.10	1.14	0.98	1.06	1.04	1.02	1.07	0.93	0.99	0.93	0.92
Wet motorway	0.98	1.11	1.05	0.95	0.95	1.01	1.06	1.11	1.02	0.92	0.94	0.99	1.11	1.04	0.83	1.00	0.91	1.10	1.03	0.90
Wet concrete	0.90	1.09	1.00	0.87	0.93	0.91	1.08	1.08	1.00	1.01	0.97	1.01	1.09	0.96	0.93	1.03	0.83	1.29	1.19	0.84
Dry FTA	0.96	0.99	1.02	1.03	1.05	1.01	1.00	0.99	0.97	0.93	1.04	1.00	1.10	1.05	0.98	0.99	0.88	1.07	1.03	0.90
Average	0.95	1.05	0.99	0.97	1.02	0.97	1.03	1.04	0.98	0.99	1.02	1.00	1.09	1.02	0.94	1.02	0.89	1.11	1.05	0.89

adhesion. Two tyre types were used to assess the effect of axle load. The load was varied between 4320 kg and 3150 kg on the wet motorway surface and between 4920kg and 3150kg on the wet concrete surface. For each test the inflation pressure was set to a value appropriate for the load being carried. The results, in the form of locked wheel BFCs, are shown in Figure 10 (wet motorway surface) and Figure 11 (wet concrete).

Neither tyre gave any consistent difference in locked wheel BFC between different loading conditions. It is therefore reasonable to conclude that the effect of axle load on locked wheel BFC is negligible.

One tyre type, Tyre T, was used to examine the effect of varying the inflation pressure. With a fixed axle load of 3150kg, the inflation pressure was varied from 65 psi (4.5 bar) to 120 psi (8.3 bar). The results are shown in Figure 12. The results indicate that on the wet motorway surface, the over-inflated tyre gives slightly lower BFCs across the tested speed range. On the concrete, however, there is very little difference between the two pressures.

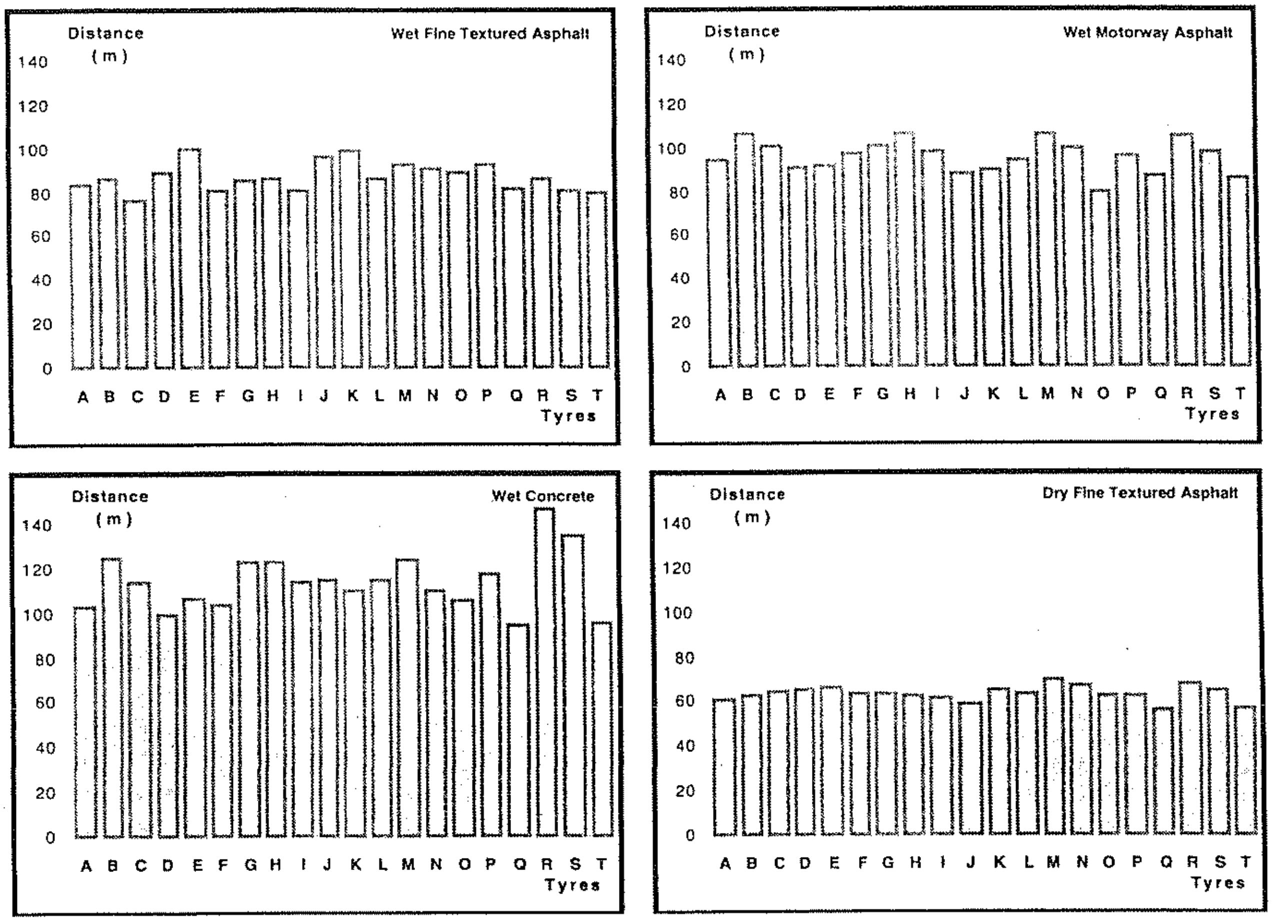

Figure 8 - Calculated locked wheel stopping distances from 90 km/h

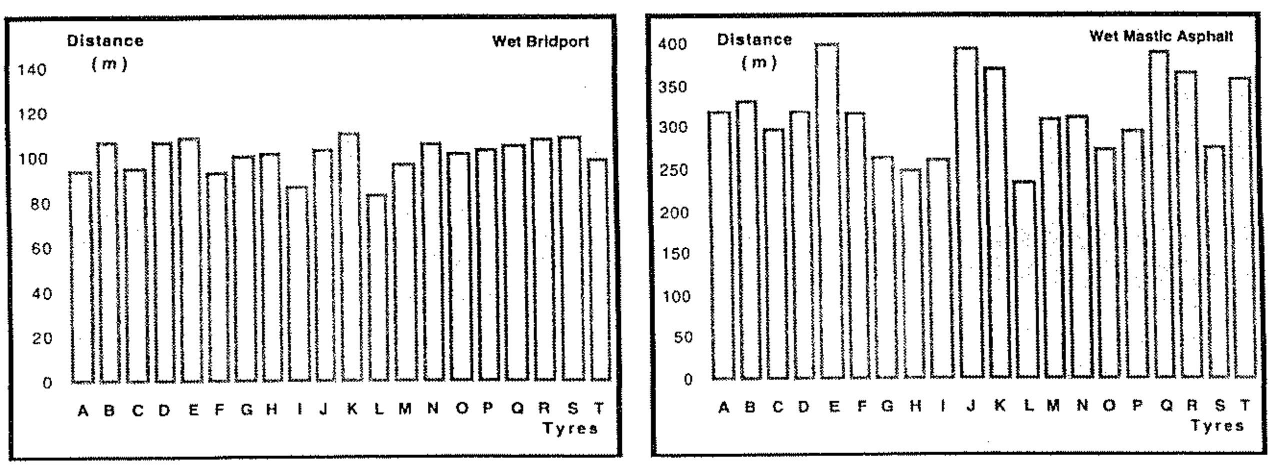

Figure 9 - Calculated locked wheel stopping distances from 80 km/h

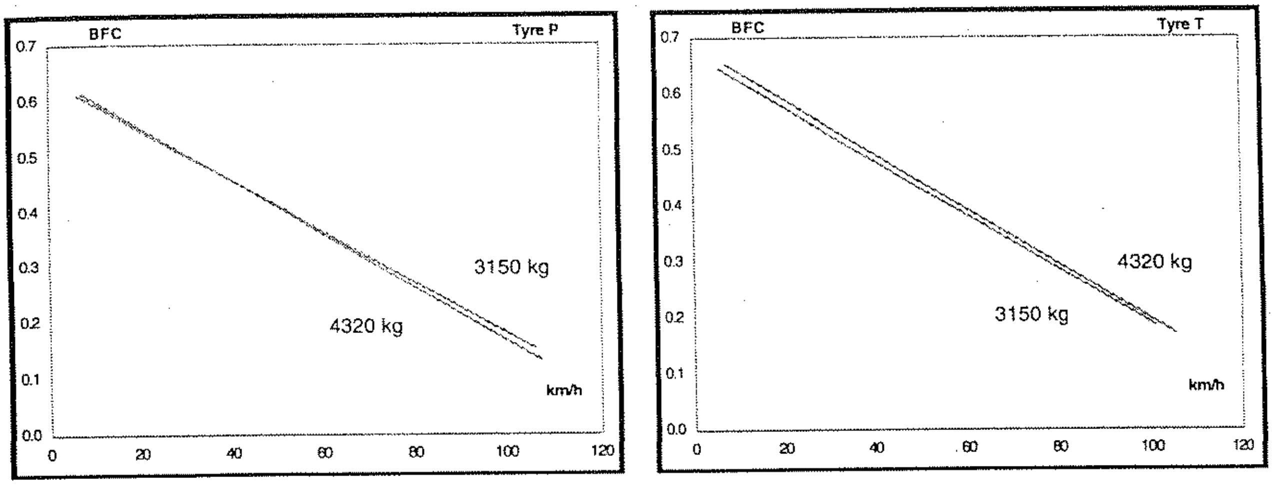

Figure 10 - Effect of axle load on locked wheel BFCs on wet motorway

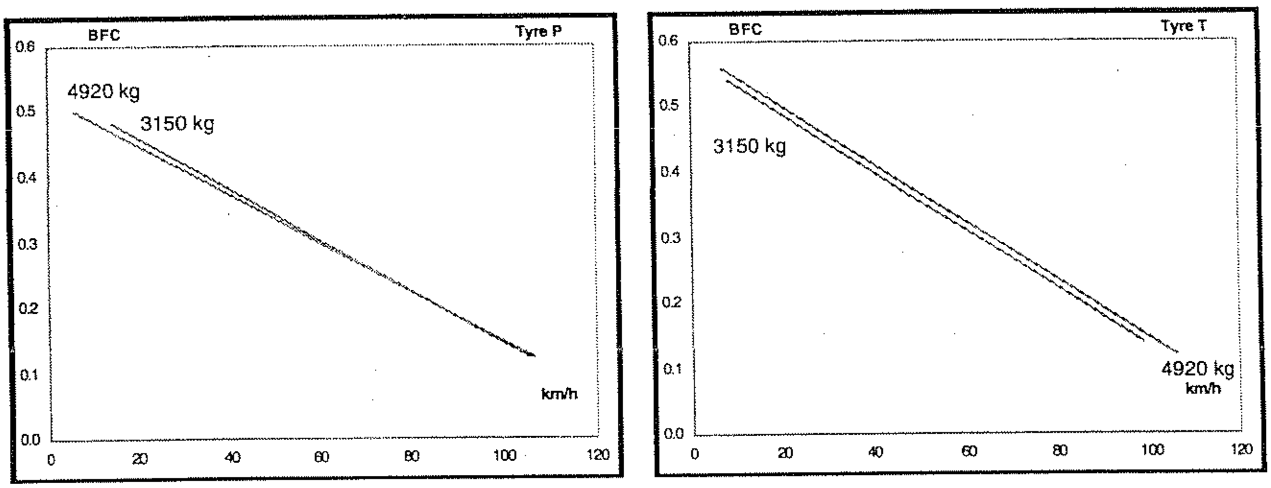

Figure 11 - Effect of axle load on locked wheel BFCs on wet concrete

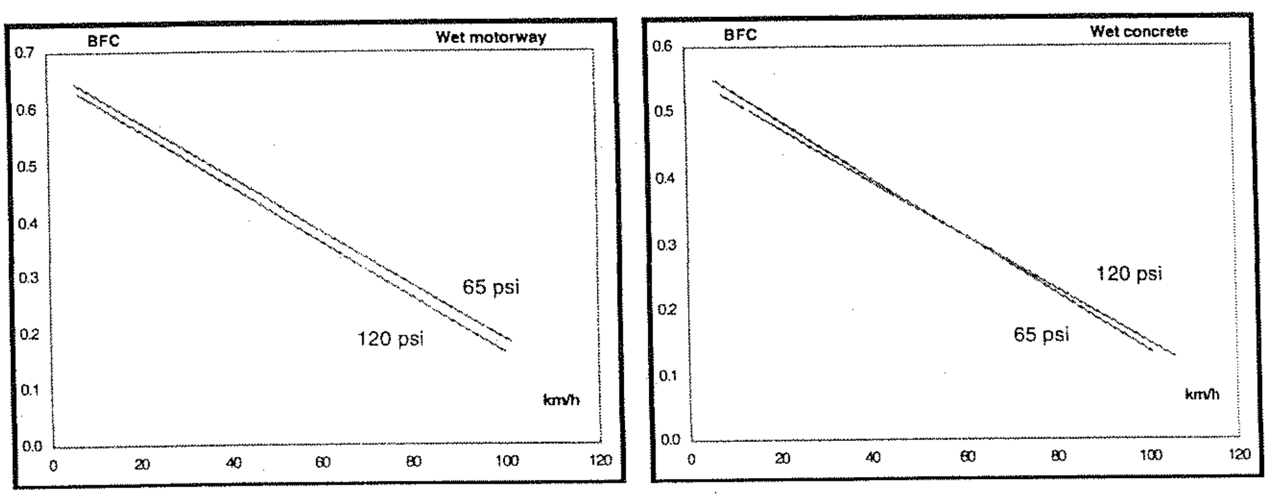

Figure 12 - Effect of inflation pressure on locked wheel BFCs, Tyre T, 3150kg axle load

7.2 CORNERING TESTS

The average peak Side Force Coefficient results from the wet cold rolled asphalt and wet Bridport surfaces are shown in Table 5. Time and track availability constraints meant that testing on the asphalt surface was limited to the twelve tyres used as part of the Tyre Noise and Safety project.

The results on the cold rolled asphalt vary between 0.56 and 0.67, with the two wide trailer tyres (R and S) giving the highest SFCs. In general, however, there was very little variation across the range of tyres, with two-thirds of the tyres giving values in the range 0.58 to 0.61. There was a similar amount of variation on the Bridport surface, with values ranging from 0.42 to 0.53, half being in the range 0.48 to 0.51.

8. COMPARISON WITH 1970s TEST DATA

Reference 2 reports that in the late seventies, seven radial ply commercial vehicle tyres were tested using broadly the same procedures as described in Section 6 of this report. Many of these braking tests were conducted on similar surfaces to those used for the recent (1993/94) tests. The surfaces now are not exactly the same as were in place in the late seventies so direct comparison of the two sets of results is very difficult. SCRIM readings are now routinely taken on many parts of the TRL track each year, but this was not the case in the late seventies. The only surface where SCRIM data is available for both sets of tests is the motorway asphalt surface. In the late seventies this surface gave a SCRIM coefficient (measured at 50 km/h) of 0.51. The surface was replaced during the eighties and in 1993 the value was 0.59. With some corrections to allow for the change in SCRIM value, the BFC results should be comparable.

To correct for the change in SCRIM, the peak and locked wheel BFCs from the late seventies have been multiplied by the ratio of SCRIM values (0.59/0.51 = 1.16). The resulting corrected BFCs should all be similar if there was no difference in tyre performance between the two sets of tests. Because of the uncertainties in comparing data from tests on two different, if similar, surfaces, a detailed analysis would not be appropriate. It has been found, however, that when the locked wheel BFCs obtained in the late seventies are multiplied by the SCRIM ratio of 1.16, the calculated stopping distances on the motorway asphalt surface become very similar to those obtained with the 1993/94 tyres.

There was found to be a significant difference in the results obtained on the dry FTA surface. The modern tyres produced calculated stopping distances that were approximately 10% shorter than those obtained from their late seventies counterparts. It is believed that the surface is the same now as was in place then, but this cannot be established conclusively. There is some evidence, therefore, that modern commercial vehicle tyres have better dry surface adhesion levels than their late seventies equivalents.

Another difference between the two sets of tests is that all the late seventies tyres were the same size, 10.00 x 20. It is known from previous TRL research that the difference between tyres on a wet harsh micro-textured surface is usually attributable to the difference in tyre compound whereas the difference between tyres on a wet smooth surface is a function of tread pattern. Tyre size does not appear to affect the results significantly. This is confirmed by the recent tests, in which tyres of several different sizes were tested (see Table 1). Tyre size was found to have no significant effect on BFC on any surface. Tread pattern, however, was found to be a significant factor on the two smooth micro-textured surfaces (Bridport and Mastic), but not on the other, harsh micro-texture surfaces. On both Bridport and Mastic, a statistical analysis of the stopping distance results reveals that tyres with a predominantly circumferential tread pattern (typically used on steer axles and trailers) give significantly longer stopping distances than tyres with a lateral pattern (drive axles and on/off highway use). There was no significant variation in stopping distances between these two tyre groups on the wet FTA, dry FTA, motorway and concrete surfaces.

The only other difference between the late seventies and 1993/94 tests is the instrumentation used. In 1977 the vehicle deceleration was recorded onto paper using a UV recorder and vehicle speed was measured using a fifth wheel. The accuracy of the measurements are comparable, but the modern instrumentation allows the raw data to be analysed far more quickly.

TABLE 5.

Peak Side Force Coefficients

Tyre	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
Average peak SFC																				
Cold rolled asphalt	0.59	0.60	0.58			0.56	0.58	0.61	0.63			0.60	0.58	0.61				0.64	0.67	
Bridport	0.47	0.43	0.43	0.50	0.48	0.50	0.43	0.44	0.42	0.51	0.53	0.50	0.51	0.51	0.48	0.45	0.51	0.44	0.45	0.52

9. FEASIBILITY OF STANDARD TEST METHOD

The results indicate that it would be very difficult to measure tyre adhesion by using one test procedure. The tyres generally gave different rank orderings with the braking tests on the various surfaces tested. Another different rank ordering was produced from the cornering tests. It would therefore be difficult to conclude too much from any one test, as a tyre that performs well in that test might perform badly in another. There were found to be statistically significant correlations between the stopping distances calculated on the wet concrete and wet motorway, wet Bridport and wet Mastic, wet Bridport and wet FTA, dry FTA and wet concrete and dry FTA and wet motorway surfaces, but not between any other surface pairs. Although some patterns emerged on some surfaces, for example retreaded tyres were found to give significantly shorter stopping distances (typically 8-13% from 90 km/h) than new tyres on the wet motorway, wet concrete and dry FTA surfaces, there were no consistent variations across all the surfaces tested, or even across all the high-grip surfaces typical of normal roads.

The locked wheel braking tests are likely to be most representative of the emergency conditions which current CV tyres are required to encounter. The increasing trend to fit anti-lock brakes to heavy commercial vehicles, however, may mean that locked wheel performance is becoming less important. Reference 5, which describes tests conducted by Cambridge University at TRL, reports that stopping distances with an air-suspended semitrailer and anti-lock brakes were much (up to 50%) shorter than with an ABS equipped, steel spring suspended semitrailer. The increased use of air suspension, in combination with ABS, would thus further tend to reduce the importance of locked wheel tyre performance.

Cornering adhesion is not usually very important for commercial vehicle tyres, as the vehicles tend to overturn before the limiting adhesion of the tyres is reached. It can be important in some situations, for example on a very slippery road surface or with an unladen vehicle with a low centre of gravity, and thus cannot be ignored altogether.

The tyre noise and safety study (Reference 1) found a significant negative correlation between truck tyre braking distances (measured on two high-grip, wetted surfaces) and noise levels (measured on two high-grip dry surfaces). The correlation is broadly equivalent to a one metre increase in stopping distance from 80 km/h for every 1dB reduction in coast-by noise level at 80 km/h. The potential effect of reducing truck tyre noise is thus quite small, but might be sufficient to warrant concern if large reductions in tyre noise limits were proposed. The tyre noise and safety study also concluded that since some tyres were found not to comply with the general trend, that is they gave both low noise levels and good braking characteristics, there was scope for tyre designers to maintain adhesion levels despite reductions in permitted noise levels.

When one considers the results of all the studies into truck tyre performance, it is clear that the factors which influence tyre and vehicle adhesion levels are many and complex. The trend to reduce tyre noise levels could possibly lead to an increase in commercial vehicle emergency stopping distances, but other trends, such as fitting anti-lock brakes and electronic braking systems ("brake by wire"), may well have the opposite effect. The crucial question is whether legislation (or international standards) should be used to ensure that tyre adhesion levels are maintained or improved, despite all the other influencing factors. It is important that any legislative test is simple to perform and representative of the conditions tyres encounter in normal use. It is also important that the design of tyres which easily meet any adhesion level requirements are not changed (to give better noise or tread wear characteristics for example) in such a way that the adhesion performance is reduced to nearer the minimum standard required. It is also of vital importance that improvements in tyre adhesion levels translate into improvements in commercial vehicle braking and cornering performance. This can only be done through further legislation or standards governing all the various vehicle components and systems (brakes, suspension etc.) which influence this performance.

In summary there are various factors which influence tyre adhesion. The substantial international pressure to reduce tyre noise levels is likely to result in legislation in the very near future. Such legislation might well adversely affect tyre adhesion levels. It is therefore the author's opinion that there is a need to develop a legislative test for commercial vehicle tyre adhesion with some urgency. It is likely that whatever form of test may be proposed or developed, the main problem will be in specifying what road surfaces the adhesion levels are to be measured on. The research described in this report has found that any given tyre can perform quite differently, relative to another tyre, from one surface to another. Any standard test must therefore cover a variety of surfaces. Further research into the effect of road surface on tyre adhesion needs to be carried out in preparation for the development of a standard test. One surface which ought to be considered is porous asphalt, which is becoming increasingly common on British and European roads.

9.1 THE CHOICE OF TEST PROCEDURE

The choice of which test procedure(s) to use will doubtless be another topic of considerable international debate. This research has found that the front wheel straight-line braking test along the lines of that prescribed in ASTM F403 - 86 offers a simple and effective means of measuring peak and locked wheel tyre adhesion levels on a wide variety of surfaces.

There are other methods of measuring tyre braking force coefficients. The first uses a drum rig and is a method employed extensively by tyre manufacturers. The drum can usually be rotated at speeds up to the maximum rating for high speed tyres. A test tyre is placed on the drum at a required load and the wheel is braked as the drum is rotated at a required speed. The method is good for comparing performance of tyres but is not so good at producing accurate values of BFC - the curvature of the drum produces a different pressure distribution within the tyre contact patch compared with that produced on a flat road surface. Another objection with drum rigs is that, with rare exceptions, they cannot be used wet. On the other hand they are usually under cover and can be used continuously in comfortable working conditions.

The second method can be used on any road surface and probably gives the most accurate and repeatable results but uses a facility which is expensive to build. An instrumented wheel is fitted between the axles of a test-bed vehicle, or a large multi-wheeled trailer (as used by TNO, Delft for example). Wheel vertical load can usually be varied and the peak and locked wheel braking forces, produced by normal braking means or by some other method of slowing the wheel, are measured by a force transducer. The vehicle is driven over a test surface at a required speed and where wet surface measurements are needed, a metered quantity of water is deposited in front of the wheel, or the surface may be wetted by other means. Because of the cost of such equipment, there are very few of this type of test rig throughout the world. A similar type of instrumented wheel can be mounted in a small, single wheel trailer and be towed by a separate vehicle but the system is not as versatile in use as the test-bed type of rig.

Computer simulation is another possible method of measuring tyre adhesion. Though a great deal of work has gone into developing usable tyre models in recent years, much of this work has concentrated on car tyres. It is likely that a great deal more modelling and validation work will be needed before a truck tyre computer simulation model can be used confidently to replace the more traditional test methods.

In future, the development of computer models or more useful and accurate drum rigs may make the testing of real tyres on real vehicles on real road surfaces unnecessary. At present, however, it is the author's opinion that the straight line front-wheel braking method, as described in this report, offers the best combination of cost, simplicity, accuracy, reliability and repeatability.

A standard cornering test would be difficult to justify, as the ultimate cornering grip of commercial vehicle tyres may be rarely needed in real situations. It is considered unlikely that the difficulties in developing a simple, safe and realistic cornering test for commercial vehicle tyres will be justified by the improvements in vehicle safety that might result. Other test methods, such as use of a drum rig or computer simulation, may allow the safe study of cornering adhesion of commercial vehicle tyres on high grip surfaces, but a legislative test using even these methods may be difficult to justify. It has been established for many years (Reference 6, for example) that for car tyres, peak BFCs are very similar in magnitude to maximum SFCs. This was found to be the case for the truck tyres tested during 1993/94, where the maximum SFCs at 30 km/h on the Bridport were very similar to the peak BFCs at the same speed. The average SFC for all the tyres tested was 0.476, exactly the same value as the average peak BFC. There was also found to be a strong correlation between the two measurements over the range of tyres tested, indicating that peak BFC can be used to predict maximum SFC, thus avoiding the need for any separate test to measure cornering adhesion. It should be stressed, however, that only one surface was used to compare BFC and SFC measurements. It is not known if there is such a correlation on other types of surface.

10. CONCLUSIONS

1.The commercial vehicle tyres tested recently generally gave similar Braking Force Coefficients (BFCs, peak and locked wheel) on any one surface. The most consistent surface was found to be the wet FTA, where, for example, the lowest peak BFC at 60 km/h was only 11% less than the highest (both readings found from plotting a best-fit regression line through the measured data points) and the equivalent figure for the locked-wheel BFCs at 60 km/h was 22% less. The Mastic surface gave the greatest variability, with the lowest peak BFC at 60 km/h being about 50% lower than the highest and the lowest locked-wheel BFC being about 40% lower than the highest.

2. The peak BFCs on the wet Fine Textured Asphalt surface showed a slight reduction as speed increased. The tyres generally gave values ranging from around 0.7 to 0.8 at 20 km/h down to roughly 0.6 at 100 km/h. The locked wheel BFCs were all highly linearly related to vehicle speed. Values of 0.55 - 0.6 at 20 km/h down to 0.25 at 90 km/h were typical.

3. The peak BFCs on the wet motorway asphalt surface were only slightly speed dependent. The tyres generally

gave values of around 0.75 at low speed (20-30 km/h) reducing to 0.65 at 100 km/h. The locked wheel BFCs were all highly linearly related to vehicle speed. Values of 0.5 - 0.6 at 20 km/h down to 0.2 - 0.25 at 90 km/h were typical.

4. The peak BFCs on the wet concrete surface showed a slight reduction as speed increased, typically from around 0.65 - 0.7 at 20 or 30 km/h down to 0.5 - 0.6 at 100 km/h. Once again, there was a clear linear speed dependence apparent with the locked wheel BFCs. Values of 0.40-0.5 at 20 km/h and 0.15 - 0.25 at 90 km/h were typical.

5. The peak BFCs on the Bridport surface were only slightly speed dependent. The tyres generally gave values of around 0.5 at low speed (20-30 km/h) reducing to 0.4 at 80 km/h. The locked wheel BFCs were all linearly related to speed, but the relationship was not as strong as on the wet FTA, motorway and concrete surfaces. Values of 0.3 - 0.4 at 20 km/h down to 0.2 at 80 km/h were typical.

6. The peak BFCs on the wet Mastic asphalt surface generally showed a reduction as speed increased, typically from around 0.2 - 0.3 at 20 km/h down to 0.1 at 100 km/h. There was evidence for a linear speed dependence with the locked wheel BFCs, but the dependence was much lower than on any of the other surfaces tested. Values of 0.1 - 0.15 at 20 km/h and 0.05 at 90 km/h were typical.

7. The peak BFCs on the dry FTA surface remained fairly constant as speed increased. The tyres generally gave values ranging from around 0.7 to 0.9 at all speeds. The locked wheel BFCs are all linearly related to vehicle speed, but the strength of the relationship is lower than was found on the wet FTA surface. Values of 0.6 at 20 km/h down to 0.4 at 90 km/h are typical.

8. There was found to be a large difference between peak and locked wheel BFCs on all the surfaces, indicating the potential for improvement in braking performance if greater use was made of the peak adhesion by means of systems designed to prevent wheel lock.

9. On both Bridport and Mastic, a statistical analysis of the stopping distance results reveals that tyres with a predominantly circumferential tread pattern (typically used on steer axles and trailers) give significantly longer stopping distances than tyres with a lateral pattern (drive axles and on/off highway use). There was no significant variation in stopping distances between these two tyre groups on the wet FTA, dry FTA, wet motorway and wet concrete surfaces.

10. The peak Side Force Coefficients, measured on a wet asphalt surface and a wet Bridport gravel surface, did not vary much between tyres, though the wide trailer tyres gave slightly better results than the other, narrower tyres on the asphalt surface.

11. Changes in test surface characteristics made comparisons between the recent series of tests and a similar series conducted in the late seventies very difficult. Where tentative comparisons were possible, there was some evidence that the adhesion levels of current generation tyres on a dry surface, as indicated by straight line braking tests, are significantly better than those from CV tyres tested previously, but this trend was not evident when comparing results on a wetted surface.

12. The results indicate that it would be very difficult to measure tyre adhesion by using one test procedure. The tyres tested generally gave different rank orderings with the braking tests on the various surfaces tested. Another different rank ordering was produced from the cornering tests. It would therefore be difficult to conclude too much from any one test, as a tyre that performs well in that test might perform badly in another.

13. Although some patterns emerged on some surfaces, for example retreaded tyres were calculated to give significantly shorter stopping distances than new tyres on the wet motorway, wet concrete and dry FTA surfaces, there were no consistent variations across all the surfaces tested, or even across all the high-grip surfaces typical of normal roads.

14. There are various factors which influence tyre adhesion. The substantial international pressure to reduce tyre noise levels is likely to result in legislation in the very near future. Such legislation might well adversely affect tyre adhesion levels. It is therefore the author's opinion that there is a need to develop a legislative test for commercial vehicle tyre adhesion with some urgency.

15. In future, the development of computer models or more useful and accurate drum rigs may make the testing of real tyres on real vehicles on real road surfaces unnecessary. At present, however, it is the author's opinion that the straight line front-wheel braking method, as described in this report, offers the best combination of cost, simplicity, accuracy, reliability and repeatability.

16. A standard cornering test would be difficult to justify, as the ultimate cornering grip of commercial vehicle tyres may be rarely needed in real situations. It is considered likely that the difficulties in developing a simple, safe and realistic cornering test for commercial vehicle tyres will not be justified by the improvements in vehicle safety that might result. There was some evidence from the recent tests that peak BFC can be used to predict maximum SFC, thus avoiding the need for any separate test to measure cornering adhesion.

11. REFERENCES

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12. ACKNOWLEDGEMENTS

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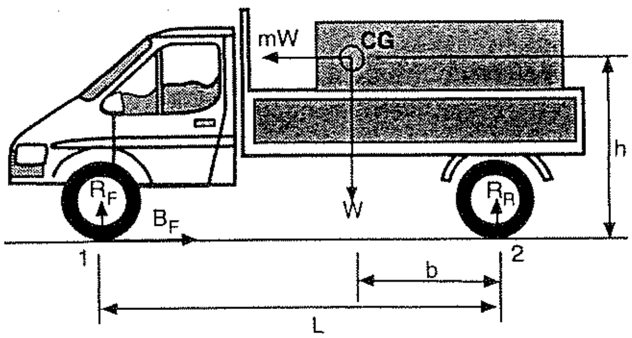
APPENDIX - CALCULATION OF BRAKE FORCE COEFFICIENT, SIDE FORCE COEFFICIENT AND STOPPING DISTANCE

A.1 BRAKE FORCE COEFFICIENT (BFC)



CG = Centre of total mass of vehicle

L = Vehicle wheel base (metres)

b = Distance from CG to rear axle (metres)

h = Distance of height of CG above ground (metres)

W = Total vehicle weight (N)

m = Vehicle braking deceleration (metres/sec^2) divided by 9.81 (gravitational acceleration). "m" has non-dimensional units of "g".

R_F = Front axle dynamic vertical load (N)

R_R = Rear axle dynamic vertical load (N)

B_F = Braking force at front axle (N)

Figure A1. Calculation of BFC

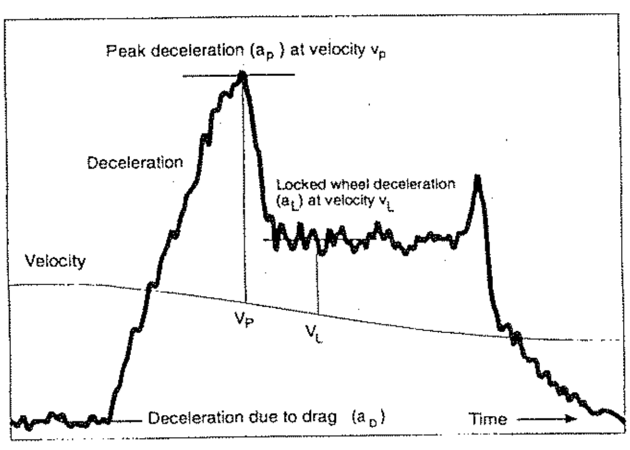


Figure A2.Typical vehicle deceleration/velocity traces

From Figure A1,

Equating longitudinal forces gives .... $B_F = mW$

Taking moments about point 2 gives .... $R_F L = Wb + mWh$

Braking Force Coefficient (BFC) is defined as .... $BFC = B_F / R_F$

Therefore .... $BFC = mW / \{(Wb + mWh)/L\}$

and thus:

$$BFC = \frac{mL}{(b + mh)}$$

From Figure A2, Peak BFC occurs when $m = a_P - a_D$ at velocity v_P and locked wheel BFC occurs when $m = a_L - a_D$ at velocity v_L.

A.2 SIDE FORCE COEFFICIENT (SFC)

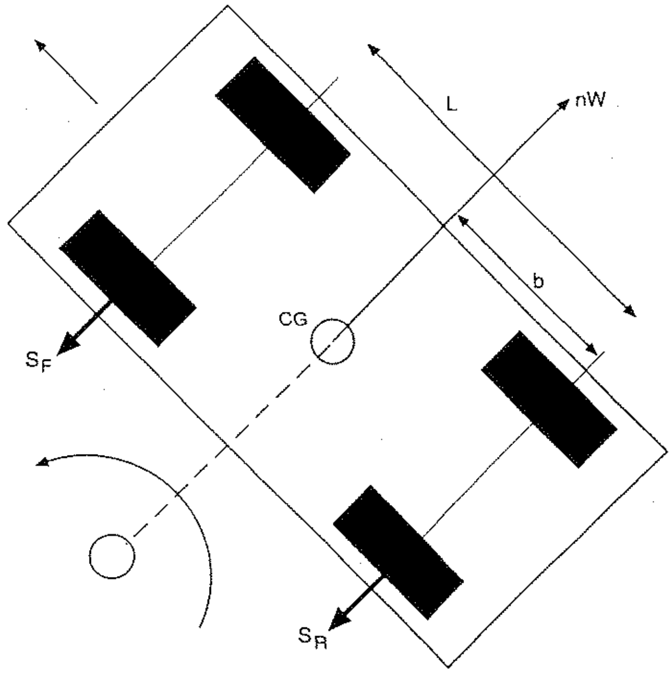


Figure A3. Calculation of SFC

In Figure A3,

CG = Centre of total mass of vehicle

L = Vehicle wheel base (metres)

b = Distance from CG to rear axle (metres)

W = Total vehicle weight (N)

n = Vehicle cornering acceleration (metres/sec^2) divided by 9.81 (gravitational acceleration). "n" has non-dimensional units of "g".

R_F = Front axle average vertical load (N)

R_R = Rear axle average vertical load (N)

S_F = Front axle average sideways force (N)

S_R = Rear axle average sideways force (N)

When the vehicle is in steady state cornering, the following applies,

Rear axle vertical load .... $R_F = bW/L$

Taking moments about rear axle .....$nWb = S_F L$

Cornering acceleration is given by..$n = S_F L/Wb = S_F/R_F$

Side Force Coefficient (SFC) is defined as .... $SFC = S_F / R_F$

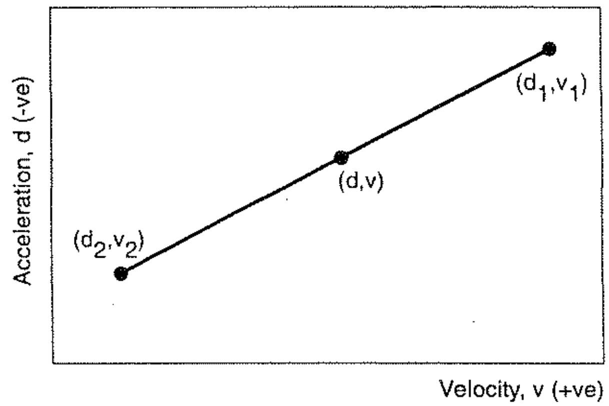
Therefore:

SFC = n

An accelerometer which is fixed on the body of the vehicle so that it is in a horizontal plane when the vehicle is stationary will record a component of gravitational acceleration equal to [sin Φ] when the vehicle is at a roll angle Φ . To correct for this it is necessary to use the expression $n = SFC = (a_m - \sin \Phi)$, where a_m is the measured cornering acceleration.

A.3 STOPPING DISTANCE

Consider the graph of deceleration, d (BFC x g) against velocity, v. The aim is to calculate the distance travelled between the point 1 (velocity v_1, deceleration d_1) to point 2 (velocity v_2, deceleration d_2), assuming that the deceleration varies linearly with velocity between the two points.



$$\frac{d_2 - d}{v_2 - v} = \frac{d_2 - d_1}{v_2 - v_1}$$

$$\therefore (d_2 - d)(v_2 - v_1) = (d_2 - d_1)(v_2 - v)$$

$$\therefore (d_2 - d) = \frac{(d_1 - d_2)}{(v_2 - v_1)} + \frac{(d_2 - d_1)v_2}{(v_2 - v_1)}$$

$$\therefore d = \frac{(d_2 - d_1)}{(v_2 - v_1)} + d_2 \frac{(d_2 - d_1)v_2}{(v_2 - v_1)}$$

$$\therefore d = kv + A, \text{ where } k = \frac{(d_2 - d_1)}{(v_2 - v_1)} \text{ and A } = \frac{(d_1v_2 - d_2v_1)}{(v_2 - v_1)}$$

but $d = \dfrac{d^2x}{dt^2}$ and $v = \dfrac{dx}{dt}$

$$\therefore \frac{d^2x}{dt^2} - k\frac{dx}{dt} = A$$

This is a second order differential equation with complementary function of the form $x = A_1 + A_2e^{kt}$ where A_1 and A_2 are constants.

The particular integral is found by solution of the equation $(D^2 - kD)x = A$:

$$x = \frac{A}{D(D-K)} = \frac{1}{D}\left(\frac{-A}{k}\right) + \frac{1}{D-k}\left(\frac{A}{k}\right)$$

By binomial expansion,

$$\therefore x = \frac{-A}{k}t + \left(\frac{-1}{k} - \frac{D}{k^2}\dots\right)\frac{A}{k}$$

and $x = \dfrac{At}{k} - \dfrac{A}{k^2}$

The complete solution is therefore :

$$x = A_1 + A_2 e^{kt} - \frac{At}{k} - \frac{A}{k^2}$$

$$\therefore \frac{dx}{dt} = kA_2 e^{kt} - \frac{A}{k}$$

and $\frac{d^2x}{dt^2} = k^2 A_2 e^{kt}$

Substituting end conditions :

when $t = 0, x = 0$ and $dx/dt = v_1$

$$\therefore A_2 = \left(\frac{v_1 + A/k}{k}\right)$$

and $A_1 = \frac{A}{k^2} - \left(\frac{v_1 + A/k}{k}\right) = \frac{v_1}{k}$

The time taken to reach deceleration d_2 from d_1 is given by :

$$d_2 = (kv_1 + A)e^{kt}$$

therefore, the braking time, t_o is given by :

$$t_o = \frac{1}{k}\log_e\left(\frac{d_2}{kv_1 + A}\right)$$

and hence the distance travelled between the two conditions is :

$$x = \frac{-v_1}{k} + \left(\frac{v_1 + A/k}{k}\right)e^{\log_e(d_2/kv_1+A)} - \frac{A}{k^2}\log_e(d_2/(kv_1 + A)) - \frac{A}{k^2}$$

$$\boxed{\therefore x = \left(\frac{v_1 + A/k}{k}\right)(e^{kt_o} - 1) - \frac{At_o}{k}}$$

where

$$t_o = \frac{1}{k}\log_e\left(\frac{d_2}{kv_1 + A}\right)$$

$$k = \left(\frac{d_2 - d_1}{v_2 - v_1}\right) \text{ and } A = \frac{(d_1 v_2 - d_2 v_1)}{(v_2 - v_1)}$$