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Abstract

The science of accident reconstruction has undergone continual improvements since its inception. With the advent of computer-based reconstruction and spreadsheet analysis, among other techniques, it is now possible for the reconstructionist to examine vehicle behavior during accident situations in great detail. In this work, we examine some fundamental issues that are often misunderstood and/or misapplied by reconstructionists and discuss the implications of the issues on reconstruction accuracy and validity.

Introduction

The reconstruction of a vehicular accident consists of observing and documenting physical evidence, deriving equations of motion, implementing conservation principles of energy and momentum, and making calculations in order to determine vehicle (and sometimes driver) behavior before, during and after the accident sequence. From the point of view of physics, this involves calculating the motion of (assumed) rigid or nearly-rigid bodies.

The Newtonian equations of motion for any rigid body free to move in 3-space can be represented in state vector form by the set of equations [1-3]:

$$\dot{\underline{x}} = f\left(\underline{x}, \underline{u}, t\right) \tag{1}$$

In which: $\underline{x}$ = nx1 state vector (n=12 if there are no constraints)
$\underline{u}$=mx1 control vector
t=time (a scalar)
f=the functional relationships among the variables

with m≤n and n = system order. For rigid body motion the state vector $\underline{x}$ will contain position and velocity coordinates for each translational and rotational degree of freedom of the body(s) involved in the calculation. The mx1 control or input vector contains the time history of the totality of all of the inputs to the bodies involved. These can be divided into:

- Intentional inputs (driver commands such as steering, braking and acceleration which result on forces developed at the vehicle tire/road interface), and
- Unintentional inputs (collision forces applied to the external surfaces of the vehicles, aerodynamic forces, *etc.*). In control theory unintentional inputs are often termed *noise* and are oftentimes formally separated out.

In nearly all mathematical research into systems and control behavior (of which vehicle dynamics is a subset, and accident reconstruction a subset of vehicle dynamics), the problem posed is almost universally structured in the form of *forward control*. Formally:

> Given a set of specified, known set of control inputs $\underline{u}$(t), a known set of initial conditions or *initial state* $\underline{x}$(t=0) and the functional relationships *f* among the variables, compute the trajectory of the state vector x(t) for t>t_o, *i.e.,*

$$\left[\underline{x}(t), t \in (0, t_{final}\right] \tag{2}$$

One example of the forward control problem is a racing simulation. The driver's control inputs $\underline{u}$(t) are assumed, the characteristics of the vehicle are known (inertias, engine power, gear ratios and gear selection, etc.), as well as the desired path and constraints (the race track). The task is to compute a trajectory for a given set of controls so as to minimize lap time while simultaneously satisfying the constraints (staying on the track). Ingenious methods have been developed using optimal control, Markov chain theory, etc., to solve this *forward control* problem successively for repeated variations in the choice of driver controls.

In accident reconstruction situations, the forward control problem formulation is inappropriate. In all three phases of a collision (pre-impact, impact and post-impact phases) control inputs of both intentional and unintentional types are usually not known *a priori*, nor is the initial state $\underline{x}_{t=0}$ With measurements of the final rest position or state of the vehicles $\underline{x}_t = t_{final}$, the problem is instead the opposite of the forward control paradigm and is termed *inverse control*:

> Given a known *final state* $\underline{x}(t = t_{final}) \equiv \underline{x}_{rest}$ and (possibly from scene measurements) known or partially known trajectory histories, and unknown or only partially known intentional and unintentional control inputs, compute the value of the initial state $\underline{x}$(t=0) and the subsequent trajectory time histories of the states.

It is immediately obvious that while the forward control problem has a unique solution, the inverse control problem may have no readily-derivable solution, multiple solutions or an infinite number of solutions, depending on a number of issues. We outline these issues next.

Issues Important to Reconstruction

The above problem formulation in mathematical terms is instructive but it is also helpful in addition to pose the reconstruction problem verbally. The fundamental questions which arise in almost every accident reconstruction can be described as:

- What were the roadway *positions* of the vehicles when the accident sequence can be considered to have begun?

- What were the translational and rotational *velocities* of the vehicles when the accident sequence can be considered to have begun?
- What were the control forces exerted during the accident sequence (intentional and unintentional)?

In performing a reconstruction, the inverse dynamical mathematics and formulation described above are employed to determine answers to these questions.

In addition to mathematical structure and analysis, there is also physical evidence in the form of roadway signatures, vehicle damage, video and event data recorder data and perhaps witness testimony to consider. Not only must any proposed reconstruction be mathematically viable, it must also match the physical evidence as closely as possible in order to be considered accurate to a reasonable degree of scientific and engineering certainty.

Motor vehicle accidents occur on roadways which are never perfectly level and involve large-motion displacements and rotations, so the detailed structure of Eq (1) consists of a highly nonlinear set of three-dimensional ordinary differential equations. The solution of these equations constitutes what is termed accident reconstruction.

Properties of the equations of motion

Because the equations of motion involve describing Newtonian three-dimensional rigid body motion, their structure will have all the following properties:

- They will be geometrically nonlinear because of the potential for large-angle motion of the vehicles, hence the need for an Euler angle description of position with the attendant trigonometric coordinate transformations.
- They will be mechanically nonlinear because of the presence of hard nonlinearities such as friction and vehicle crush.
- Both types of nonlinearities will be at least piecewise continuous, though perhaps not smoothly continuous, that is, they may possess discontinuities and discontinuous derivatives at one or more locations.
- They will be coupled unless vehicle motion is unidirectional and/or if there is more than one body to be analyzed (for example, wheels on a vehicle), and therefore will have to be solved simultaneously.
- Because of their nonlinearities and complexities, they will have to be solved/integrated numerically, as no general analytical solution method for nonlinear equations exists.

Methodology

The reconstructionist often starts by making an estimate of the initial state vector $\underline{x}(t=0)$ and perhaps some driver control inputs, then performs a computation involving numerical integration of the equations of motion forward in time. Once the predicted vehicle motions are computed, (s)he compares the congruence between the simulation results at the final state when all motion has ceased and available measured scene data and physical evidence.

It is sometimes possible to know some of the initial conditions of state vector from vehicle event data recorders and, recently, dashboard video cameras. Along with this information may be the roadway location of the point(s) of impact. At this point, the reconstruction essentially becomes a boundary-value problem but the initial-value numerical integration process is still used to produce a reconstruction.

It is usually the case that the first estimates for initial conditions, control inputs and roadway location do not produce a solution that replicates scene physical evidence. Then, an iterative procedure must be used to try to produce a better match.

Underlying such an iterative procedure are three fundamental assumptions:

1. <u>The first assumption</u> is that a "solution" *actually exists*, that is, if continual iterative adjustments are made to the assumed control inputs and initial conditions enough times, the simulation will eventually match the measured scene physical evidence and the resulting "solution" will then faithfully represent a reconstruction to a reasonable degree of scientific and engineering certainty.
2. <u>The second assumption</u> is that when good agreement is obtained between computed result and scene evidence that this "solution" is the *only solution*.
3. <u>The third assumption</u> is that during the iterative process, each successive adjustment of control inputs and initial conditions will produce an *improved agreement* between evidence and computation.

Mathematicians refer to Assumptions 1 and 2 as *existence and uniqueness* or E/U criteria. While there are methods for ensuring E/U for linear systems, there is no similar general theory for nonlinear systems. Thus, speaking strictly from a mathematical point of view, there is no guarantee that there is a solution or that it is the only solution when dealing with nonlinear equations.

The existence question of Assumption 1 can be dismissed out of hand from the viewpoint of physical evidence. The accident happened, the vehicles moved through their trajectory histories and eventually came to rest. The event took place. Clearly, there must exist a mathematical solution, *i.e.*, a reconstruction, that represents what happened to a reasonable degree of scientific and engineering certainty.

It is well to recall what the solution of a differential equation set is: a function (or in the case of numerical integration, a trajectory history or set of functions) that satisfies all of the equations' state variable equalities at each and every instant in the independent variable time, including at the initial condition vector. For numerical results in accident reconstruction, this can loosely be stated verbally as "matches the scene evidence."

It is not so obvious that we can be certain of the answer to Assumption 2. The dynamics of a collision (or for that matter, for a vehicle being driven near its performance limit but not colliding with anything) are highly nonlinear. During numerical integration the reconstructionist will only know values of the state variables at discrete instants of time. Depending on the discretization or step size used, a lot could be happening in between integration steps!

A little numerical experimentation quickly shows that Assumption 3 is not valid. Linearity and superposition are synonyms; nonlinear systems need not exhibit any superposition properties at all. Experience shows that improving an initial guess about the dynamics of a collision can result in a frustratingly difficult series of "improvements" that match some evidence better but degrade agreement with others.

An algorithm can only be called an algorithm if (a) each successive iteration produces an *improvement* in the final result, and (b) if the final result is the *correct* result in the sense that it matches scene evidence. Nonlinear systems need not, and often do not, exhibit either of these properties. Each adjustment of control inputs and initial positions to a nonlinear system may produce no improvement, some improvement or a lot of improvement/degradation when the result is compared to scene evidence. Worse, what adjustments will produce the biggest (or any, for that matter!) improvement is not obvious from the current state of affairs. The analogy is that of rolling a ball down a multi-dimensional hill with differing slopes in every direction. It is not possible to predict *a priori* which direction to roll the ball to get to the bottom fastest (in skiing, this is called the fall line), nor is it possible to determine where any midcourse corrections can or should be made, or even what sort of correction to make.

Complicating Issues

Some complicating issues that are typically involved in a reconstruction are:

- Tire behavior: In the absence of aerodynamic and collision forces all of the external forces applied to vehicles to change their trajectories come from the tires. Tire data are experimentally obtained using a flat track test machine as shown in Figure 1:

Figure 1: Calspan T.I.R.F. tire test machine

Typical data obtained include lateral force vs. slip angle, camber angle and vertical load, along with many other forms of data. As one example, a typical lateral force vs. slip angle curve obtained from this machine is shown in Figure 2. During normal driving maneuvers, slip angles seldom exceed 5º and at slip angles much larger than this, tires usually exhibit saturating/nonlinear behavior. Unfortunately, in many pre- and post-collision trajectories, tires often run at very large slip angles (>>5º) and (sometimes) camber angles and may even be flat due to collision damage. Essentially no data exist for tire behavior under such circumstances. A braked tire or sliding tires is a pure frictional device but if the tire is being maneuvered through non-sliding inputs, realistic tire behavior must be obtained by testing or estimated.

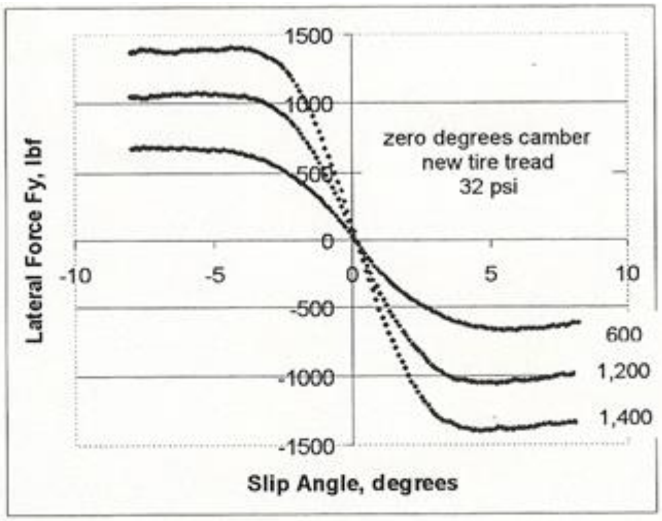

Figure 2: Typical slip angle vs. sideforce curve

- Roadway signatures: Roadway signatures consist of tire marks and gouge marks on or in the pavement. The difficulty of measuring the starting point of tire marks (both skid and yaw) and their significant effect on results has been well-documented [4]. Shadow skids, skip skids, yaw marks and other braking and cornering signatures due to road asperities, uneven driver control inputs and local tire/road µ-values have caused calculational difficulties since the beginning of scientific accident reconstruction [5].
- Tire-Road µ-values: Because tire friction plays such an important role in many reconstructions, accurate information about it has a strong effect on the validity of any reconstruction. Despite the fact that the information contained in [6] was obtained using now-obsolete bias-ply tires and has wide ranges of estimates for various combinations of tire and road, many reconstructionists continue to employ the data there. The use of VBox data acquisition systems and experimental braking and cornering performance on the accident roadway allows improved tire µ-values and is a much-preferred methodology.
- Vehicle stiffness values: In reconstructions involving use of crush/energy methods *e.g.,* CRASH/EDCRASH), vehicle A & B stiffness values are employed to estimate energy dissipation. The methods use crush measurements done according to an established protocol [7], combined with crash test data that provide vehicle stiffness estimates [8]. Stiffness coefficients are developed from controlled crash tests. Most published values do not differentiate stiffness variations due to damage height, crush nonlinearity, front/rear/side impacts or other localized stiffness variations. Because of these issues, the use of dense point clouds for crush measurement, for example, is essentially only cosmetic due to the crudeness of the stiffness values.

- Numerical solution error: The nonlinear nature of vehicle accident reconstruction often involves use of a high-end reconstruction software suite such as HVE [8]. This software allows the reconstructionist to input scene and damage measurements, vehicle data, driver control inputs and other information, formulates the nonlinear differential equations of motion then solves them. The final step is a visualization of the solution in the form of a movie. Because the analog Newtonian motion represented by the differential equations is discretized during the numerical integration process, there are inevitable errors due to truncation and roundoff. As integration step size is decreased, roundoff error increases but truncation errors decreases, leading to the conclusion that there is an optimum integration step size. In practice, the reconstructionist seldom tries to determine this because of other, larger errors in the reconstruction process.

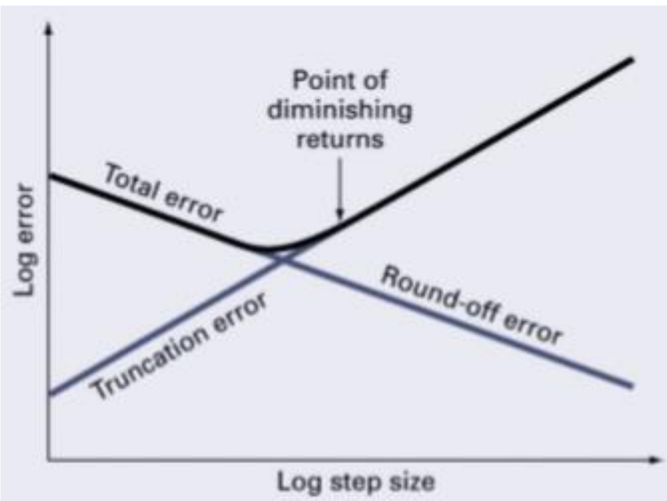

Figure 3: Roundoff and truncation error representation

- Occupant dynamics: From a modeling point of view, the human body is a three-dimensional kinematic chain [9]. The interconnections among the various links vary in approximate type (hinge joint, revolute joint, *etc*.), and each connection contains elastic and damping properties. As with all nonlinear three-dimensional motion, initial conditions are critical, but in the case of occupant dynamics, initial conditions include not only position and velocity information, *i.e.*, the initial state vector, but also muscle tension, joint properties and strength, as well as other factors impossible to know or adequately model. Consequently, while it is possible to model gross motion in situations of pedestrian/vehicle impacts for example, predicting pre-collision position within a vehicle is not scientifically valid. Occasionally pre-collision position may be known *a priori* from testimony or vehicle interior and/or exterior conditions however.
- Vehicle aerodynamics: In most accident reconstructions vehicle aerodynamic properties and their influence on the dynamics of an accident can be ignored. As a rule of thumb, vehicle drag and tire rolling resistance are approximately equal at speeds in the range of $\{u(t) \in (100 - 120\ kph)\}$, the approximate range of current interstate highway speed limits. For some large vehicles with significant frontal and side area, however, aerodynamics can play a part. Unfortunately, aerodynamic lift and drag data are typically only available for zero yaw and free stream air conditions, making the inclusion of aerodynamic effects in a reconstruction impossible for all practical purposes.
- Driver control inputs: Driver steer and brake inputs in both the pre- and post-collision phases of an accident are difficult to model without recourse to on-board data such as that obtainable from ECM units. Additionally, human physiological limits, although studied in many scenarios, are highly individual. Driver models have been successfully proposed to model certain pre-impact and path following maneuvers [10,11] and these have proven useful in some situations. The problem is exacerbated when control inputs are not severe enough to leave roadway signatures.
- Perception/reaction estimates: In many accidents, p/r times for the drivers involved are critical to a reconstruction of the accident. Perception/reaction is a well-studied science [12] and in many controlled circumstances, relatively predictable. Reconstructionists often employ “typical” times to include p/r analysis in a reconstruction, but fail to note that, while there is a lower limit to how fast a driver can respond to a stimulus, it is also possible that the driver did not respond at all.
- Time: Often the reconstructionist becomes involved in a case months or even years after the accident occurrence. In such situations, (s)he must rely on scene photographs and police measurements and investigations. Independent photographs and measurements may or may not be possible. At times the road itself is either no longer available or is in a different condition from its characteristics at the time of the accident.

Summary/Conclusions

As reconstruction science advances, it is more and more important that a reconstructionist understand the physics, assumptions and limitations of the various methodologies employed. In this work, we highlight some of the factors that come into play. While software advances, data acquisition systems, measurement methods and scanning technology get better and better and improve at an ever-increasing rate, the above complicating issues, rather than becoming less important, take on a more forward and important role in analyzing any reconstruction effort.

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