TRAINING MATERIALS
FOR
ACCIDENT RECONSTRUCTION
VOLUME II - STUDENT NOTEBOOK
VOLUME III - INSTRUCTOR'S MANUAL

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Advanced Accident Reconstruction for NASS

Student Exercises

Solutions

name

Compiled by
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Highway Safety Research Institute
The University of Michigan

January, 1979

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Daily Calendar

MONDAY	8:30	1.	Introduction	-Campbell & McDole
	9:00	2.	Review Exercises	-McDole
	9:45		Break	
	10:00	3.	Physics & Dynamics	-Hess
	11:45		Lunch	
	12:45		Lesson 3, cont.	
	2:45		Break	
	3:00	9.	Data Presentation	-Cooley
	4:45		End of Day	
TUESDAY	8:30	5.	Vehicle Force-Deflection	-Campbell
	9:30		Break	
	9:45	4.	Vehicle Dynamics	-Winkler
	11:45		Lunch	
	12:45	10.	Classroom Reconstruction	-Hess
	2:45		Break	
	3:00	8.	Vehicle Examination	-Cooley
	4:45		Dinner	
	7:00	8L.	Vehicle Examination Lab	-Cooley @ HSRI
WEDNESDAY	8:30		Classical Reconstruction, cont.	
	10:00		Break	
	10:15		Classical Reconstruction, cont.	
	11:45		Lunch	
	12:45	6.	Skid Marks	-Cooley
	2:45		Break	
	3:00	7.	Scene Examination	-Cooley
	4:45		Dinner	
	7:00	7L.	Scene Examination Laboratory	-Cooley @ HSRI
THURSDAY	8:30	11.	Applications of Classical Accid. Recon.	-Hess
	10:00		Break	
	10:15		Applications, cont.	
	11:45		Lunch	
	12:45		Applications, cont.	
	2:45		Break	
	3:00	11E	Exercises	-Hess, Staff
	4:00		Dinner	
	7:00		CRASH Lab.	-Hess @ HSRI

FRIDAY	8:30		CRASH Lab, cont.	-@ HSRI
	10:00	12.	CRASH Lab Discussion	-@ Chrysler Center
	11:45		Lunch	
	12:45	13.	Collision Severity Measures	
	1:45		Break	
	2:00	14.	Final Examination	
	3:30	15.	Summary & Closure	
	4:00		Dismissal	

Lesson Outlines

and

Supplementary Lecture Notes

Faculty

Kenneth L. Campbell, Ph.D. - Project Director

Thomas L. McDole, Ph.D. - Course Coordinator

Robert L. Hess, Ph.D. - Director, HSRI and Professor,
Engineering Mechanics, U of M

Peter Cooley, M.S. - Coordinator, Accident Investigation at HSRI

Christopher Winkler, MSME - Vehicle Dynamics Research

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 1, Introduction

Presented by: Kenneth L. Campbell
Thomas L. McDole
Highway Safety Research Institute

Description:

The introduction to the course. Presented will be the overall course organization, course objectives, lesson schedule, and calendar, distribution of lesson material, other course organizational information, and an overview of Accident Reconstruction.

Topics:

Welcome - T. McDole
Course Organization
Course Materials
Announcements
Word from Washington - S. Lee
Course Introduction - K. Campbell
Course Objectives
Overview of Accident Reconstruction

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 2, Review Exercises

Presented by: Thomas L. McDole

Description:

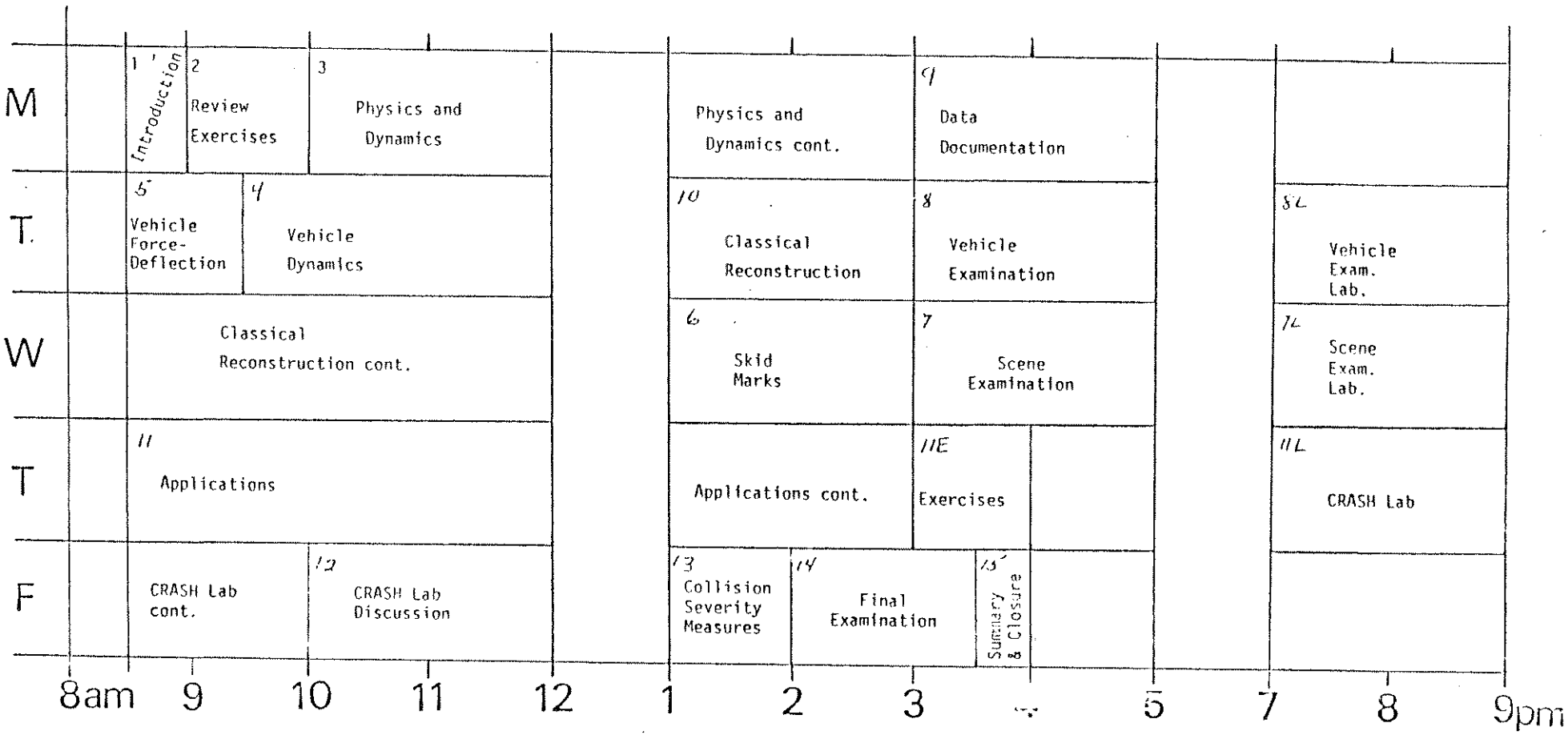
A review of the pre-course student mathematical exercises. The graded exercises will be returned to the student and problem areas will be discussed. The solutions to the problems will be presented and student's questions answered.

Topics:

Discussion of the graded pre-course exercises.

NOTES

Advanced Accident Reconstruction
Lesson Schedule



List of Lessons

Unit I. Course Overview

1. Course Introduction
2. Review Exercises

Unit II. Basic Principles

3. Physics and Dynamics
4. Vehicle Dynamics
5. Vehicle Force-Deflection Characteristics
6. Skid Marks and Analysis

Unit III. Data Collection and Documentation

7. Scene Inspection
7L. Scene Inspection Laboratory
8. Vehicle Inspection
8L. Vehicle Inspection Laboratory
9. Data Documentation

Unit IV. Reconstruction Techniques

10. Classical Reconstruction
11. Applications of Classical Accident Reconstruction
11E. Classical Reconstruction Exercises
11L. CRASH Laboratory
12. CRASH Laboratory Discussion
13. Collision Severity Measures

Unit V. Summation

14. Final Examination
15. Summary & Closure

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Technical Report Documentation Page

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15. Supplementary Notes		

16. Abstract

This Student Notebook is Volume II of the training materials for a course in Accident Reconstruction for the National Accident Sampling System (NASS). Volume I is a Reference Manual, while Volume III is an Instructor's Manual containing lesson outlines. The Student Notebook contains solutions to the pre-course exercises, lesson outlines, and supplementary lecture notes.

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Highway Safety Research Institute

HSRI
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PREFACE

This Student Notebook is Volume II of the training materials for a course in accident reconstruction developed for investigators in the National Accident Sampling System (NASS). Volume I is a Reference Manula and Volume III is an Instructor's Manual. The material in this volume is supplementary information which was distributed to students in a color-coded binder for in-class use. Included are the lesson schedule and copies of various notes which the lecturers provided during the classroom sessions. Also included are a set of pre-course exercises which were reviewed during the first class period.

The course was conducted for senior NASS investigators in May of 1979. A course evaluation was reported separately (Report No. UM-HSRI-79-81).

HSRI
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Technical Report Documentation Page

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9. Performing Organization Name and Address Highway Safety Research Institute The University of Michigan Ann Arbor, Michigan 48109		10. Work Unit No. 11. Contract or Grant No. DOT-HS-8-01968
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Research Institute

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PREFACE

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Advanced Accident Investigation
Student Review and Study Exercises

To the Student:

You should study the following exercises prior to attending the course on Advanced Accident Reconstruction. The problems are representative of the types of computational and geometric construction problems you will encounter in the class.

By completing these exercises, you will have the opportunity to practice your computational skills and thus be prepared to participate in the classroom exercises and discussions.

You will need the following items to complete these exercises. You will also be expected to bring these items to the course.

1. Slide-Rule-Level electronic calculator
2. Engineer's or Architect's scale
3. Straight edge
4. Protractor
5. Compass
6. Dividers
7. Pencil

All computations and solutions are to be completed in this exercise booklet. Show all steps and constructions. If additional space is needed, use the back of the preceding page.

Part I -- Computation Exercises:

A. Using your calculator. Solve these problems, and where appropriate, round off the answer to 3 decimal places.

1. $(3+1)^2 = 4^2 = \underline{\underline{16}}$

2. $18-10+3\times35 = 18-10+(3\times35) = 8+105 = \underline{\underline{113}}$

3. $36\pi \div 12 = \frac{36\times3.142}{12} = \frac{113.097}{12} = \underline{9.425}$

4. $\frac{(3\times12)+(9\times(-19))}{32+\frac{10}{8}\times4} = \frac{36+(-171)}{32+(1.25\times4)} = \frac{-135}{32+5} = -\frac{135}{37} = \underline{\underline{-3.649}}$

5. $\frac{16-\sqrt{32.5}}{108} = \frac{16-5.701}{108} = \frac{10.299}{108} = \underline{\underline{.095}}$

6. $\sqrt{8-\frac{(5)^2}{3}} = \sqrt{8-\frac{25}{3}} = \sqrt{8-8.33} = \sqrt{-.33} = \underline{\underline{\text{Undefined}}}$

7. $\frac{(\tan 30^\circ + 8.25)^2}{2\pi} + 31 = \frac{(.577+8.25)^2}{2\times3.142}+31 = \frac{(8.827)^2}{6.293}+31 =$

$\frac{77.916}{6.293}+31 = 12.381+31 = \underline{\underline{43.381}}$

8. $8.25\times10^6 \div 4.13\times10^5 = \frac{8.25\times10^6}{.431\times10^6} = \frac{8.25}{.413} = 19.976$

9. $\sqrt{(6)^2+(7)^2} = \sqrt{36+49} = \sqrt{85} = \underline{\underline{9.22}}$

10. $\frac{33-45}{45} \times 100 = \frac{-12}{45} \times 100 = -.267 \times 100 = \underline{\underline{26.7}}$

11. Convert 3/8 to its decimal equivalent $\frac{3}{8} = \underline{\underline{.375}}$

12. 3 x 25% = $3 \times .25 = \underline{\underline{.75}}$

B. <u>Fractions.</u> Solve the following and show your calculations.

13. $\frac{11}{16} + \frac{3}{8} = \frac{11}{16} + \frac{6}{16} = \frac{17}{16} = \underline{\underline{1\frac{1}{16}}}$

14. $\frac{4}{8} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = \underline{\underline{1}}$

15. $1\frac{3}{8} - \frac{11}{32} = \frac{11}{8} - \frac{11}{32} = \frac{44}{32} - \frac{11}{32} = \frac{33}{32} = \underline{\underline{1\frac{1}{32}}}$

16. $\frac{7}{16} \div \frac{3}{8} = \frac{7}{16} \times \frac{8}{3} = \frac{7}{2} \times \frac{1}{3} = \frac{7}{6} = \underline{\underline{1\frac{1}{6}}}$

17. $\frac{8}{10} \times \frac{3}{8} = \frac{1}{10} \times \frac{3}{1} = \underline{\underline{\frac{3}{10}}}$

C. <u>Solve the following equations.</u> Substitute the values where given. Use your calculator where appropriate. Show the units for the answer.

18. $C^2 = A^2 + B^2$
Solve for B

$$C^2 = A^2 + B^2$$
$$C^2 - A^2 = B^2$$
$$\underline{\underline{\sqrt{C^2 - A^2} = B}}$$

19. $E = \frac{1}{2}\left(\frac{M_1 M_2}{M_1 + M_2}\right)V^2 = \frac{1}{2}\left(\frac{78 \text{ slugs} * 109 \text{ slugs}}{78 \text{ slugs} + 109 \text{ slugs}}\right)(25 \text{ ft/sec})^2$

Solve for E where
M_1 = 78 slugs
M_2 = 109 slugs
V = 25 ft/sec

$= \frac{1}{2}\left(\frac{8502 \text{ slugs}^2}{187 \text{ slugs}}\right) 625 \text{ ft}^2/\text{sec}^2$

$= \frac{1}{2}(45.465 \text{ slugs}) 625 \text{ f}^2/\text{sec}^2$

$= \underline{\underline{14207.888 \text{ slug ft}^2/\text{sec}^2}}$

20. D=VT

a) solve for V given:

D = 100 miles
T = 90 minutes

$V = \frac{D}{T} = \frac{100 \text{ mi}}{90 \text{ min}} = \underline{\underline{1.111 \text{ mi/minute}}}$

b) Give the value at V in miles per hour

$\frac{1.111 \text{ mi}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hour}} = \underline{\underline{66.660 \text{ mi/hour}}}$

21. $°F = \frac{9}{5}°C + 32$

Solve for °C where
°F = 85 degrees

$\frac{9}{5}°C = °F - 32$

$°C = \frac{5}{9}(°F - 32)$

$= \frac{5}{9}(85 - 32) = \frac{5}{9}(53) = \underline{\underline{29.444°}}$

22. $CF = \frac{w\,V^2}{gr}$

See next page

Solve for CF where

w = vehicle weight in pounds
V = velocity (in feet per second)
g = gravity (32 feet per second/second)
r = radius of circle or turn (in feet)

Given w = 1905.088 kg
V = 32.5 mph
r = 128.016 meters

Conversion factors 1 lb. = 0.454 kg., 60 mph = 88 ft/sec.,
1 foot = 30.48 cm

Solution for #22

a) change of units

$$1905.088 \text{ kg} \times \frac{1 \text{ Lb}}{.454 \text{ kg}} = \underline{\underline{4196.229 \text{ Lb}}} \quad (4200\#)$$

$$128.016 \text{ meters} \times 100 \text{ cm/m} \times \frac{1 \text{ ft}}{30.48 \text{ cm}} = \underline{\underline{420 \text{ ft}}}$$

$$\frac{32.5 \text{ mph}}{x \text{ ft/sec}} = \frac{60 \text{ mph}}{88 \text{ ft/sec}} \qquad x = \underline{\underline{47.67 \text{ ft/sec}}}$$

$$CF = \frac{wV^2}{gr} = \frac{4196.229 \text{ Lb} \times (47.67 \text{ ft/sec})^2}{32 \text{ ft/sec}^2 \times 420 \text{ ft}}$$

$$= \frac{4196.229 \text{ Lb} \times 2272.4289 \text{ ft}^2/\text{sec}^2}{32 \times 420 \text{ ft}^2/\text{sec}^2}$$

$$= \frac{9535632.1 \text{ Lb ft}^2/\text{sec}^2}{13440 \text{ ft}^2/\text{sec}^2}$$

$$= \underline{\underline{709.496 \text{ Lb}}} \quad (710.134\#)$$

D. Solve these equations by factoring, i.e., obtain the roots.

23. $x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x - 2 = 0$ $\quad$ $x + 2 = 0$

$x = 2$ $\quad$ $x = -2$

24. $x^2 + 10x + 16 = 0$

$(x+8)(x+2) = 0$

$x = -8$

$x = -2$

25. $5x^2 = 8$

$x^2 = \frac{8}{5}$ $\quad$ $x^2 = \frac{8}{5}$ $\quad$ $x = \pm\sqrt{\frac{8}{5}}$ $\quad$ $x = \pm\frac{2\sqrt{2}}{\sqrt{5}}$ $\quad$ (1.265)

E. Solve for x

26. $\frac{3}{x} = \frac{11}{13}$

$11x = 3 \times 13$

$11x = 39$

$x = 3.545$

27. $ax + by + c = 0$

$ax = -by - c$

$ax = -(by + c)$

$x = \frac{-(by+c)}{a}$

F. Solve for x and y where:

28. $x + y - 1 = 0$

and $3x + 7y - 35 = 0$

$-3x - 3y + 3 = 0$

$3x + 7y - 35 = 0$

$+4y - 32 = 0$

$4y = 32$

$y = 8$

$x + 8 - 1 = 0$

$x + 7 = 0$

$x = -7$

29. $2X - 3y = 12$
and $5X + 2y + 8 = 0$

$$10X - 15y = 60$$
$$-10X - 4y = 16$$
$$-19y = 76$$
$$y = -4$$

$$2X - 3(-4) = 12$$
$$2X + 12 = 12$$
$$2X = 0$$
$$X = 0$$

G. Miscellaneous problems

30. Solve for $\bar{x}$

$$\bar{x} = \frac{\sum_{n=1}^{n=N} x_n}{N}$$

where:

n	x
1	3.2
2	6.5
3	1.7
4	8.0
5	0.5
6	3.2
7	3.0

$$\bar{x} = \frac{3.2 + 6.5 + 1.7 + 8.0 + 0.5 + 3.2 + 3.0}{7} = \frac{26.1}{7} = 3.7297$$

31. Solve

N! = where N = 6

$$N = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

32. Given that AB = -32.2

give the value of |AB|

$$|AB| = 32.2$$

Part II Geometric Construction, Coordinates and Plots, Trigonometry, and Vectors

A. Measuring

Using a scale and straight edge, construct the following line segments. Place a "|" at the beginning and end of each segment.
Example: 1 in =

1. 4 in.

2. 2 7/16 in.

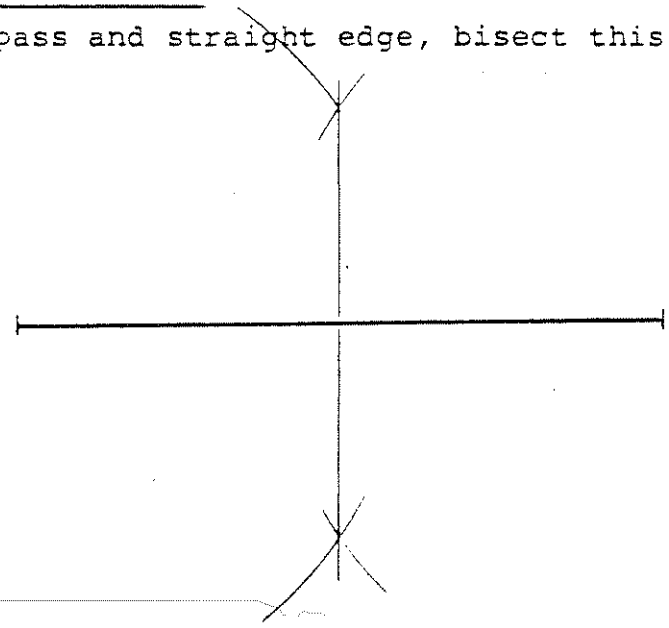
3. 1.125 in.

4. 16'-10" using a scale of 1/4" = 1'-0"

5. 11/32 in.

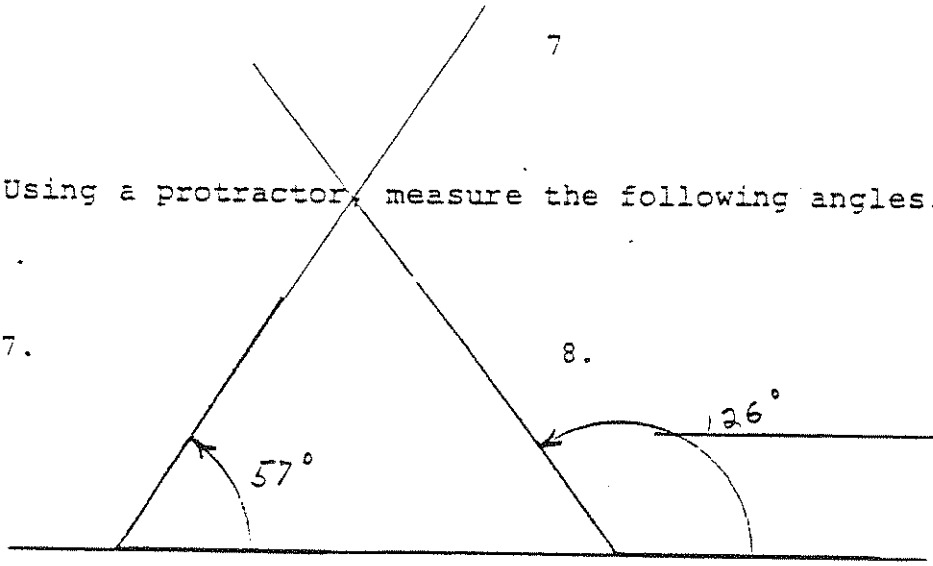
B. Geometric Construction

6. Using a compass and straight edge, bisect this line segment.

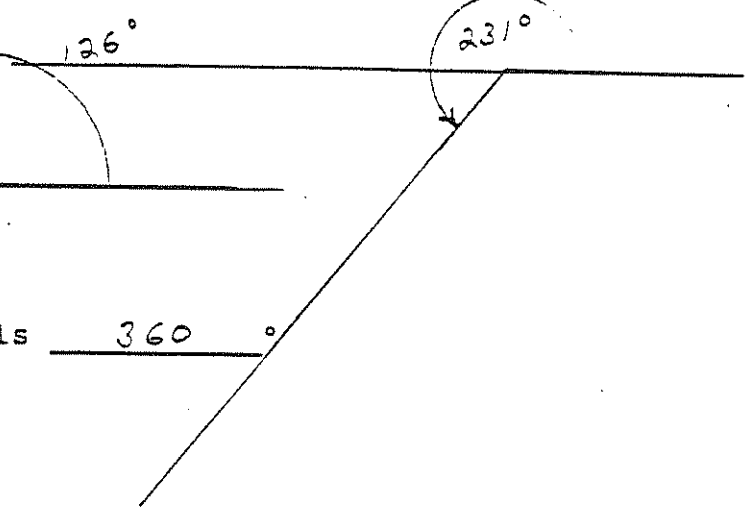


Using a protractor, measure the following angles.

7.

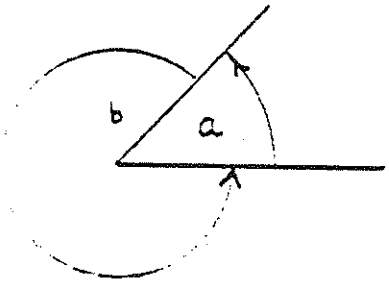


8.

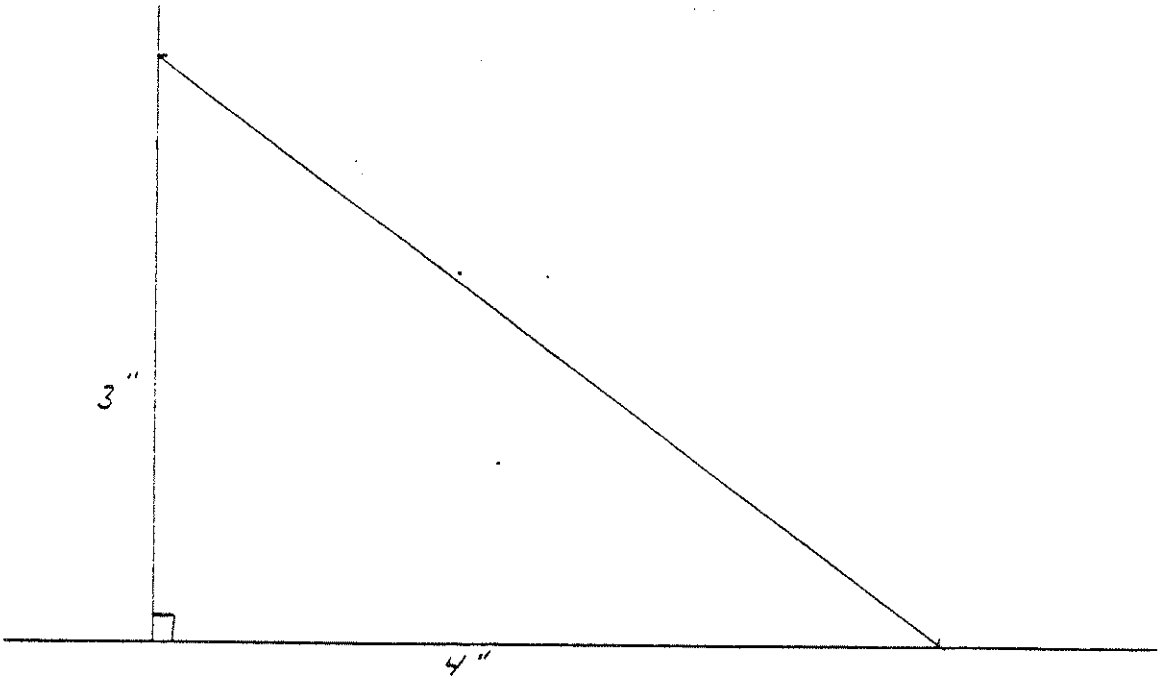


9.

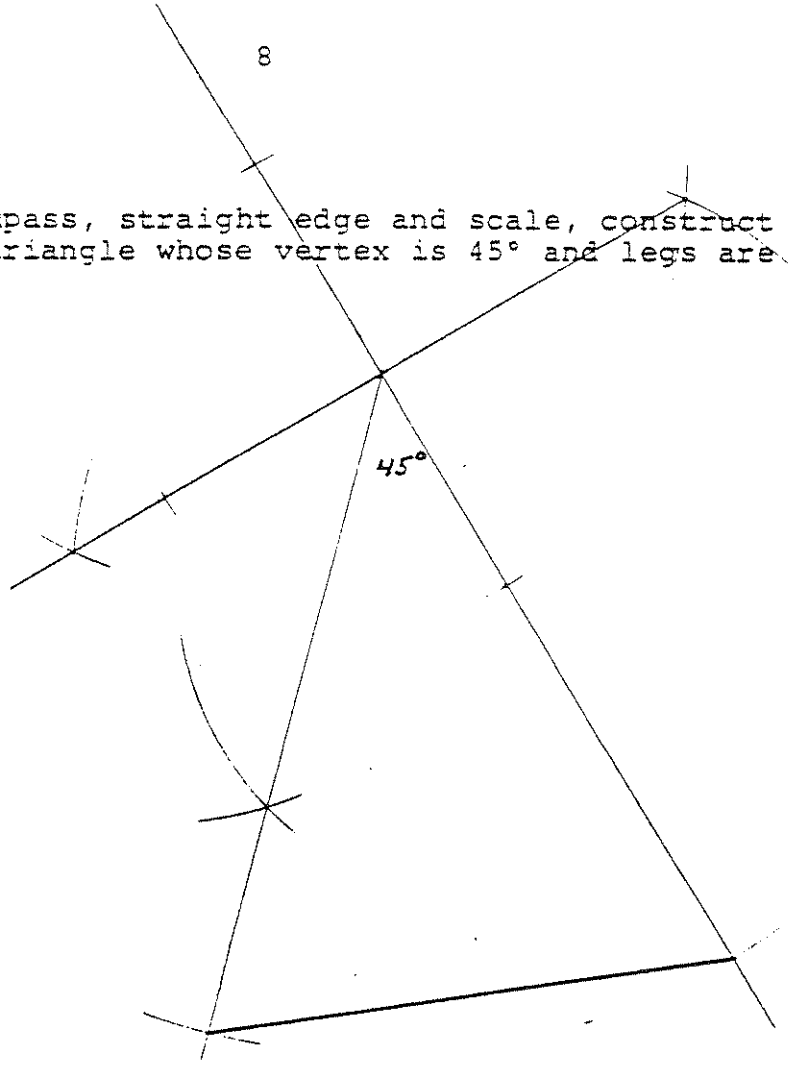
10. The sum of angles a + b equals 360 °



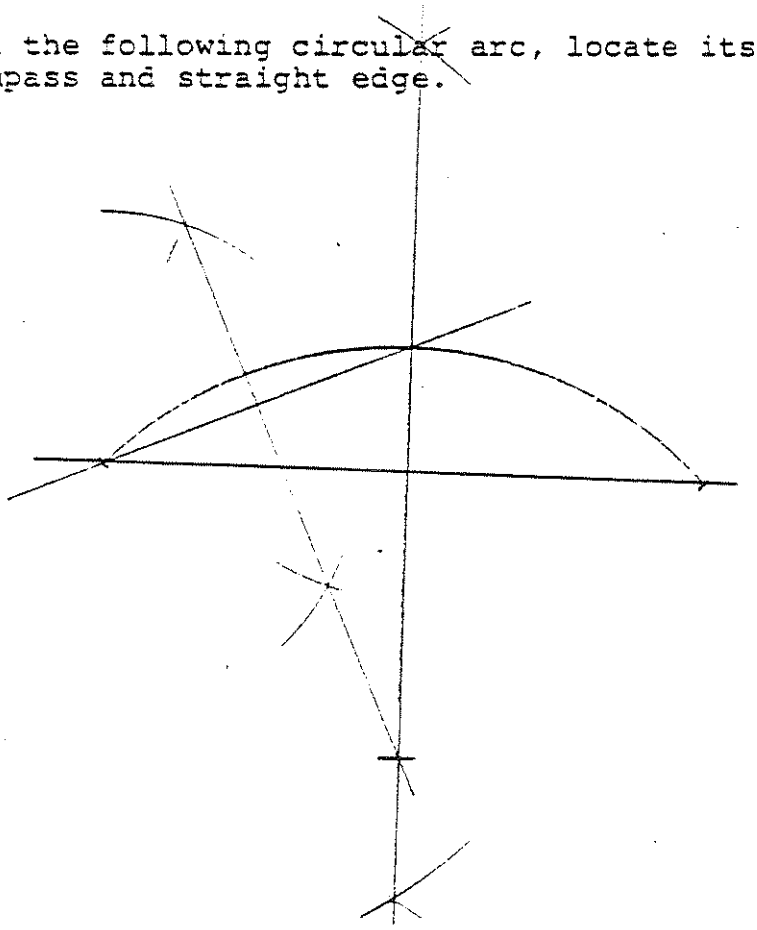
11. Using a protractor, straight edge and scale, construct a right triangle whose legs are 3" and 4" long.



2. Using a compass, straight edge and scale, construct an isosceles triangle whose vertex is 45° and legs are 3.5" long.

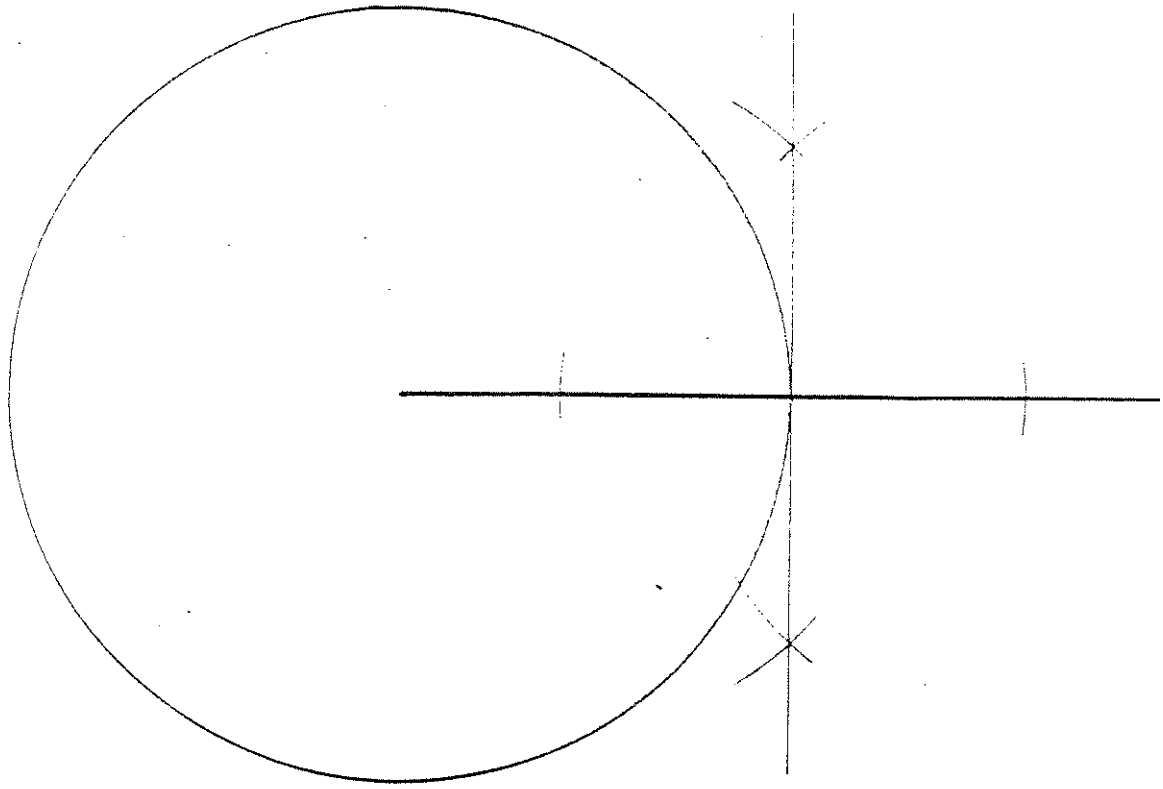


3. Given the following circular arc, locate its center using a compass and straight edge.



14. Using a compass, straight edge, and scale:

a) construct a circle with a radius of 2"



b) at any point on the circle, construct a tangent to the circle, using only a straight edge and compass.

c) using the formula $C = 2\pi r$, calculate the circumference of the circle.

$$C = 2\pi(2\text{ in}) = 4\pi\text{ in} = 4 \times 3.142\text{ in}$$

$$= \underline{\underline{12.566\text{ in}}}$$

d) using the formula $A = \frac{\pi d^2}{4}$, calculate the area of the circle

$$A = \frac{\pi\, 4^2}{4} \quad \text{where } d = 2r = 2 \times 2 = 4$$

$$A = \frac{\pi\, 16\text{ in}^2}{4} = \underline{\underline{12.566\text{ in}^2}}$$

C. Coordinates and Plots

15. On the attached rectangular coordinate paper, construct a set of cartesian coordinates such that:

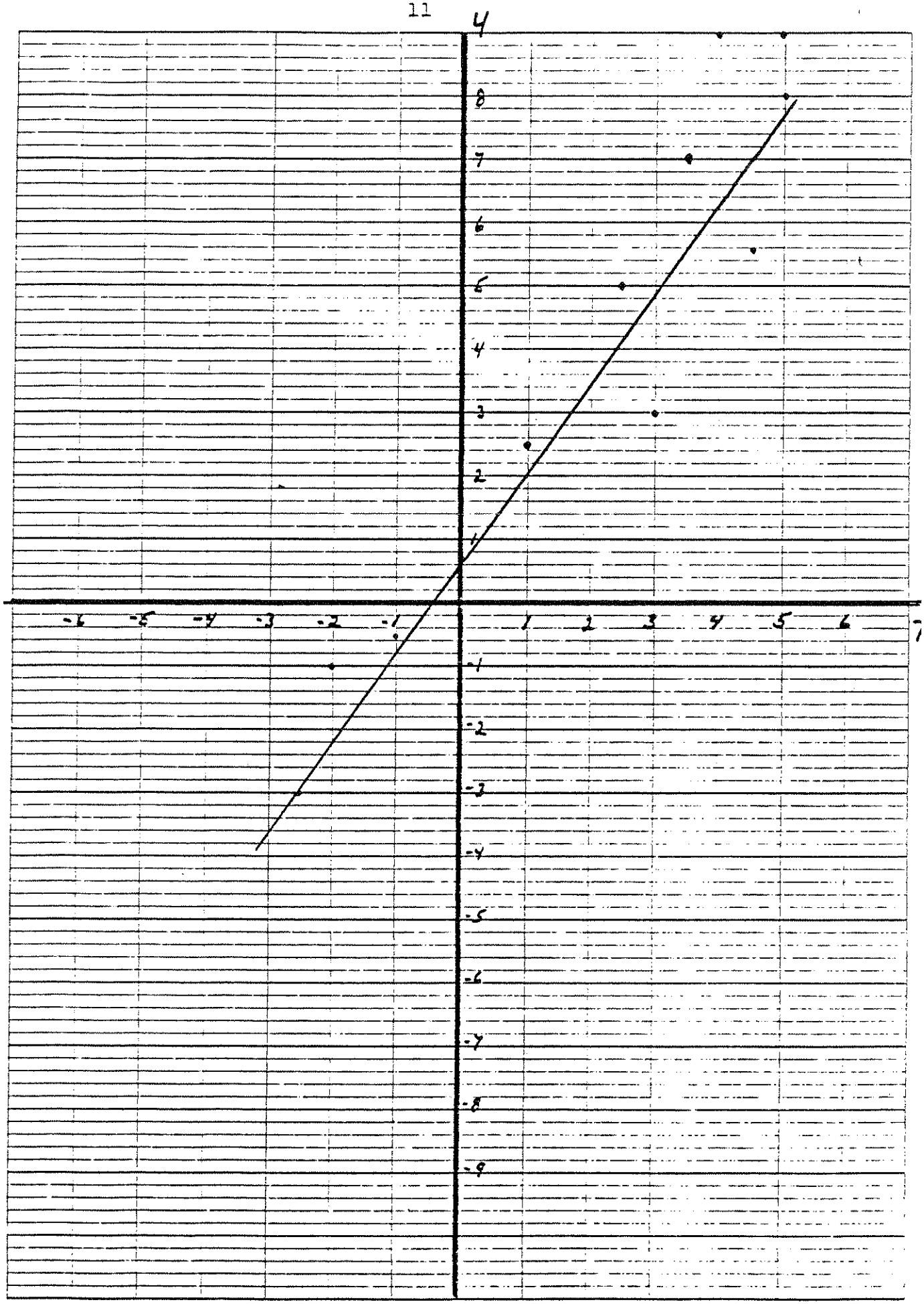
a) the intersection of the axes is in the approximate center of the page. Label the horizontal axis as the x axis and the vertical axis as the y axis.

b) Label the axes with a scale such that each large division of the paper represents 1 unit.

c) Plot the following pairs of points.

x	y
-2.5	-3.0
-2.0	-1.0
-1.0	-0.5
0.0	+0.5
+1.0	2.5
3.0	3.0
2.5	5.0
4.5	5.5
3.5	7.0
5.0	8.0
4.5	9.0
5.0	9.0

d) sketch in the line that represents the best fit through the points. (Don't play dot to dot).



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16. The following equation defines a line. Plot the line on the coordinate paper below <u>and</u> complete the table showing the points you used to plot the line. Label the axes and scales.

$$3x + 6y + 18 = 0$$

$3(0) + 6y = -18$

$6y = -18$

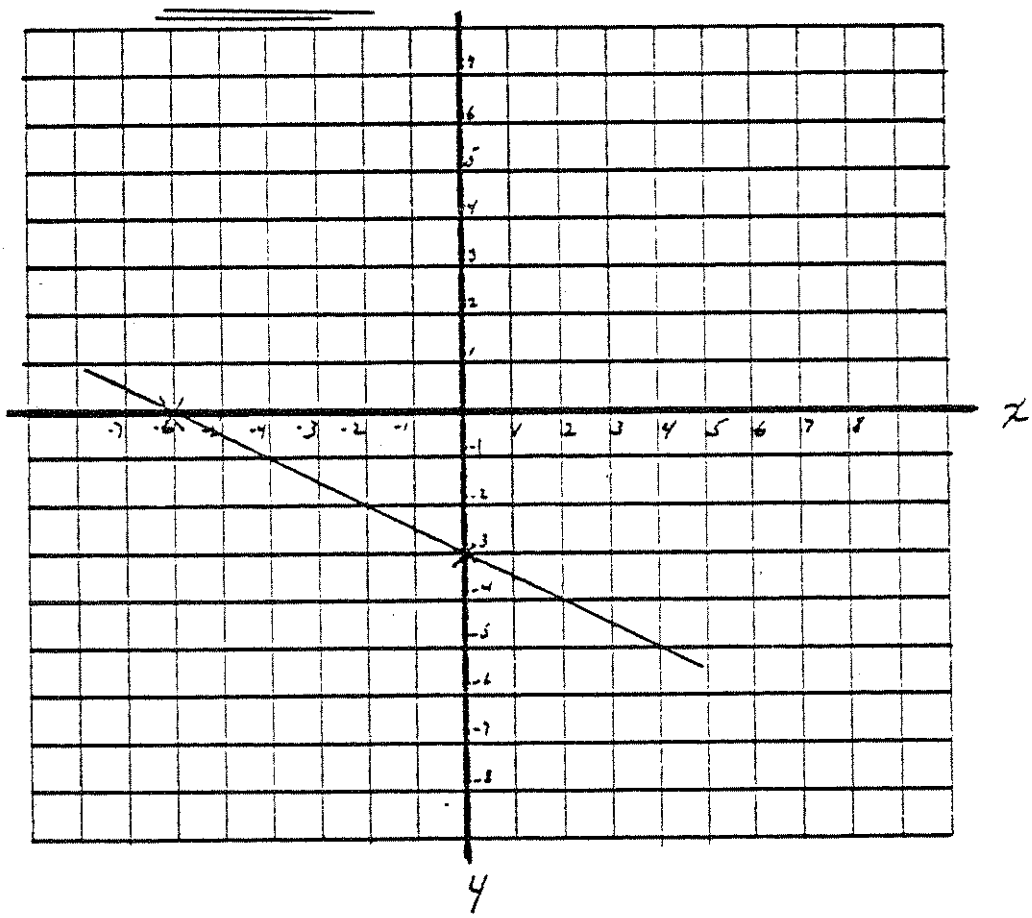
$y = -3$

$3x + 6(0) = -18$

$3x = -18$

$x = -6$

x	y
0	-3
-6	0



17. The following pairs of equations represent lines that lie in the same plane. By solving the simultaneous equations in (a) below, determine the point at which lines one and two intersect. Also determine the slope of each line. Repeat for lines 3 and 4.

(a) line #1 $x + 3y - 12 = 0$

line #2 $3x + y + 12 = 0$

$ax + by + c = 0$

2) Slope $m = -\frac{a}{b}$

line 1. $m_1 = -\frac{1}{3}$

line 2 $m_2 = -\frac{3}{1} = \underline{\underline{-3}}$

1) Intersection

$$3x + 9y - 36 = 0$$

Subt. $$3x + y + 12 = 0$$

$$8y - 48 = 0$$

$$8y = 48$$

$$\underline{\underline{y = 6}}$$

$$x + 3(6) - 12 = 0$$

$$x + 18 - 12 = 0$$

$$x + 6 = 0$$

$$\underline{\underline{x = -6}}$$

intersection $(x, y) = \underline{\underline{(-6, 6)}}$

(b) line #3 $x + 2y - 10 = 0$

line #4 $2x + 4y - 20 = 0$

2) slope $m = -\frac{a}{b}$

line 1 = line 2

$m_1 = m_2 = -\frac{a}{b}$

$= -\frac{2}{4} = \underline{\underline{-\frac{1}{2}}}$

Intersection

$$2x + 4y - 20 = 0$$

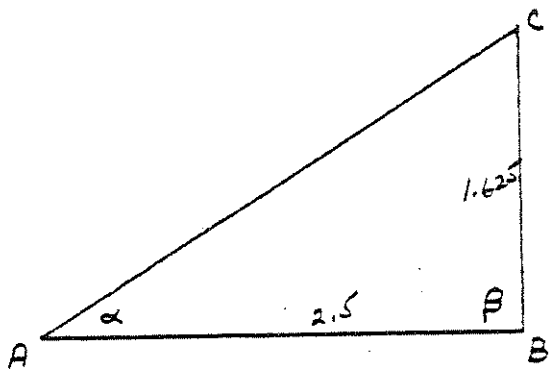
Subt. $$2x + 4y - 20 = 0$$

$$\underline{\underline{0 = 0}}$$

Both are the same line.

Trigonometry

In the right triangle ABC, side AB = 2 1/2", BC = 1 5/8"



$AC = \sqrt{(AB)^2 + (BC)^2}$

$AC = \sqrt{(2.5)^2 + (1.625)^2}$

$AC = \sqrt{6.25 + 2.641}$

$AC = \sqrt{8.891}$

$AC = 2.982''$

a) calculate the length of side AC. Measure side AC to verify your answer.

b) using the formula $\alpha = \tan\left(\frac{BC}{AB}\right)$, calculate the tangent of angle CAB (α)

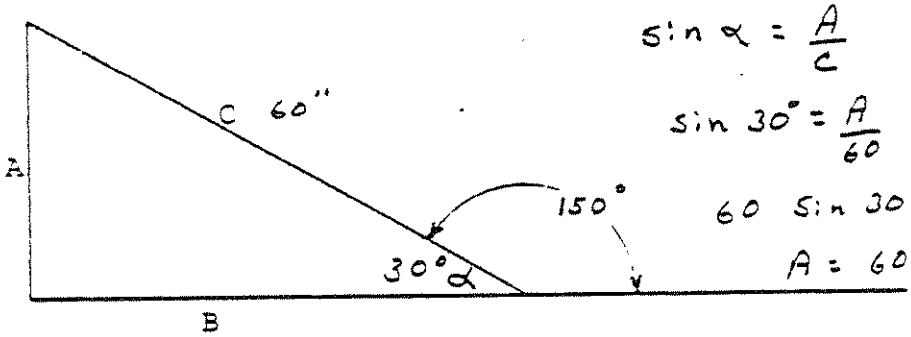
$$\alpha = \tan \frac{BC}{AB} = \tan \frac{1.625}{2.5} = \tan .650 = 33.02^\circ$$

c) calculate the sine of angle ACB $\sin \angle ACB = \frac{AB}{AC} = \frac{2.5}{2.982} = .838$

d) calculate the cosine of angle α

$$\cos \alpha = \frac{AB}{AC} = \frac{2.5}{2.982} = .838$$

In this triangle side C = 60 in. What is the length of side A?



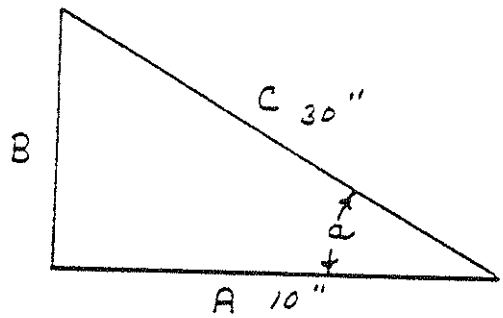
$\sin \alpha = \frac{A}{C}$

$\sin 30^\circ = \frac{A}{60}$

$60 \sin 30^\circ = A$

$A = 60(.5) = 30''$

20. Side A = 10 inches, side C = 2.5 feet. What is the value of angle a in degrees? in radians?



$\cos a = \frac{A}{C}$

$\cos a = \frac{10}{30} = .33$

$a = 70.731°$

1 radian = 57.295°

$\frac{1}{57.295°} = \frac{r}{70.731}$

$r = 1.235$

21. Express these angles in degrees

a) $\frac{5\pi}{3} = \frac{5(3.142)}{3} = \frac{15.710}{3} = 7.855$ $\angle = 7.855r \times 57.295 = 299.995$ $= 300°$

b) $3.5^{(r)*}$: $200.535°$

c) $\frac{7\pi}{6} = 3.665^{r} = 210°$

d) $-4.2^{(r)} = -240.642°$

22. Express these angles in radians

a) 30° $\frac{1r}{57.295°} = \frac{r}{30°}$ $57.295°r = 30°$ $r = .524$

b) 185° 3.229^{r}

c) 420° 7.330^{r}

d) -155° -2.705^{r}

* (r) = Radians

23. CDC (Collision Damage Classification) direction of principal force (PDOF) is usually espressed in terms of a clock direction.

a) Convert these CDC clock values into the units specified.

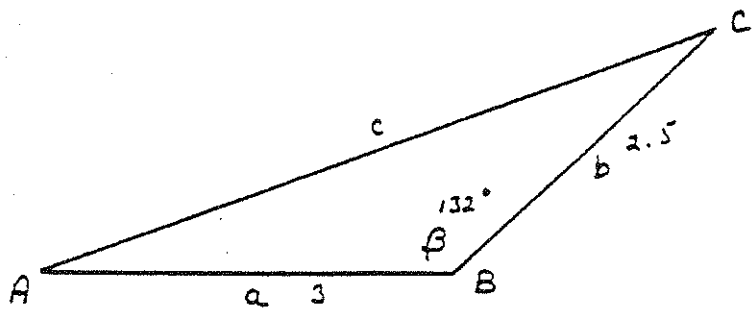
Assume 12 o'clock = 0°

CDC		Angle in	
11 o'clock	$\frac{11}{12} \times 360$	330°	degrees
9 o'clock	270°	4.71	radians
12 o'clock	0°	0	radians
4 o'clock	$\frac{4}{12} \times 360 = 120°$	2094	~~milliradius~~ milliradians

b) Given the angle, convert it to the nearest clock direction

Angle		CDC Clock Number
35°	$\frac{12 \times 35}{360} =$	$\frac{12 \times 35}{360} = 1.167$ 1 o'clock
45°		$= 1.5$ 1 or 2 o'clock
283°		$= 9.43$ 9 o'clock

24. In triangle ABC, side a = 3, side b = 2.5 and angle β = 132°. Find the value of side c.



$c^2 = a^2 + b^2 - 2ab \cos\beta$

$c^2 = (3)^2 + (2.5)^2 - 2(3)(2.5) \cos 132°$

$c^2 = 9 + 6.25 - 15(-.669)$

$c^2 = 15.25 - (-10.035)$

$c^2 = 15.25 + 10.035$

$c^2 = 25.285$

$c = 5.028$

E. Polar Coordinates

25. A hunter leaves his base camp and walks south for 1 mile. He turns east and proceeds for 1.5 miles. At this point he shoots a bear and returns to camp by walking 1 mile north. What color was the bear? white

26. Using the attached polar coordinate paper plot the following vectors:

θ^*	r (magnitude)	scale
30°	100#	1 division = 10#
45°	300#	1 division = 30#
185°	-32	1 division = 2
260°	12	1 division = 1
90°	-100	1 division = 5

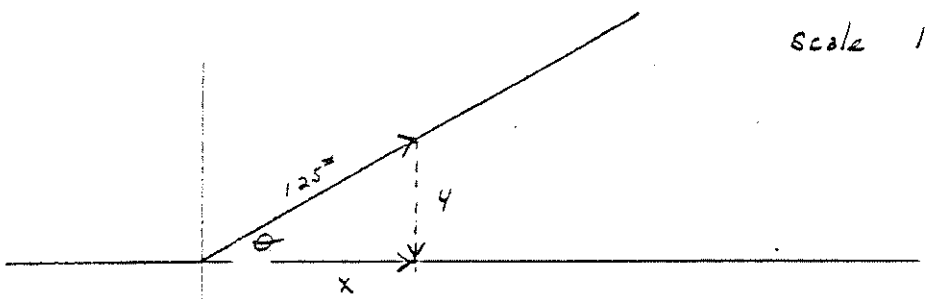
*Measured clockwise from 0°

F. Vectors

NOTE: For each of the following problems, make a sketch or scale drawing before attempting the solution.

27. A force of 125# acts at an angle of 30° CCW to the horizontal. Resolve this force into its x and y components by (1) geometric construction and (2) computation.

Scale 1" = 100#

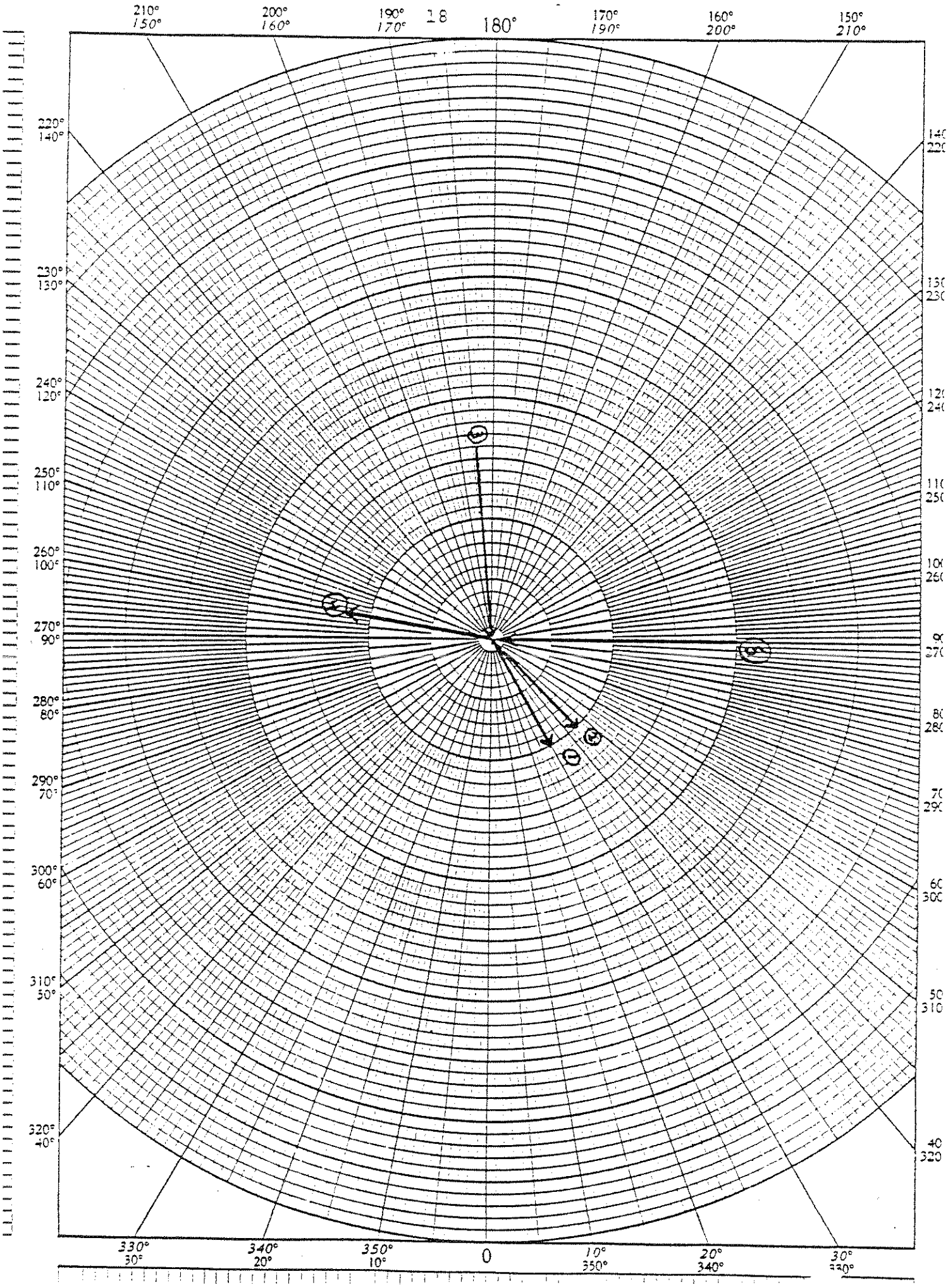


1) $y = \frac{5}{8}'' = \frac{5}{8} \times 100 = \underline{\underline{62.5\#}}$

$x = 1\frac{1}{16} = \frac{17}{16} \times 100 = \underline{\underline{106.25\#}}$

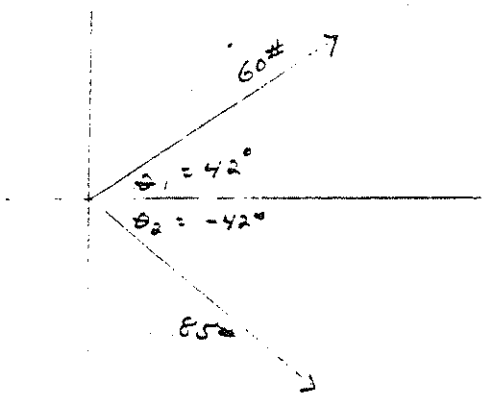
2) $x = r \cos\theta = 125(\cos 30°) = 125(.866) = \underline{\underline{108.25\#}}$

$y = r \sin\theta = 125(\sin 30°) = 125(.5) = \underline{\underline{62.5\#}}$


18
180°
210° 150°
200° 160°
190° 170°
170° 190°
160° 200°
150° 210°
220° 140°
230° 130°
240° 120°
250° 110°
260° 100°
270° 90°
280° 80°
290° 70°
300° 60°
310° 50°
320° 40°
330° 30°
340° 20°
350° 10°
0
10° 350°
20° 340°
30° 330°

Note: All angles are measured counter clockwise (CCW) from the x axis (horizontal) unless otherwise stated.

28. A force of 60# acts at an angle of 42° CCW to the x-axis with a second force of 85# acting at an angle of 318° CCW to the X-axis. What is the resultant force and angle?



$x_1 = r_1 \cos\theta_1 = 60 \cos 42 = 60(.74) = 44.59$
$y_1 = r_1 \sin\theta_1 = 60 \sin 42 = 60(.67) = 40.15$
$x_2 = r_2 \cos\theta_2 = 85 \cos -42 = 85(.74) = 63.17$
$y_2 = r_2 \sin\theta_2 = 85 \sin -42 = 85(-.67) = -56.88$

$x_r = x_1 + x_2 = 44.59 + 63.17 = 107.76$
$y_r = y_1 + y_2 = 40.15 - 56.88 = -16.73$

$r = \sqrt{(x_r)^2 + (y_r)^2} = \sqrt{(107.76)^2 + (-16.73)^2}$

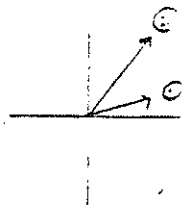
$= \sqrt{11892.11} = \underline{\underline{109.05}}$

$\tan\theta = \frac{y_r}{x_r} = \frac{-16.73}{107.76} = .16$

$\theta = \underline{\underline{-8.9°}} = 351.1°$

29. Add the following vectors:

a) 10# @ ① 30° CCW + 35# @ ② 45° CCW



$x_1 = r_1 \cos\theta_1 = 10 \cos 30 = 8.66$
$x_2 = r_2 \cos\theta_2 = 35 \cos 45 = 24.75$
$x_r = 33.41$

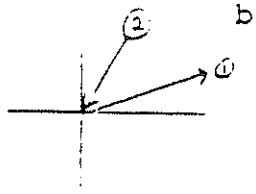
$y_1 = r_1 \sin\theta_1 = 10 \sin 30 = 5.00$
$y_2 = r_2 \sin\theta_2 = 35 \sin 45 = 24.75$
$y_r = 29.75$

$r = \sqrt{(x_r)^2 + (y_r)^2} = \sqrt{(33.41)^2 + (29.75)^2}$

$\underline{\underline{r = 44.74\#}}$

$\tan\theta = \frac{y_r}{x_r} = \frac{29.75}{33.41} = .89$ $\underline{\underline{\theta = 41.68}}$

b) 180# @ ① 15° CCW + (−65#) @ ② 75° CCW



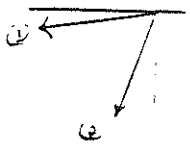
$x_1 = r_1 \cos\theta_1 = 180 \cos 15 = 173.87$
$x_2 = r_2 \cos\theta_2 = -65 \cos 75 = -16.82$
$x_r = 157.05$

$y_1 = r_1 \sin\theta_1 = 180 \sin 15 = 46.59$
$y_2 = r_2 \sin\theta_2 = -65 \sin 75 = -62.79$
$y_r = -16.20$

$r = \sqrt{(157.05)^2 + (-16.20)^2} = \underline{\underline{157.88\#}}$

$\tan\theta = \frac{y_r}{x_r} = -\frac{16.2}{157.05} = -.1$ $\theta = -5.9$ $\underline{\underline{\text{or } 354.}}$

c) 36# @ ① 185° CCW + 21# @ ② 265° CCW



$x_1 = r_1 \cos\theta_1 = 36 \cos 185 = 36(-.996) = -35.86$
$x_2 = r_2 \cos\theta_2 = 21 \cos 265 = 21(-.087) = -1.83$
$x_r = -37.69$

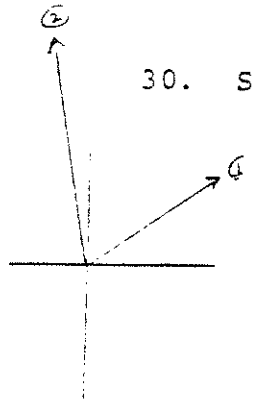
$y_1 = r_1 \sin\theta_1 = 36 \sin 185 = 36(-.087) = -3.13$
$y_2 = r_2 \sin\theta_2 = 21 \sin 265 = 21(-.996) = -20.92$
$y_r = -24.05$

$r = \sqrt{(-37.69)^2 + (-24.05)^2}$

$= \underline{\underline{44.71\#}}$

$\tan\theta = \frac{y_r}{x_r} = \frac{-24.05}{-37.69} = .63$

$\theta = 32.5 + 180 = \underline{\underline{212.5°}}$



30. Subtract the vector 100# @ 50° CCW from 350# @ 95° CCW.

$x_2 = r_2 \cos\theta_2 = 350 \cos 95 = 350(-.087) = -30.45$

$x_1 = r_1 \cos\theta_1 = 100 \cos 50 = 100(.642) = 64.20$

$x_r = 94.65$

$y_2 = r_2 \sin\theta_2 = 350 \sin 95 = 350(.996) = 348.60$

$y_1 = r_1 \sin\theta_1 = 100 \sin 50 = 100(.766) = 76.60$

$y_r = 272$

$r = \sqrt{(x_r)^2 + (y_r)^2} = \sqrt{(94.65)^2 + (272)^2} = \underline{\underline{287.998^{\#}}}$

$\tan\theta = \frac{y_r}{x_r} = \frac{272}{94.65} = 2.874 \quad \theta = 180 - 70.8 = 109.2°$ CCW

31. A force of 1050# @ 38° CCW is multiplied by a factor of 4. What is the resultant force and angle?

$1050^{\#} \times 4 = 4200^{\#}$

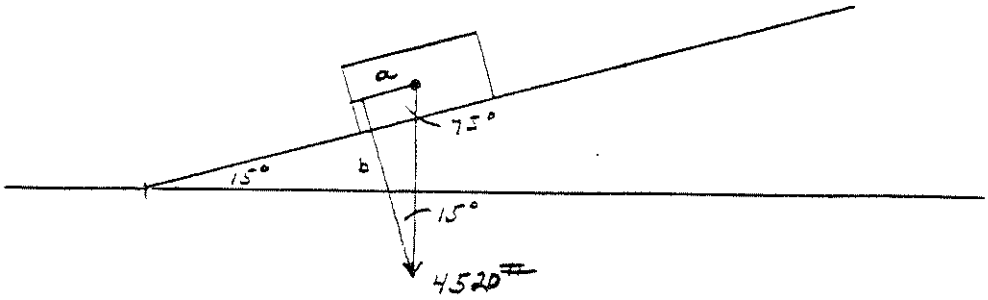
$\underline{\underline{4200^{\#} @ 38° CCW}}$

32. 35# is added to the vector 95# @ 21° CCW. What is the resultant vector?

$95^{\#} + 35^{\#} = 130^{\#}$

$\underline{\underline{130^{\#} @ 21° CCW}}$

33. A 4520# car rests on a 15° incline. What are the components of this weight acting (a) perpendicular to the incline and (b) parallel to the incline?

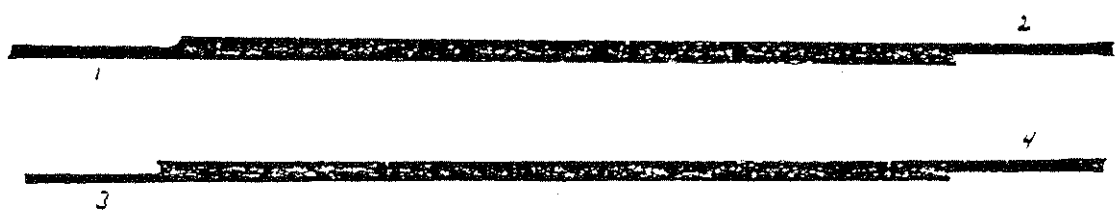


$b = r \cos\theta = 4520 \cos 15 = \underline{\underline{4365.98\#}}$

$a = r \sin\theta = 4520 \sin 15 = \underline{\underline{1169.86\#}}$

Part III - Skid Mark Analysis

A vehicle was traveling from left to right at a moderate rate of speed when its brake system malfunctioned, causing all four brakes to suddenly apply and lock all four wheels. This caused the vehicle to skid to a stop, leaving the skid marks as shown below. The vehicle had a wheel base of 102" and a front overhang of 30". The vehicle weighed 2540 pounds. A special test skid was made, using the repaired vehicle. During the test skid, a mark of 25' was made from a measured vehicle speed of 22.0 mph. Using the formula $D = \frac{S^2}{30F}$ determine the drag factor. Then, by using the Special Tape Measure (attached), measure the skid marks and determine the speed of the vehicle at the time the brakes malfunctioned. In the formula, D = distance in feet to slide to a stop; S = the vehicle speed in mph; and F = coefficient of friction or drag factor.



length 1 - 48'-6"
2 - 48'-0"
3 - 47'-6"
4 - 48'-6"

Avg length $= \frac{\Sigma l}{4} = \frac{192.50}{4} = 48.13'$

Coeff. of Friction

$D = \frac{S^2}{30F}$ $\quad F = \frac{S^2}{30D} = \frac{(22)^2}{30(25)} = \frac{484}{750} = .645$

Speed of vehicle

$D = \frac{S^2}{30F}$ $\quad S = \sqrt{30FD} = \sqrt{30(.645)(48.13)} = \sqrt{931.316} = \underline{\underline{30.5 \text{ mph}}}$

Cut it out before using. The zero point is not at the end (as with ordinary tape measures). The zero measure in multiples of six inches. By combining the divisions to the left of the zero point is possible to measure distances to the nearest three inches. The two readings (left + right) it is possible to measure distances to the nearest three inches.

This is a special tape measure for use with the skid marks. The right of the zero measure in multiples of 5 feet.

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 3, Physics and Dynamics

Presented by: Robert L. Hess, Ph.D.

Description:

An introduction to the basic principles of physics and dynamics as they apply to accident reconstruction.

Topics:

Mathematical Background
- Coordinate systems
- Lines and points in a plane
- Vectors

Plane Kinematics of a Particle
- Vector position and displacement
- Velocity
- Momentum and Force

Plane Kinematics of a Rigid Body
- Angular Velocity
- Rotation
- Centroids
- Acceleration

Kinematics of a Body in Plane Motion
- Center of Mass
- Newton's Axiom
- Rectilinear Motion of a Rigid Body
- Plane Motion of a Rigid Body
- Vehicle Force System

NOTES

SYMBOLS AND ABBREVIATIONS

Table 1. Greek Alphabet

A	α	Alpha	H	η	Eta	N	ν	Nu	T	τ	Tau
B	β	Beta	Θ	ϑ θ	Theta	Ξ	ξ	Xi	Y	υ	Upsilon
Γ	γ	Gamma	I	ι	Iota	O	o	Omicron	Φ	ϕ	Phi
Δ	δ	Delta	K	κ	Kappa	Π	π	Pi	X	χ	Chi
E	ϵ	Epsilon	Λ	λ	Lambda	P	ρ	Rho	Ψ	ψ	Psi
Z	ζ	Zeta	M	μ	Mu	Σ	σ ς	Sigma	Ω	ω	Omega

Table 2. Symbols for Mathematical Operations

References: American Standard for Mathematical Symbols (AESC Report Z10f). Mathematical Association of America (National Committee Report, 1922, Chapter 8). American Standard Mathematical Symbols (ASA Z10f—1928). American Standards Association, New York.

Addition and Subtraction

$a + b$, a plus b
$a - b$, a minus b
$a \pm b$, a plus or minus b
$a \mp b$, a minus or plus b

Multiplication and Division

$a \times b$, or $a \cdot b$, or ab, a times b
$a \div b$, or $\frac{a}{b}$, or a/b, a divided by b

Symbols of Aggregation

( ) parentheses
[ ] brackets
{ } braces
— vinculum

Equalities and Inequalities

$a = b$, a equals b
$a \approx b$, a approximately equals b
$a \neq b$, a is not equal to b
$a > b$, a is greater than b
$a < b$, a is less than b
$a \geqq b$, a equals or is greater than b
$a \leqq b$, a is less than or equals b
$a \equiv b$, a is identical to b
$a \longrightarrow b$, or $a = b$, a approaches b as a limit

Proportion

$a/b = c/d$, or $a : b :: c : d$, a is to b as c is to d
$a \propto b$, a varies directly as b
%, per cent

Powers and Roots

a^2, a squared
a^n, a raised to the nth power
$\sqrt[3]{a}$, cube root of a
$\sqrt[n]{a}$, or $a^{1/n}$, nth root of a
a^{-n}, $1/a^n$
$3.14 \times 10^4 = 31{,}400$
$3.14 \times 10^{-4} = 0.000314$

Miscellaneous

$n! = 1 \cdot 2 \cdot 3 \ldots n$, factorial n
$P(n, r) = n(n-1)(n-2)\ldots(n-r+1)$
$C(n, r) = \frac{P(n, r)}{r!} = \binom{n}{r}$ = binomial coefficients
$|a|$ = absolute value of a
i (or j) $= \sqrt{-1}$, imaginary unit
$\pi = 3.1416$, ratio of the circumference to the diameter of a circle
∞, infinity

Plane Geometry

$\angle$, angle
$\triangle$, triangle
$\parallel$, parallel
$\perp$, perpendicular
$\odot$, circle
$\square$, parallelogram
$\therefore$, therefore
$°$ $'$ $''$, degree, minute, second
$'$ $''$, feet, inches

Logarithms

$\log a = \log_{10} a$, common logarithm of a or log of a to the base 10
$\ln a = \log_e a$, natural logarithm of a or log of a to the base e ($e = 2.718$)
$\log^{-1} a$, number whose log is a

Trigonometry

sin, cos, tan
cosec or csc, sec, cot or ctn
vers, covers
} trigonometric functions
$\sin^{-1}$, $\cos^{-1}$, etc., inverse of the functions

Analytic Geometry

x, y, z; ξ, η, ζ, rectangular coordinates
ρ, s, intrinsic coordinates
ρ, radius of curvature
s, length of arc
r, θ, polar coordinates
ψ, angle from radius vector to tangent
r, θ, ϕ, spherical coordinates
θ, co-latitude
ϕ, longitude
r, θ, z, cylindrical coordinates
ϵ, eccentricity in conics
p, semi latus rectum in conics
$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$, direction cosines

Calculus

$y = f(x)$, y is a function of x
$y' = f'(x) = \frac{dy}{dx} = D_x y$, derivative of $y = f(x)$ with respect to x
$y'' = f''(x) = \frac{d(y')}{dx} = D_x^2 y = \frac{d^2y}{dx^2}$, second derivative of $y = f(x)$ with respect to x
$u = f(x, y)$, u is a function of x and y
$u_x = f_x(x, y) = D_x(u) = \frac{\partial u}{\partial x}$, partial derivative of $u = f(x, y)$ with respect to x
$u_{xy} = f_{xy}(x, y) = D_y(D_x u) = \frac{\partial^2 u}{\partial y \partial x}$, second partial derivative of $u = f(x, y)$ with respect to x and y
Δy, increment of y
dy, differential of y
δy, variation of y
$\sum_{i=a}^{b}$, summation over i from a to b
$\lim_{x \to a} (y) = b$, $y \to b$ as $x \to a$
$\int$, integral of
$\int_a^b$, definite integral of

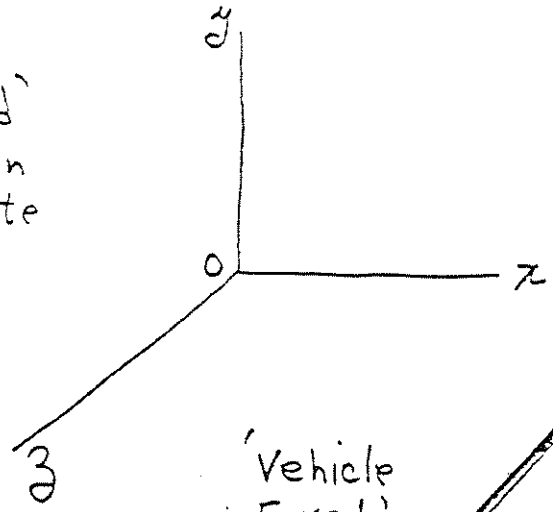
Vector Analysis

i, j, k, unit vectors along the axes (right-handed system)
$a \cdot b = (ab) = Sab$, scalar product of a and b
$a \times b = [ab] = Vab$, vector product of a and b
Vectors are indicated in print by bold-faced type.

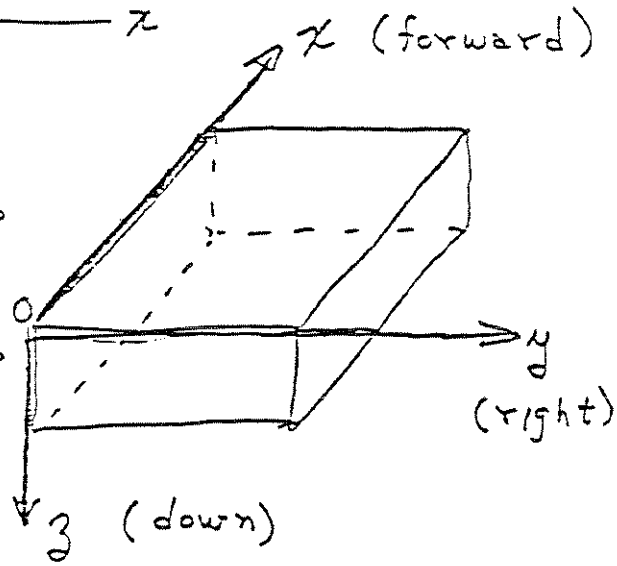
RLH

1

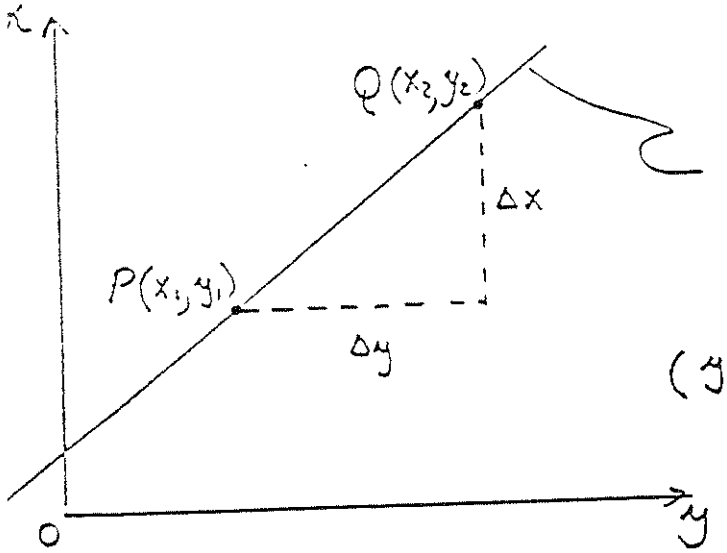
'Standard' Cartesian Coordinate Axes



'Vehicle Fixed' Cartesian Coordinate System



RLH



$$Ax + By + C = 0$$

show that

$$(y_1 - y_2)x + (x_2 - x_1)y + (x_1 y_2 - x_2 y_1) = 0$$

$P(x_1, y_1)$ & $Q(x_2, y_2)$ are each on line $Ax + By + C = 0$.

$$A x_1 + B y_1 + C = 0$$
$$A x_2 + B y_2 + C = 0$$

$$\begin{array}{l} A x_1 x_2 + B y_1 x_2 + C x_2 = 0 \\ \underline{A x_2 x_1 + B y_2 x_1 + C x_1 = 0} \end{array}$$

$$B(y_1 x_2 - y_2 x_1) + C(x_2 - x_1) = 0$$

$$B = \frac{(x_2 - x_1)}{(x_1 y_2 - x_2 y_1)}$$

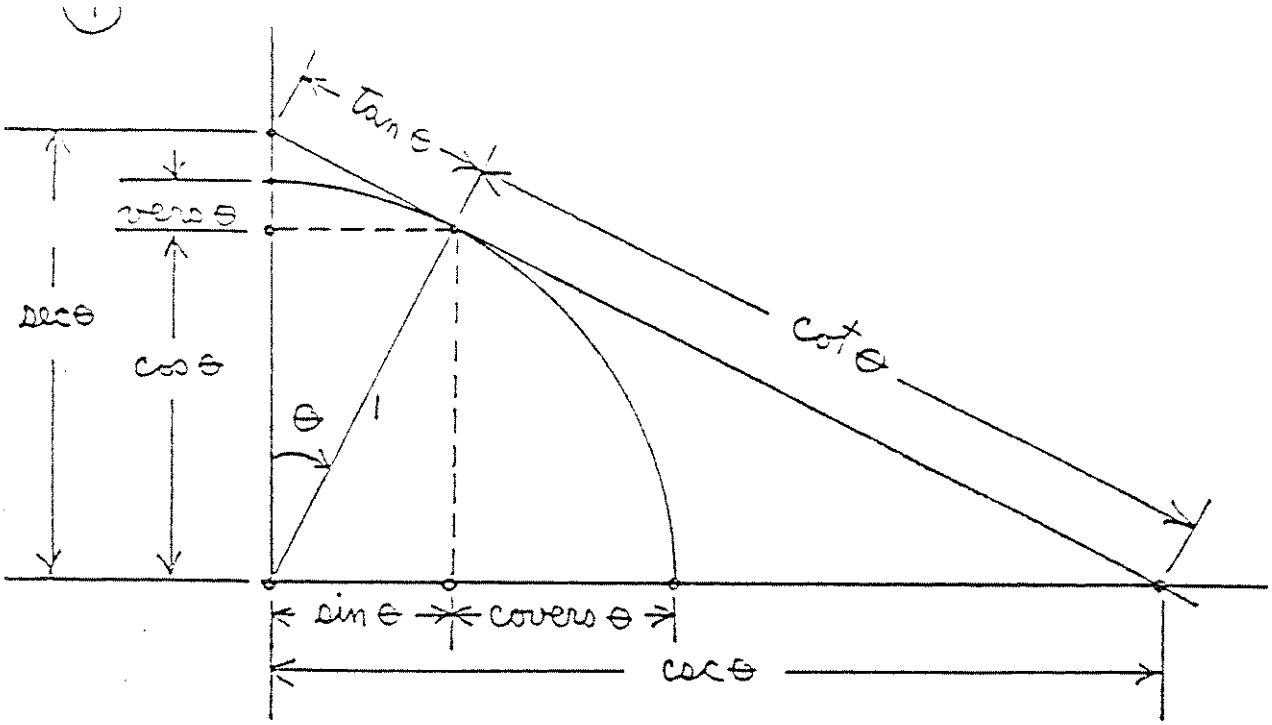
$$\begin{array}{l} A x_1 y_2 + B y_1 y_2 + C y_2 = 0 \\ \underline{A x_2 y_1 + B y_2 y_1 + C y_1 = 0} \end{array}$$

$$A(x_1 y_2 - x_2 y_1) + C(y_2 - y_1) = 0$$

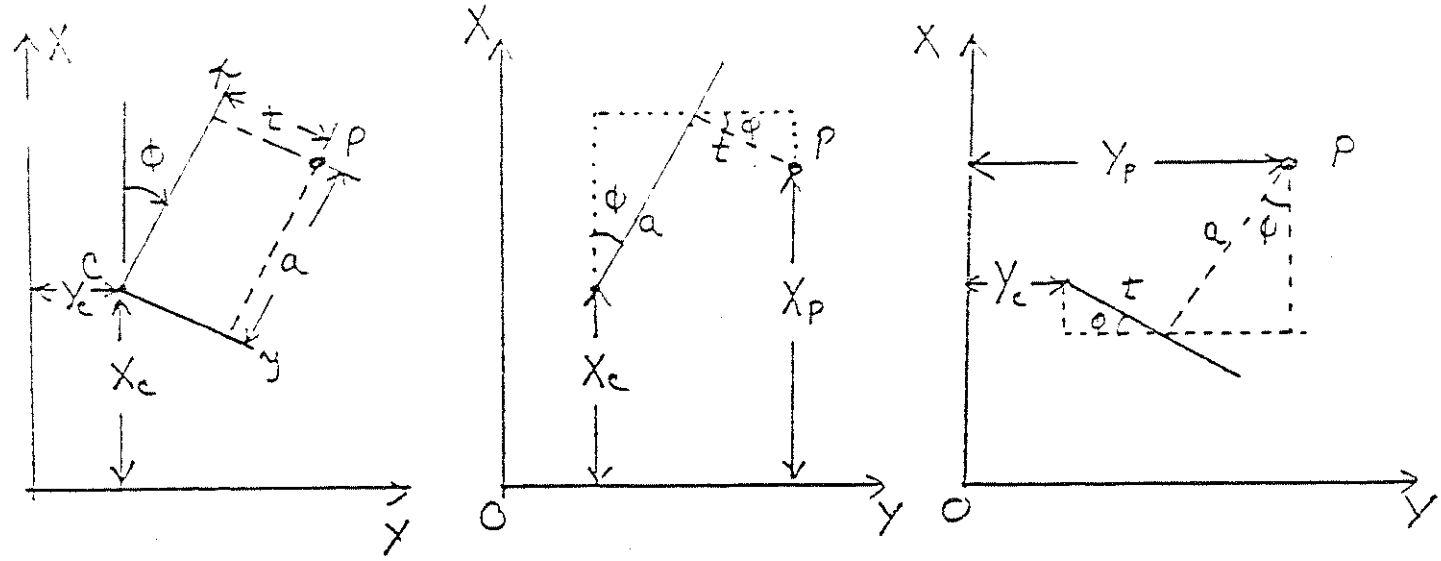
$$A = \frac{(y_1 - y_2)}{(x_1 y_2 - x_2 y_1)}$$

$$(y_1 - y_2)x + (x_2 - x_1)y + (x_1 y_2 - x_2 y_1) = 0$$

KEH



Definitions of Trigonometric Functions of θ

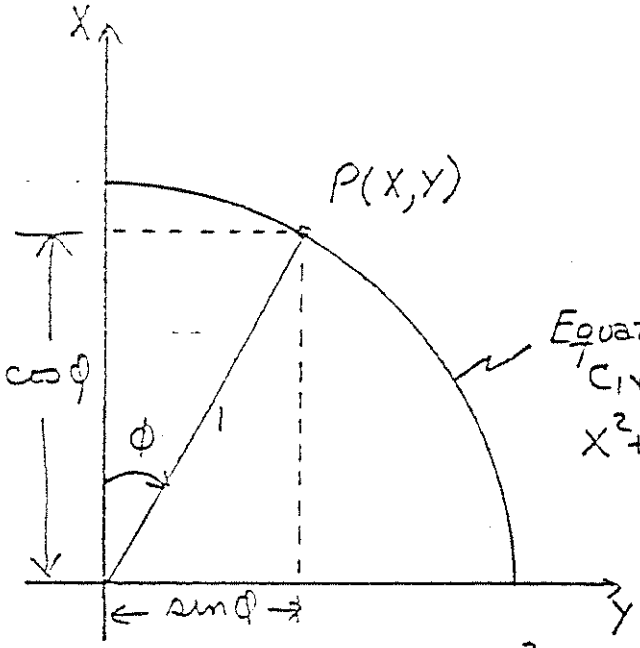


$$X_p = X_c + a * \cos\varphi - t * \sin\varphi$$

$$Y_p = Y_c + t * \cos\varphi + a * \sin\varphi$$

'Transformation' of Coordinates

R.E.H.



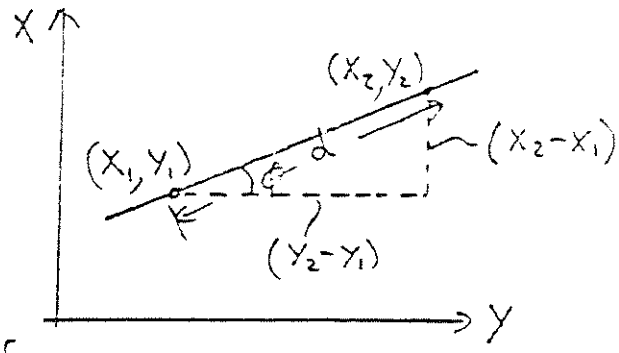
Equation of Circle

$X^2 + Y^2 = 1$

$X = \cos\phi$

$Y = \sin\phi$

$\cos^2\phi + \sin^2\phi = 1$

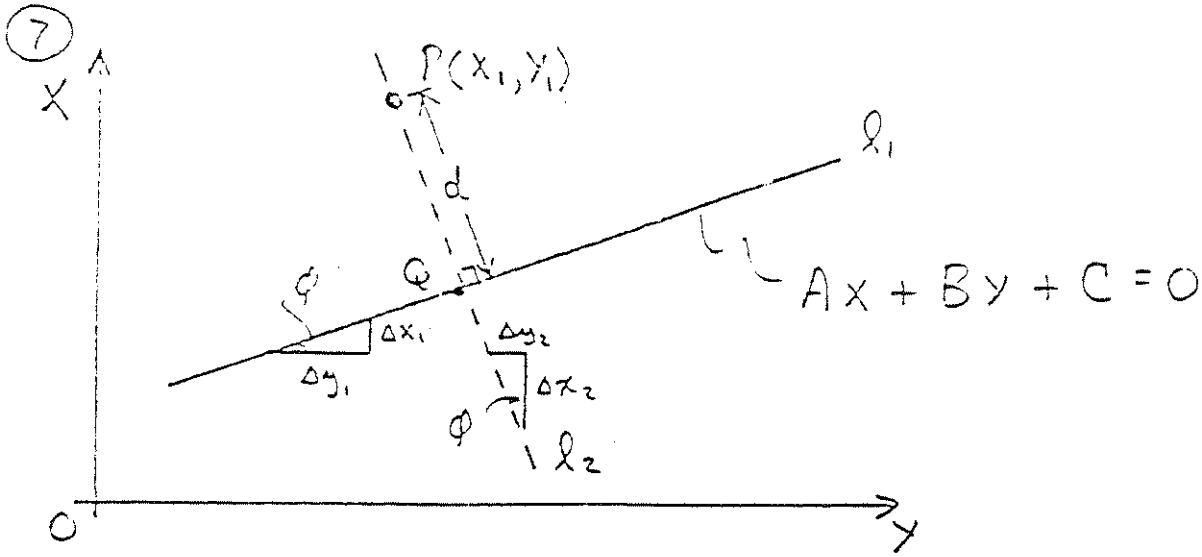


$$\cos^2\phi = \left(\frac{(Y_2-Y_1)}{d}\right)^2$$

$$\sin^2\phi = \left(\frac{(X_2-X_1)}{d}\right)^2$$

$$d^2 = (X_2-X_1)^2 + (Y_2-Y_1)^2$$

The distance between two points



The distance d of the point P from the line l_1.

$$\begin{cases} A(X+\Delta x_1) + B(Y+\Delta y_1) + C = 0 \\ A\,X \qquad + B\,Y \qquad + C = 0 \\ \hline A\,\Delta x_1 + B\,\Delta y_1 = 0 \end{cases}$$

$$\tan\phi = \frac{\Delta x_1}{\Delta y_1} = -\frac{B}{A} = \frac{\Delta y_2}{\Delta x_2}$$

The line l_2 is $\perp$ l_1. The equation of l_2 is

$$\begin{array}{r} -B\,X + A\,Y + C_1 = 0 \\ -B(X-\Delta x_2) + A(Y+\Delta y_2) + C_1 = 0 \\ \hline -B\,\Delta x_2 + A\,\Delta y_2 = 0 \end{array}$$

$P(X_1, Y_1)$ is on l_2

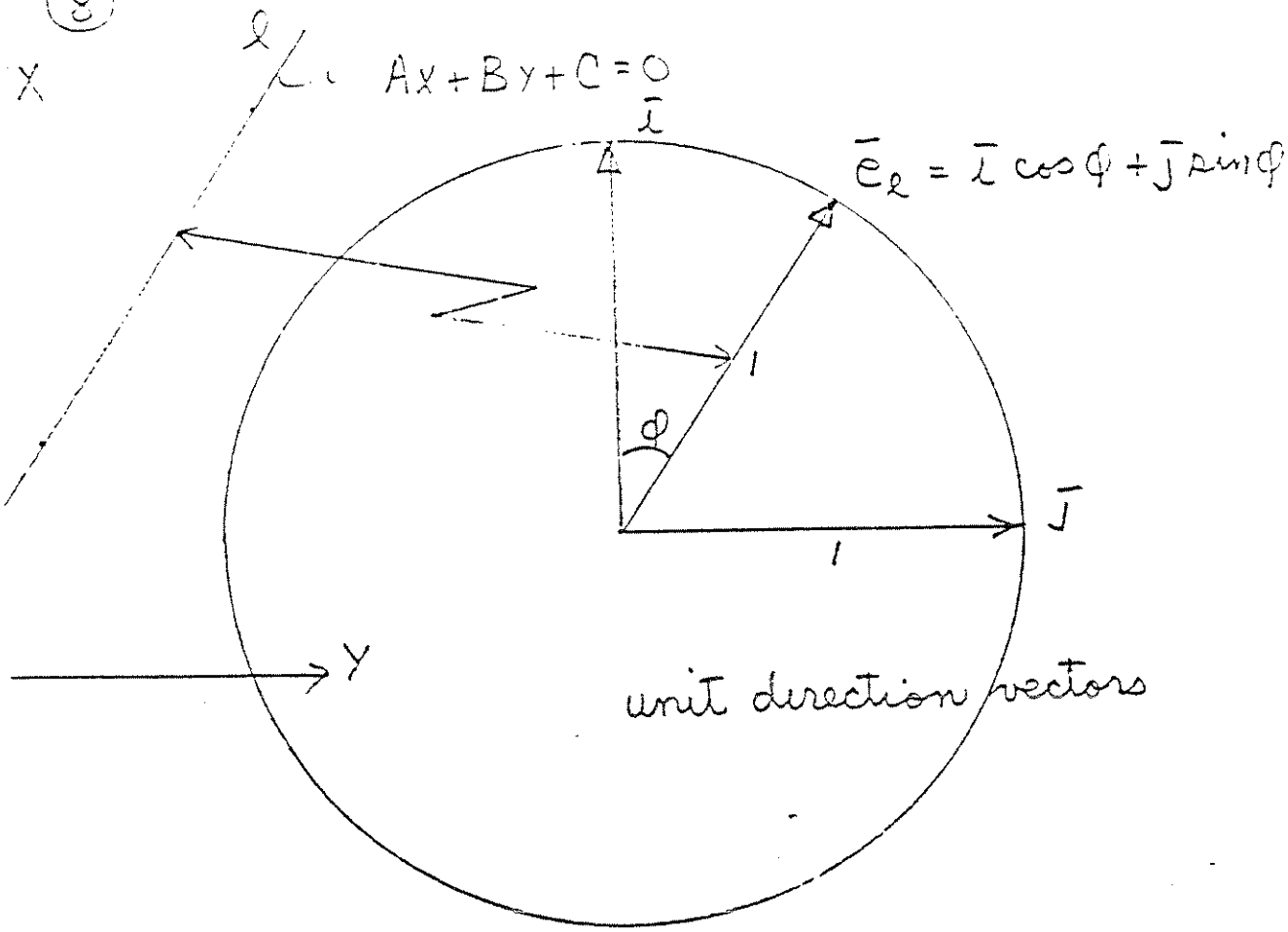
$$-B X_1 + A Y_1 + C_1 = 0 \qquad \text{or} \quad C_1 = (B X_1 - A Y_1)$$

Solve $\begin{array}{l} A\,X_Q + B\,Y_Q + C = 0 \\ -B\,X_Q + A\,Y_Q + (BX_1 - AY_1) = 0 \end{array}$ for X_Q, Y_Q

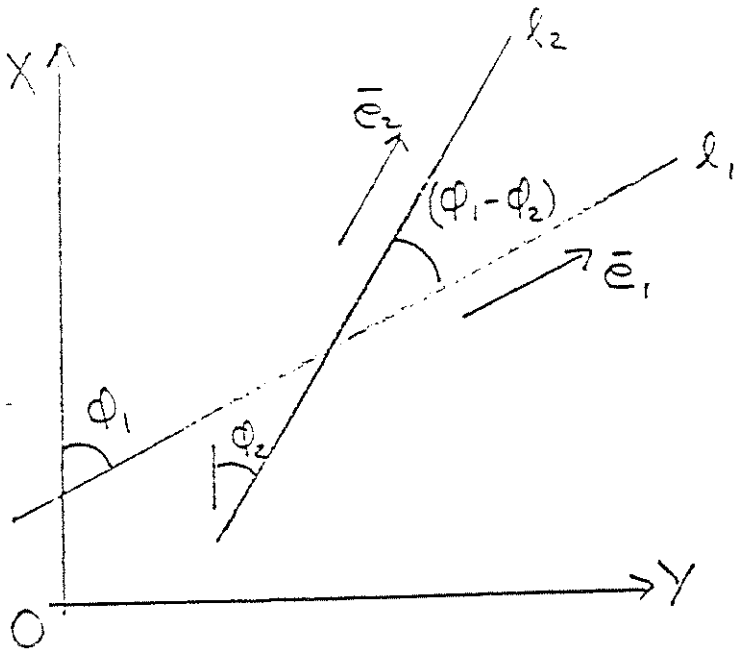
$$X_Q = \frac{-CA + B^2 X_1 - ABY_1}{A^2 + B^2}, \quad Y_Q = \frac{-ABX_1 + A^2 Y_1 - BC}{A^2 + B^2}$$

$$d = \left[(X_Q - X_1)^2 + (Y_Q - Y_1)^2\right]^{1/2} = \frac{A\,X_1 + B\,Y_1 + C}{\sqrt{A^2 + B^2}}$$

(8)


X
ℓ
$Ax+By+C=0$
$\bar{i}$
$\bar{e}_\ell = \bar{i}\cos\phi + \bar{j}\sin\phi$
1
ϕ
$\bar{j}$
1
Y
unit direction vectors

9



$$\bar{e}_1 = \bar{i}\cos\phi_1 + \bar{j}\sin\phi_1$$
$$\bar{e}_2 = \bar{i}\cos\phi_2 + \bar{j}\sin\phi_2$$

$$\bar{e}_1 \cdot \bar{e}_2 = (\bar{i}\cos\phi_1 + \bar{j}\sin\phi_1)\cdot(\bar{i}\cos\phi_2 + \bar{j}\sin\phi_2)$$
$$= \bar{i}\cdot\bar{i}\cos\phi_1\cos\phi_2 + \bar{i}\cdot\bar{j}\cos\phi_1\sin\phi_2 + \bar{j}\cdot\bar{i}\sin\phi_1\cos\phi_2 + \bar{j}\cdot\bar{j}\sin\phi_1\sin\phi_2$$
$$= \cos(\phi_1-\phi_2)$$

The angle between two lines and the vector dot product

(10)

Given l_1 $\quad 3X + 4Y - 8 = 0$
and l_2 $\quad -2X + 5Y + 12 = 0$

Find the angle between l_1 & l_2.

l_1 has $\Delta y / \Delta x = \frac{-3}{4}$, $\bar{e}_1 = \frac{4\bar{i} - 3\bar{j}}{\sqrt{4^2+3^2}}$

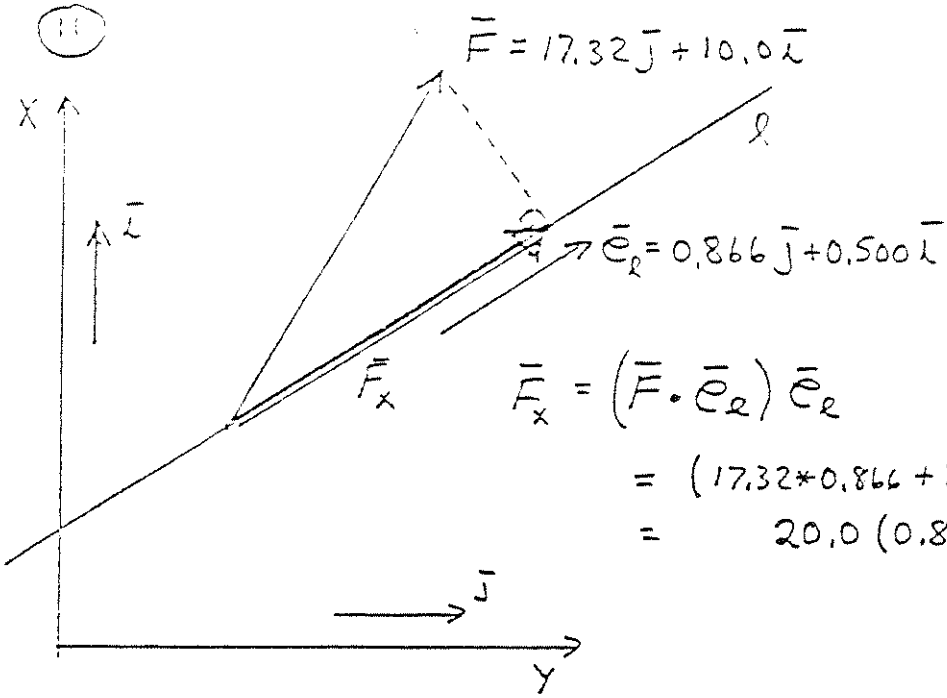
l_2 has $\Delta y / \Delta x = \frac{2}{5}$, $\bar{e}_2 = \frac{5\bar{i} + 2\bar{j}}{\sqrt{5^2+2^2}}$

$$\bar{e}_1 \cdot \bar{e}_2 = \frac{(4\bar{i} - 3\bar{j}) \cdot (5\bar{i} + 2\bar{j})}{\sqrt{25}\ \sqrt{29}} = \frac{20-6}{\sqrt{25}\sqrt{29}} = \cos(\phi_1 - \phi_2)$$

$$(\phi_1 - \phi_2) = 58.7^\circ$$

Vector Dot Product illustrated

RLH



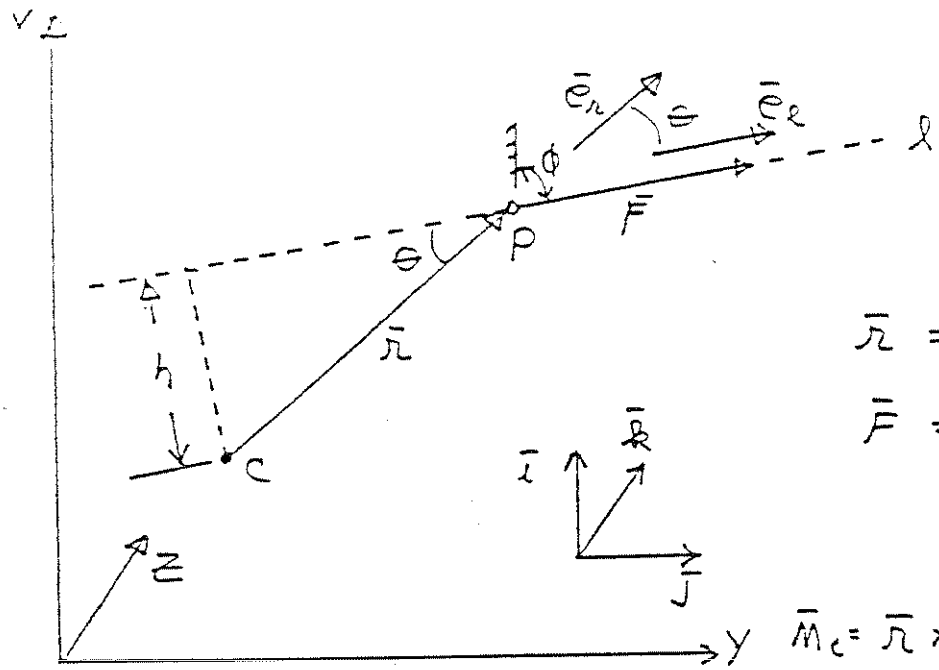
$$\bar{F}_x = (\bar{F} \cdot \bar{e}_l)\,\bar{e}_l$$

$$= (17.32 * 0.866 + 10.0 * 0.50)(0.866\bar{j} + 0.500\bar{i})$$

$$= 20.0\,(0.866\bar{j} + 0.500\bar{i})$$

The component of a vector $\bar{F}$ on a line l

(12)



$$\bar{e}_r = \bar{i}\cos\phi + \bar{j}\sin\phi$$

$$\bar{e}_r = \bar{i}\cos(\phi-\theta) + \bar{j}\sin(\phi-\theta)$$

$$\bar{r} = r\,\bar{e}_r$$

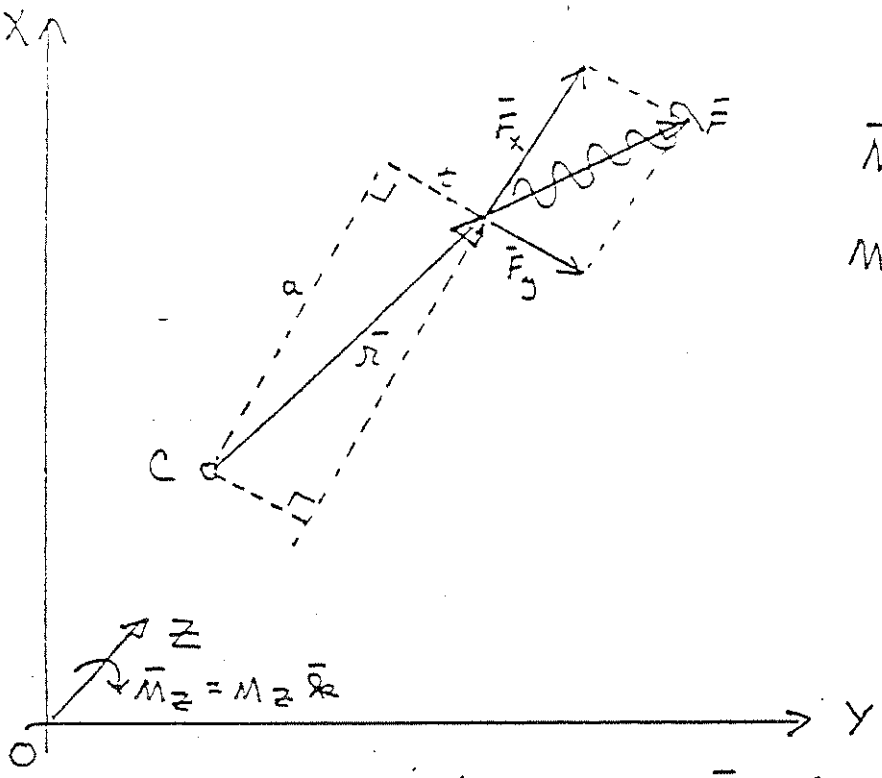
$$\bar{F} = F\,\bar{e}_\ell$$

$$\bar{e}_r \times \bar{e}_\ell = \bar{k}\sin\theta$$

$$\bar{M}_c = \bar{r} \times \bar{F} = (r * F * \sin\theta) * \bar{k}$$

$$\bar{M}_c = (h * F) * \bar{k}$$

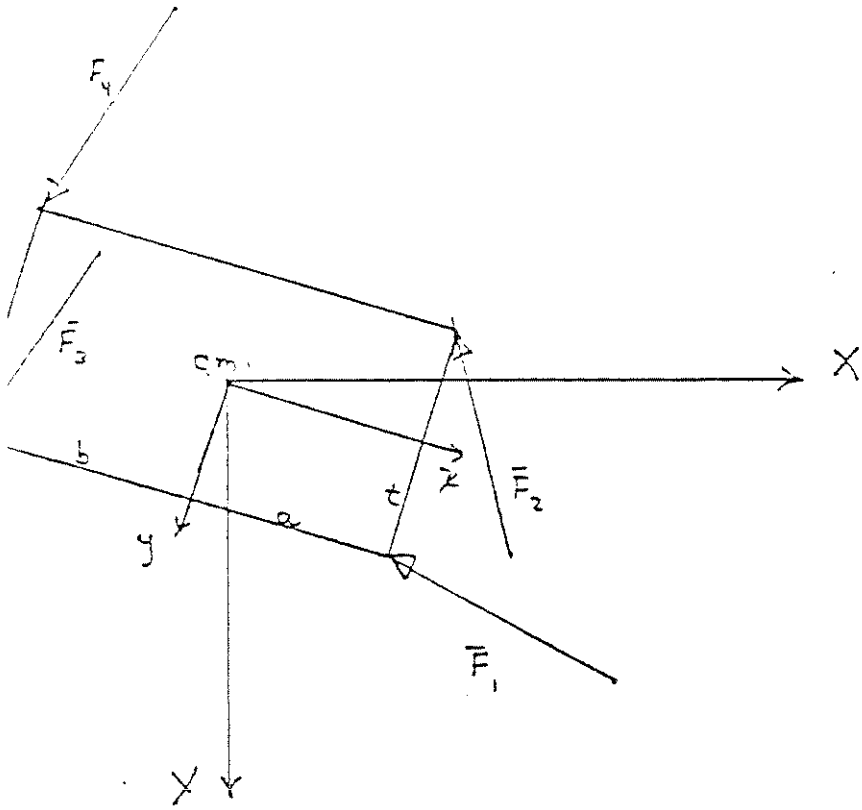
The moment of a vector $\bar{F}$ about a point C and the vector cross product.



$$\bar{M}_z^c = M^c \bar{k}$$

$$M^c \text{ or } M_c = (a * F_y - t * F_x)$$

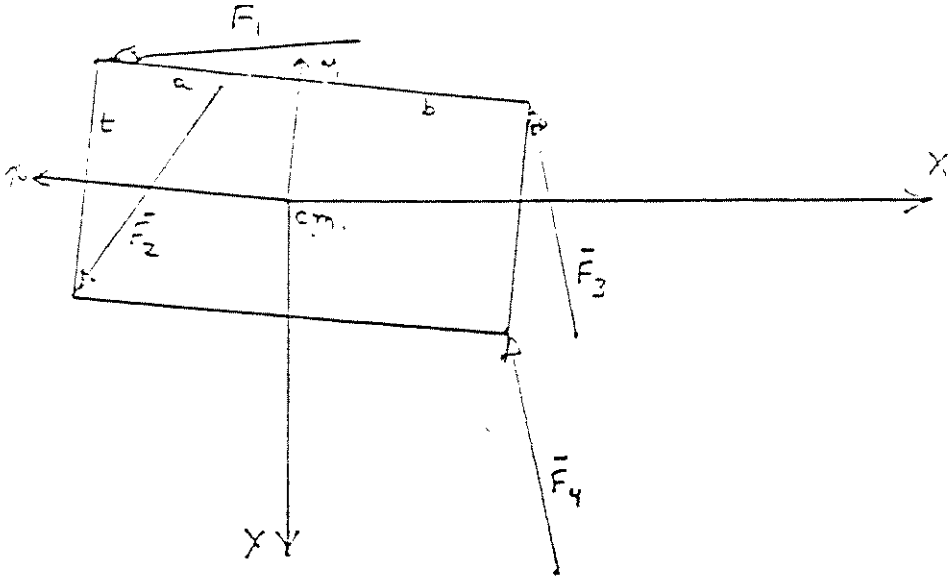
The moment of a vector $\bar{F}$ about a point C using components and perpendicular leverarms.



so having moments about point c.m.

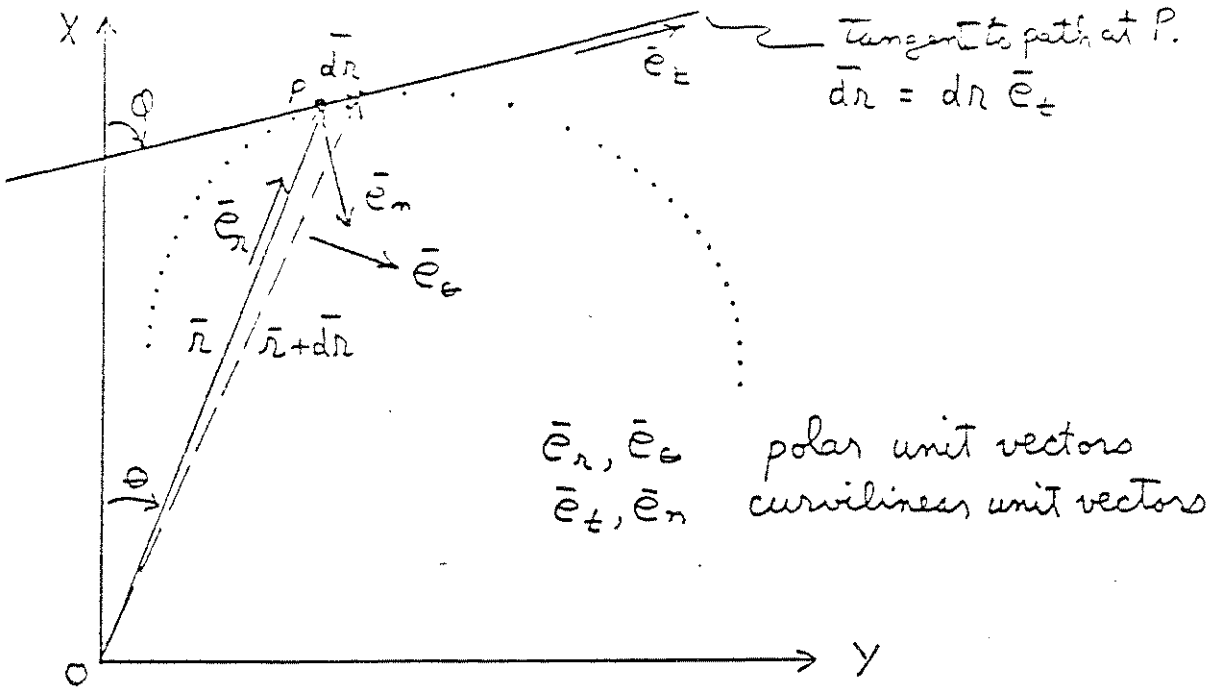
(4b)

RLH



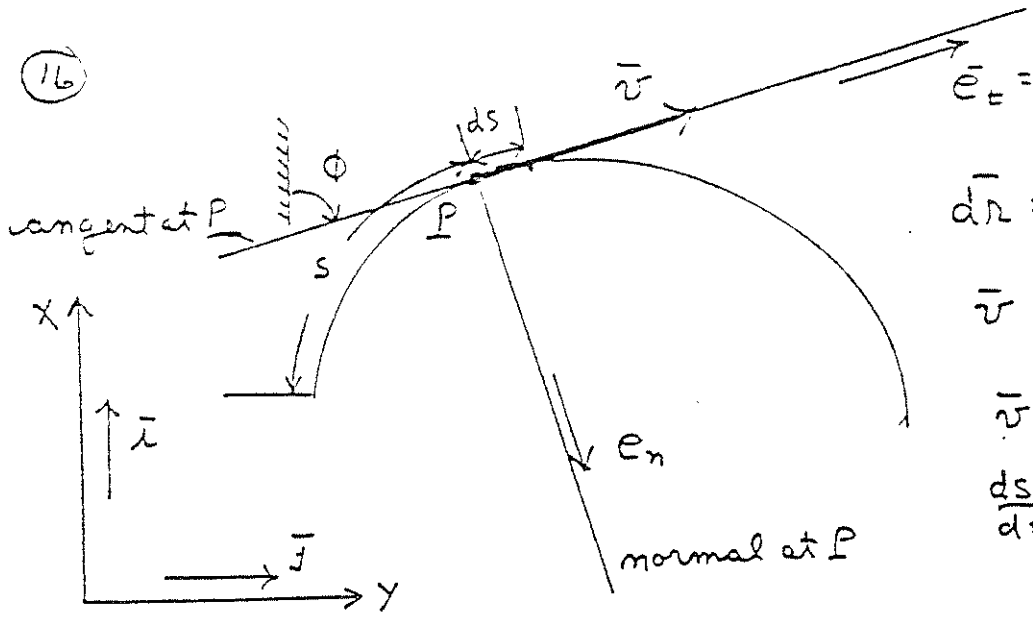
Four forces having moments about point c.m.

(15)



The position vector $\bar{r}$ of the point P at time t and the position vector $\bar{r}+d\bar{r}$ at time $t+dt$, if P is moving.

(16)



$\bar{e}_t = (\sin\phi)\bar{j} + (\cos\phi)\bar{i}$

$d\bar{r} = ds\,\bar{e}_t$

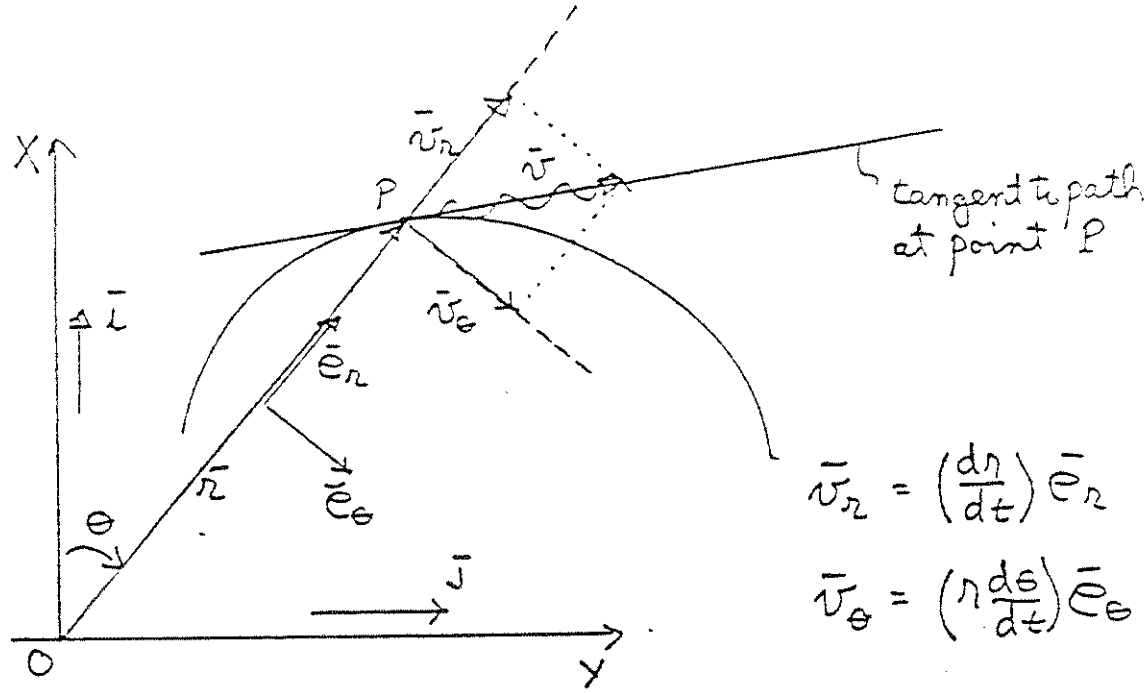
$\bar{v} = \dfrac{d\bar{r}}{dt} = \dfrac{ds}{dt}\bar{e}_t$

$\bar{v}$ — velocity vector

$\dfrac{ds}{dt}$ — speed of P

Velocity vector - tangent to path of moving point P

(17)



$$\bar{v}_r = \left(\frac{dr}{dt}\right)\bar{e}_r$$

$$\bar{v}_\theta = \left(r\frac{d\theta}{dt}\right)\bar{e}_\theta$$

$$\bar{r} = r\,\bar{e}_r$$

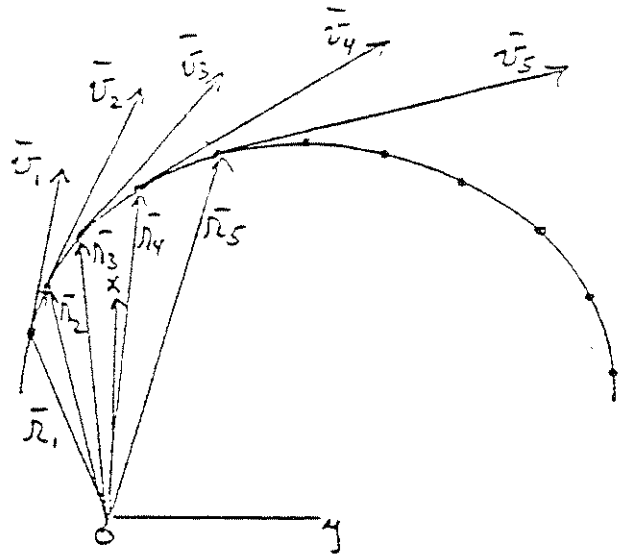
$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{dr}{dt}\bar{e}_r + r\frac{d\bar{e}_r}{dt} = \frac{dr}{dt}\bar{e}_r + r\frac{d\theta}{dt}\bar{e}_\theta$$

Velocity (the time derivative of the position vector) illustrated in component form to demonstrate that, in general, a position vector has a magnitude change <u>and</u> a directional change.

K&E 10 X 10 TO THE INCH 7 X 10 INCHES
KEUFFEL & ESSER CO. MADE IN U.S.A.

46 0720

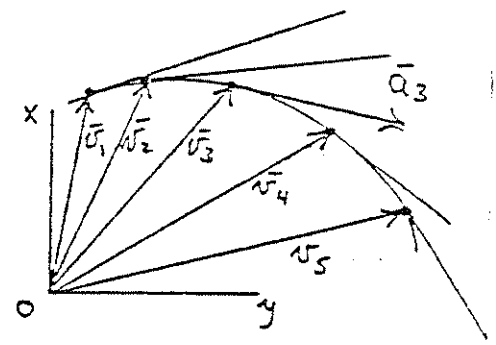
(12)



Position Vectors from Pole O → PATH

Velocity Vector $= \dfrac{d\bar{r}}{dt}$ (tangent to PATH)

$$\bar{a}_3 = \frac{d\bar{v}_3}{d\tau}$$

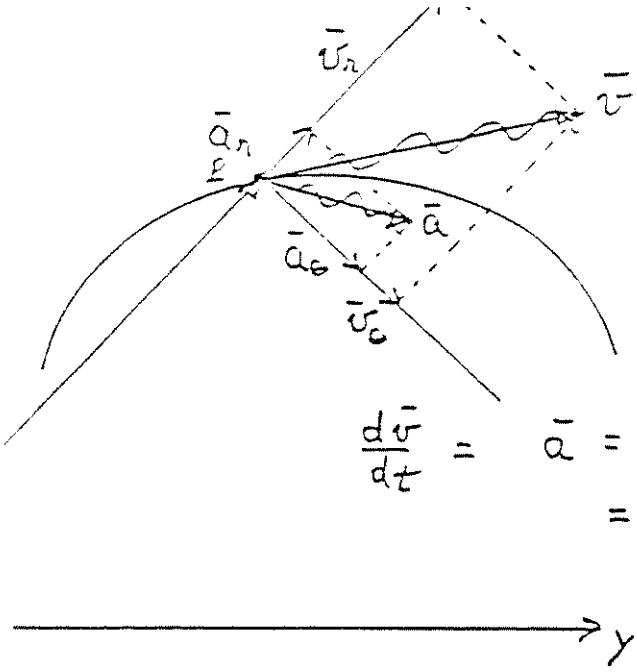


Velocity Vectors from Pole O ↓ HODOGRAPH

Acceleration Vector $= \dfrac{d\bar{v}}{dt}$

(tangent to HODOGRAPH)

RLH



$$\bar{v} = \bar{v}_r + \bar{v}_\theta$$

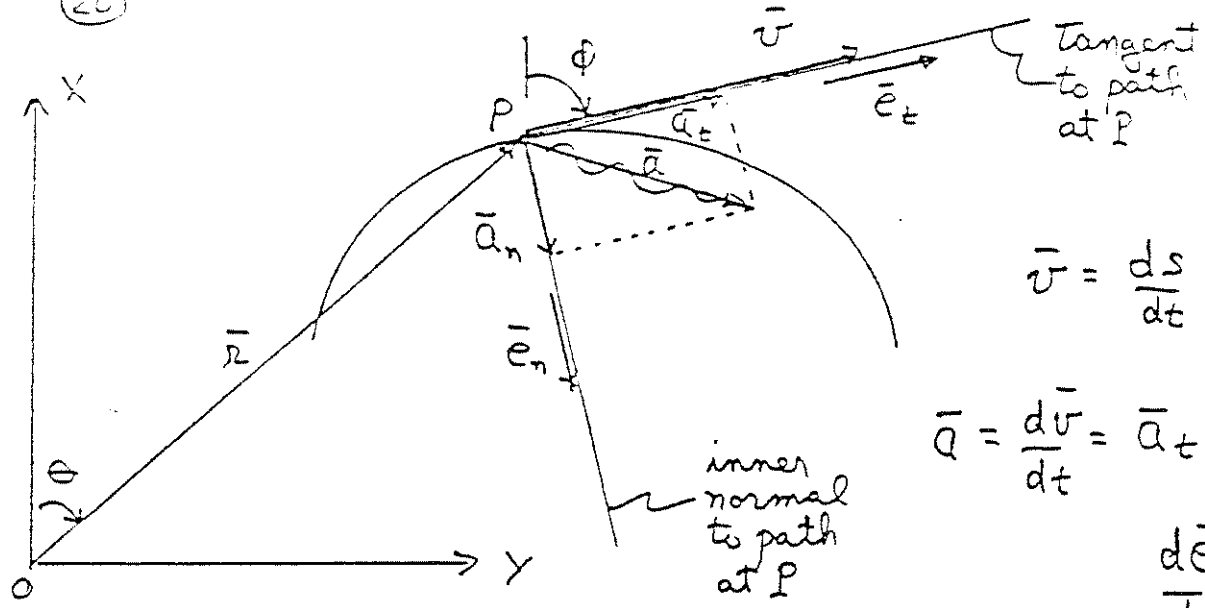
$$= \frac{dr}{dt}\bar{e}_r + r\frac{d\theta}{dt}\bar{e}_\theta$$

$$\frac{d\bar{v}}{dt} = \bar{a} = \bar{a}_r + \bar{a}_\theta$$

$$= \left[\frac{d}{dt}\left(\frac{dr}{dt}\right) - r\frac{d\theta}{dt}\frac{d\theta}{dt}\right]\bar{e}_r$$

$$+ \left[\frac{dr}{dt}\frac{d\theta}{dt} + \frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d}{dt}\left(\frac{d\theta}{dt}\right)\right]\bar{e}_\theta$$

leration vector in polar component form
ing the time rate of change of both magnitude
tion of each of the polar components of velocity.



$$\bar{v} = \frac{ds}{dt}\,\bar{e}_t$$

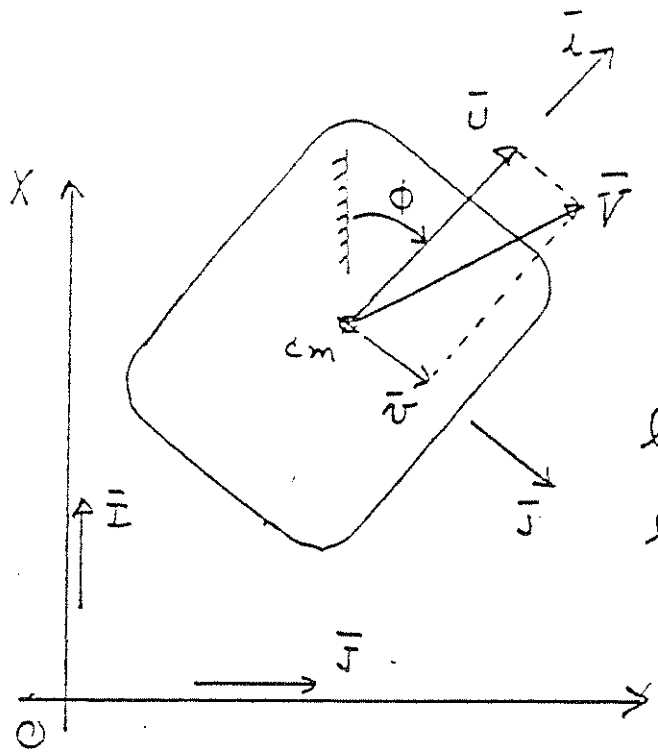
$$\bar{a} = \frac{d\bar{v}}{dt} = \bar{a}_t + \bar{a}_n$$

$$\frac{d\bar{e}_t}{dt} = \frac{d\phi}{dt}\,\bar{e}_n$$

$$\bar{a} = \frac{d}{dt}\left(\frac{ds}{dt}\right)\bar{e}_t + \frac{ds}{dt}\frac{d\phi}{dt}\,\bar{e}_n$$

The acceleration vector in curvilinear component form illustrating the time rate of change of the velocity's magnitude and the time rate of change of the velocity's direction vector.

RLH



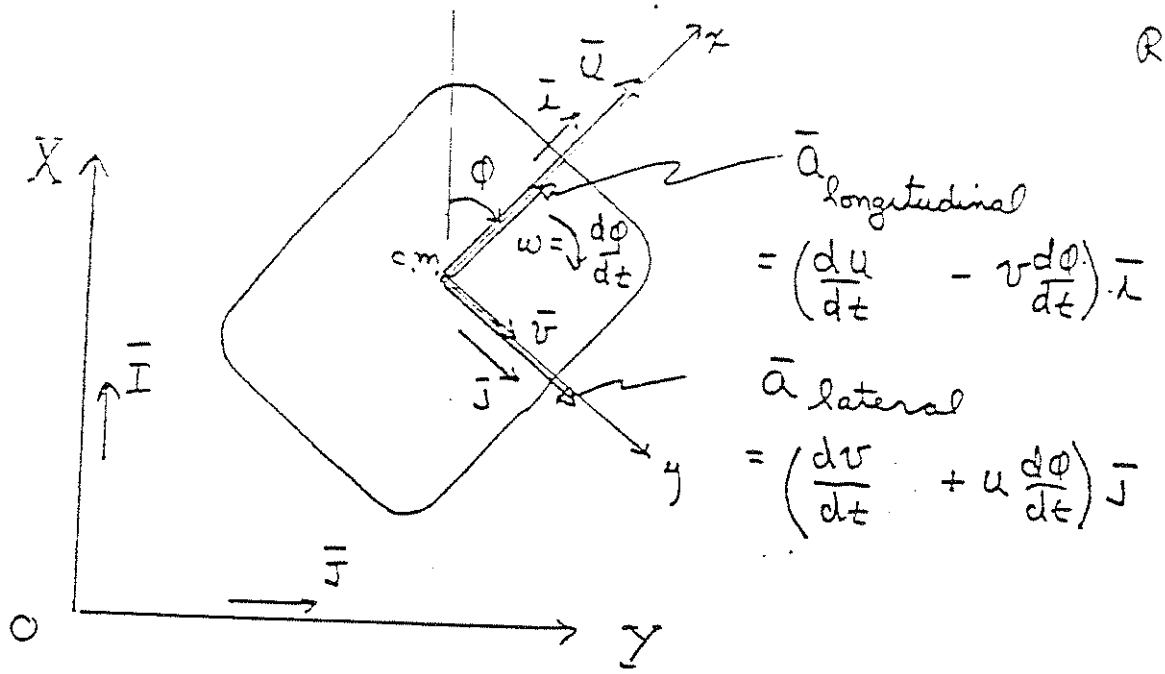
$$\bar{V} = \bar{U} + \bar{v}$$

let $|\bar{V}|$ = constant

let $\frac{d\phi}{dt} = \omega \neq$ zero

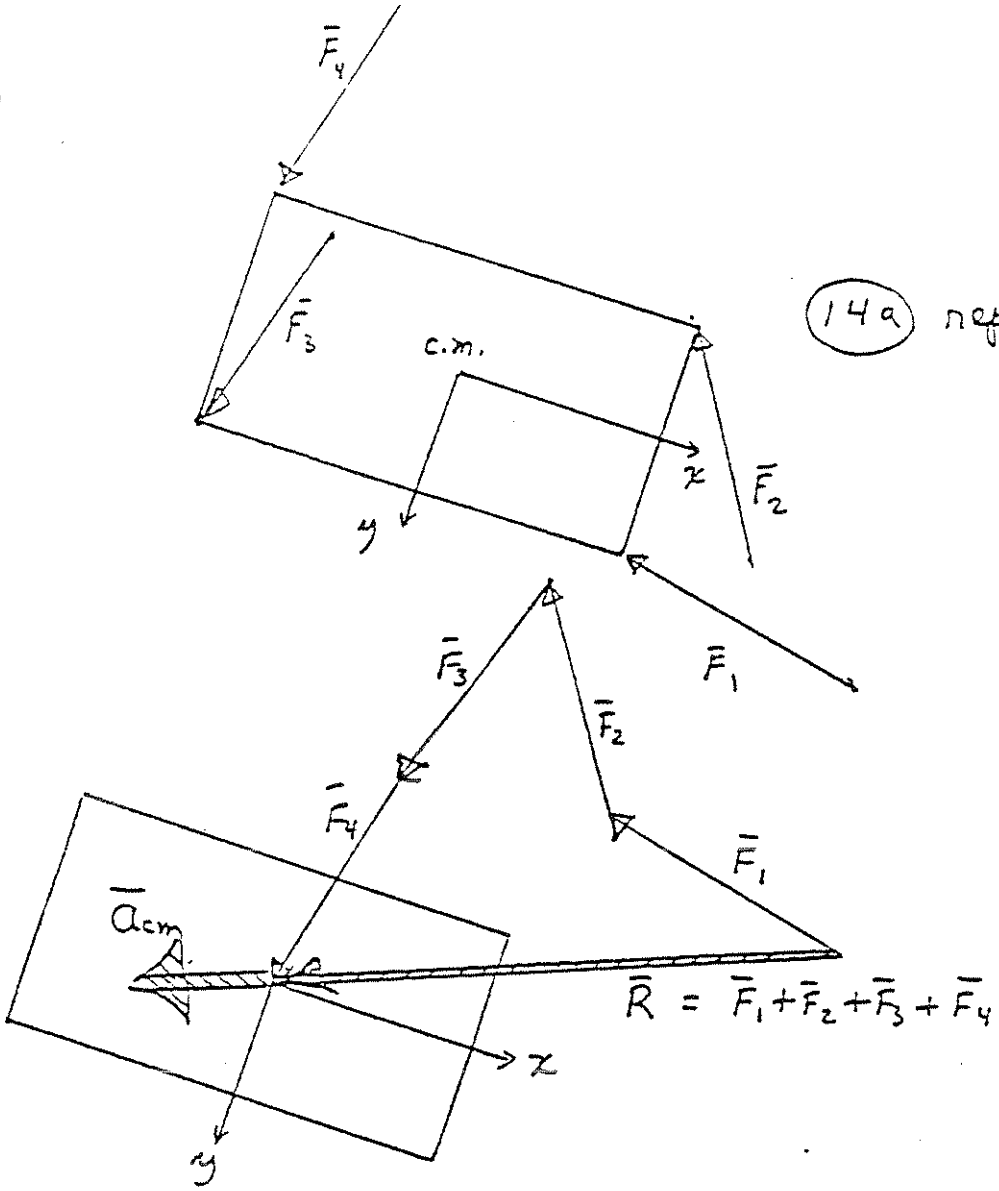
onstant linear speed vehicle's c.m. has components of
ration ($\bar{a}_{cm} = \bar{J}\left(U \cdot \frac{d\phi}{dt}\right) - \bar{I}\left(v \cdot \frac{d\phi}{dt}\right)$)
: vehicle's axes have an angular velocity.

RLH



vehicle fixed coordinate system along with
onents($\bar{u}$, $\bar{v}$) of the velocity of the c.m. point.
ration components ($\bar{a}_{longitudinal}$, $\bar{a}_{lateral}$) of the
center of mass.

23

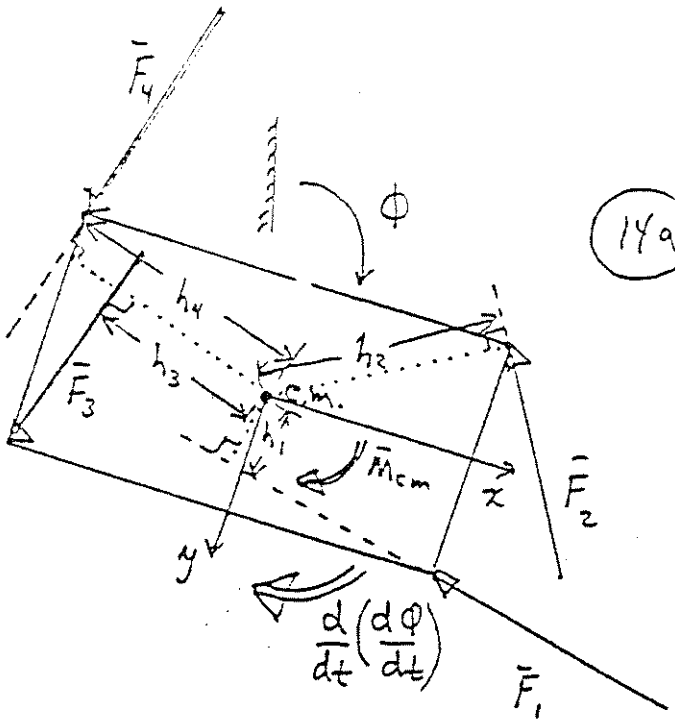


14a repeated

The sum of the four tire-road shear forces. The sum is illustrated as acting at the vehicle's center-of-mass

The center-of-mass has an acceleration parallel to the sum and of a magnitude

$$|\bar{a}_{cm}| = \frac{|\bar{R}|}{mass}$$



(14a) repeated

'oment of the four tire road-shear forces about cle's center of mass

$$|\bar{M}_{cm}| = M_{cm} = h_1|\bar{F}_1| - h_2|\bar{F}_2| - h_3|\bar{F}_3| - h_4|\bar{F}_4|$$

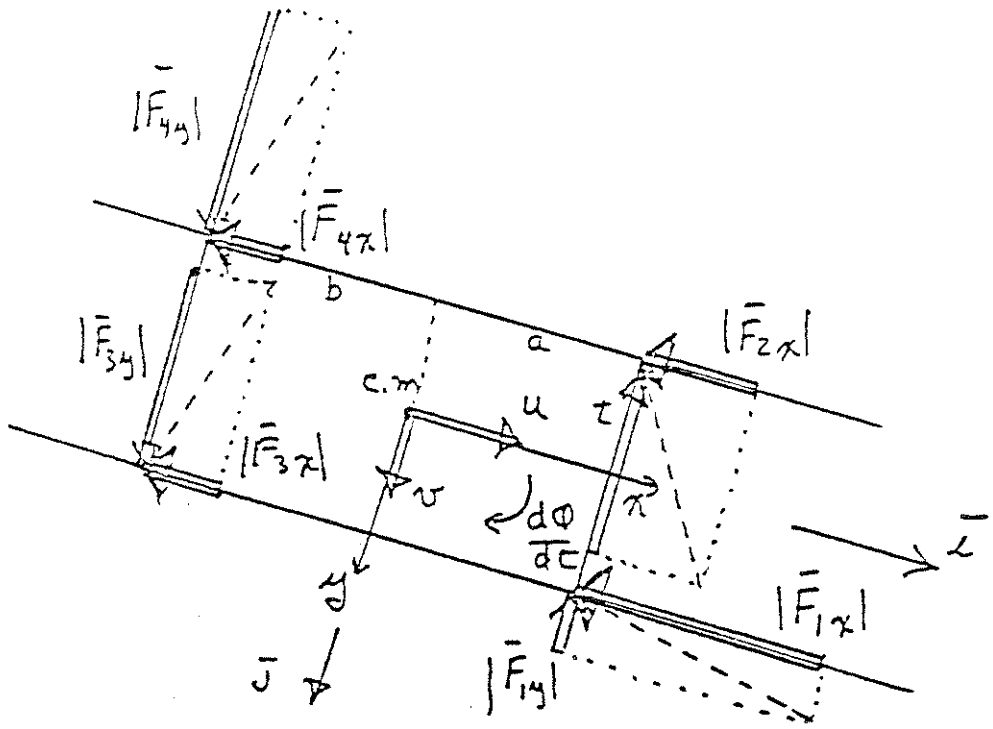
angular velocity of the vehicle, $\frac{d\phi}{dt}$, has a time-change in the direction of M_{cm} and ional to M_{cm} as stated by:

$$\frac{d}{dt}\left(\frac{d\phi}{dt}\right) = \frac{M_{cm}}{mass * k_{gy}^2}$$

H

RL

(25)



$$(1) \qquad a_x = \left(\frac{du}{dt} - v\frac{d\phi}{dt}\right) = \frac{-|\bar{F}_{1x}| - |\bar{F}_{2x}| - |\bar{F}_{3x}| - |\bar{F}_{4x}|}{mass}$$

$$(2) \qquad a_y = \left(\frac{dv}{dt} + u\frac{d\phi}{dt}\right) = \frac{-(|\bar{F}_{1y}| - |\bar{F}_{2y}| + |\bar{F}_{3y}| + |\bar{F}_{4y}|}{mass}$$

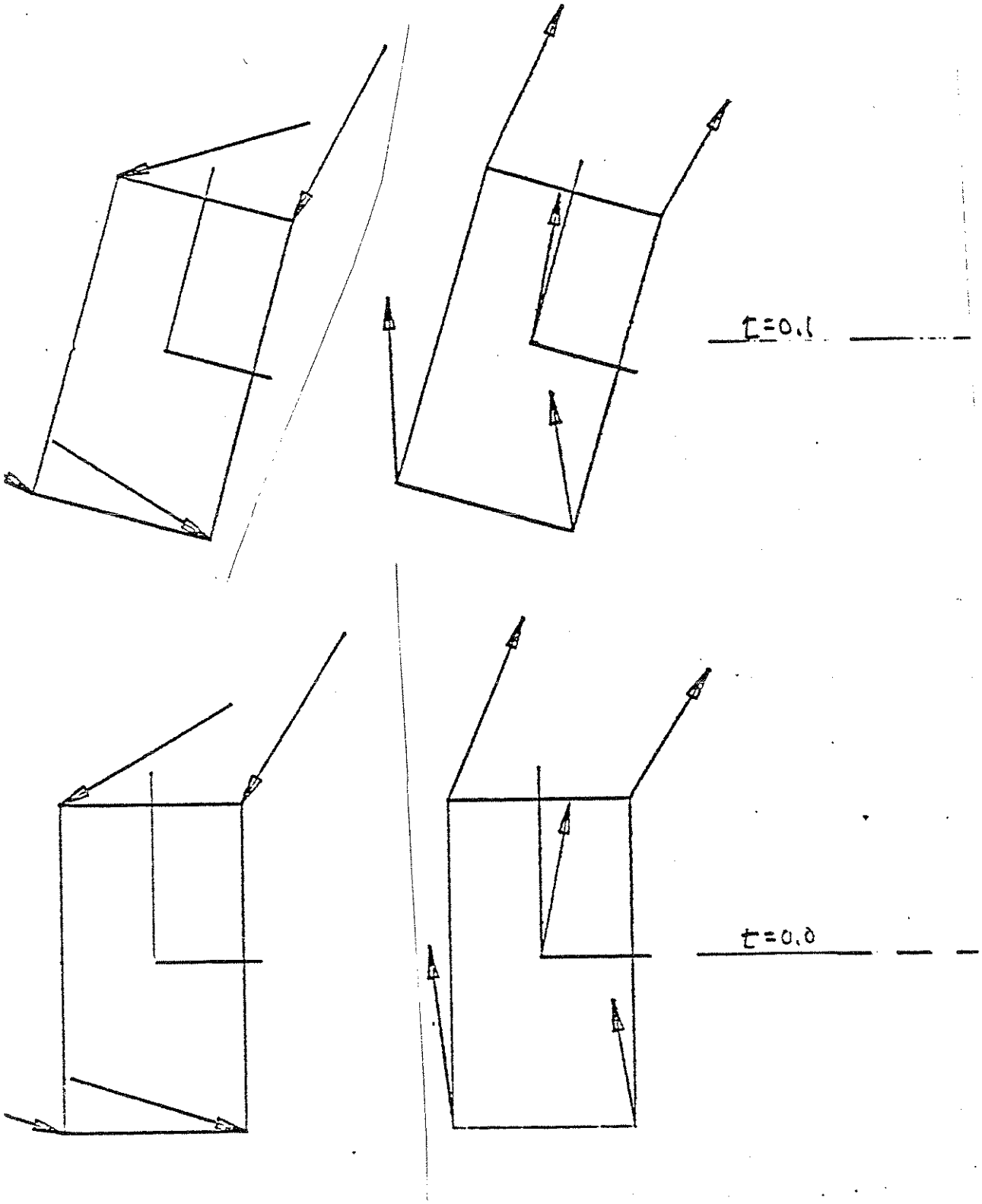
$+|F_{3x}|$

$$(3) \qquad \alpha = \frac{d}{dt}\left(\frac{d\phi}{dt}\right) = \frac{t\left[|\bar{F}_{1x}| + |\bar{F}_{3x}| - |\bar{F}_{2x}| - |\bar{F}_{4x}|\right] - a\left[|\bar{F}_{1y}| + |F_{2y}|\right] - b\left[|F_{3y}|\right.}{mass * \cancel{R_{gy}^2}}$$

Acceleration, Forces and Equations of Motion.

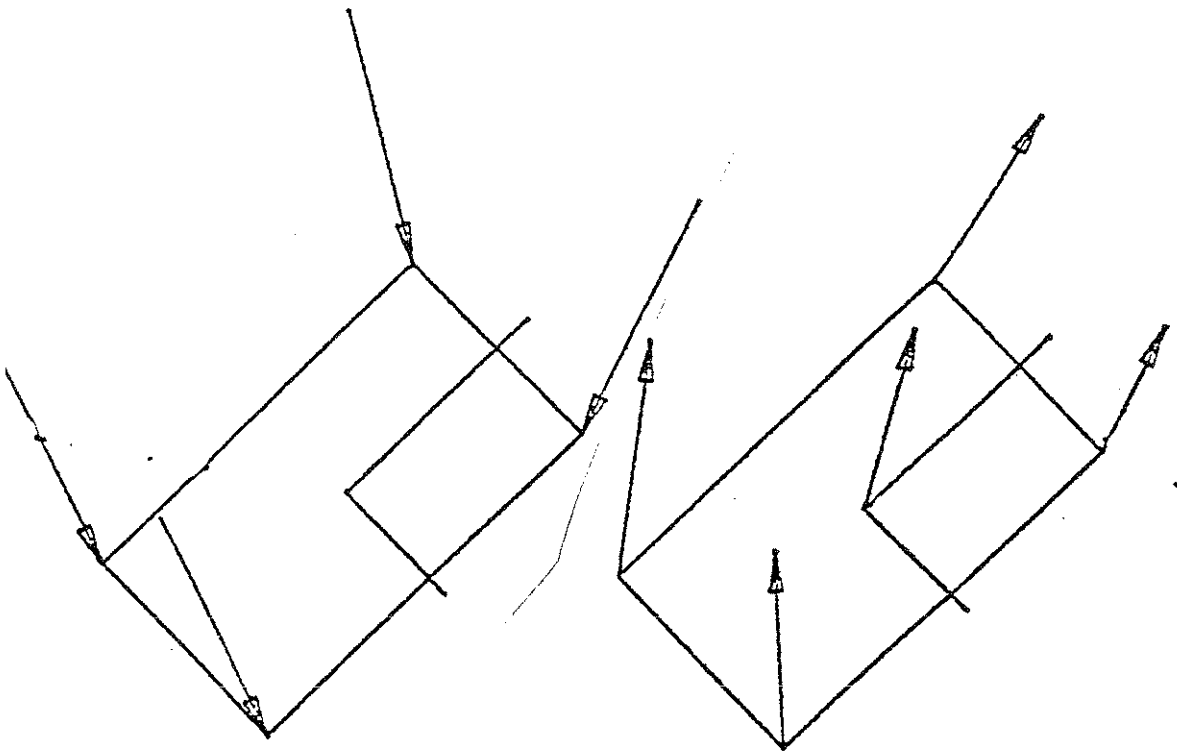
(signs of terms are specific to this figure)

R&H

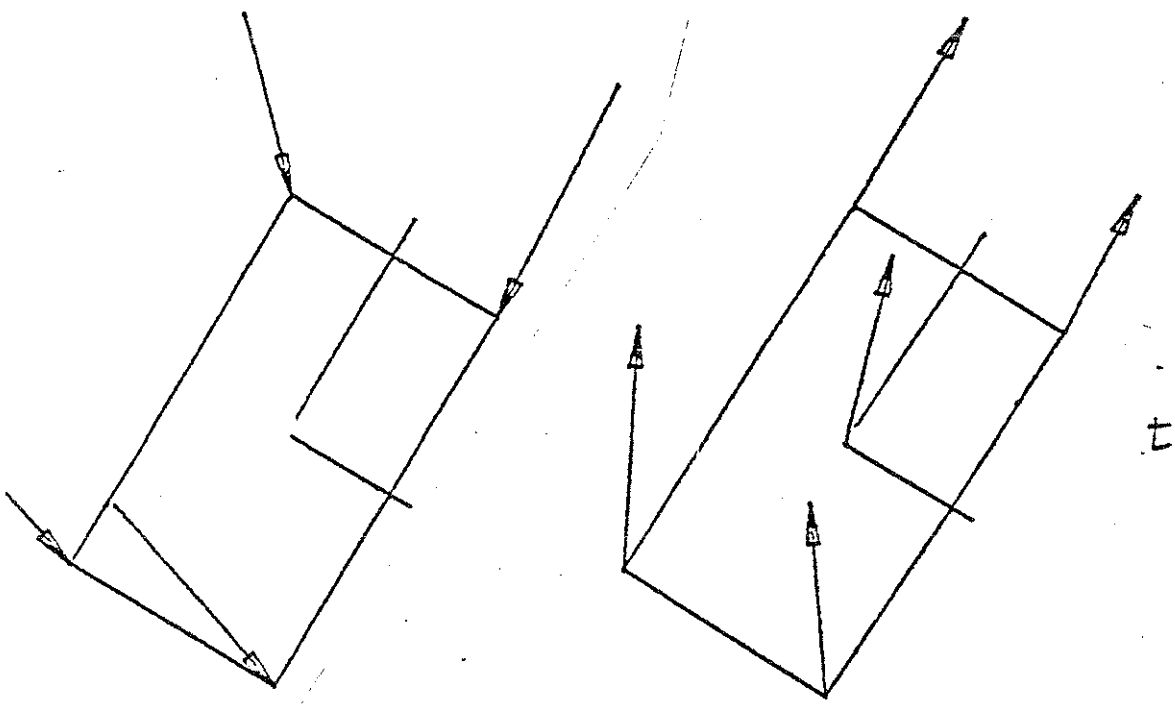


FORCE VELOCITY

RLH.



t=0.1



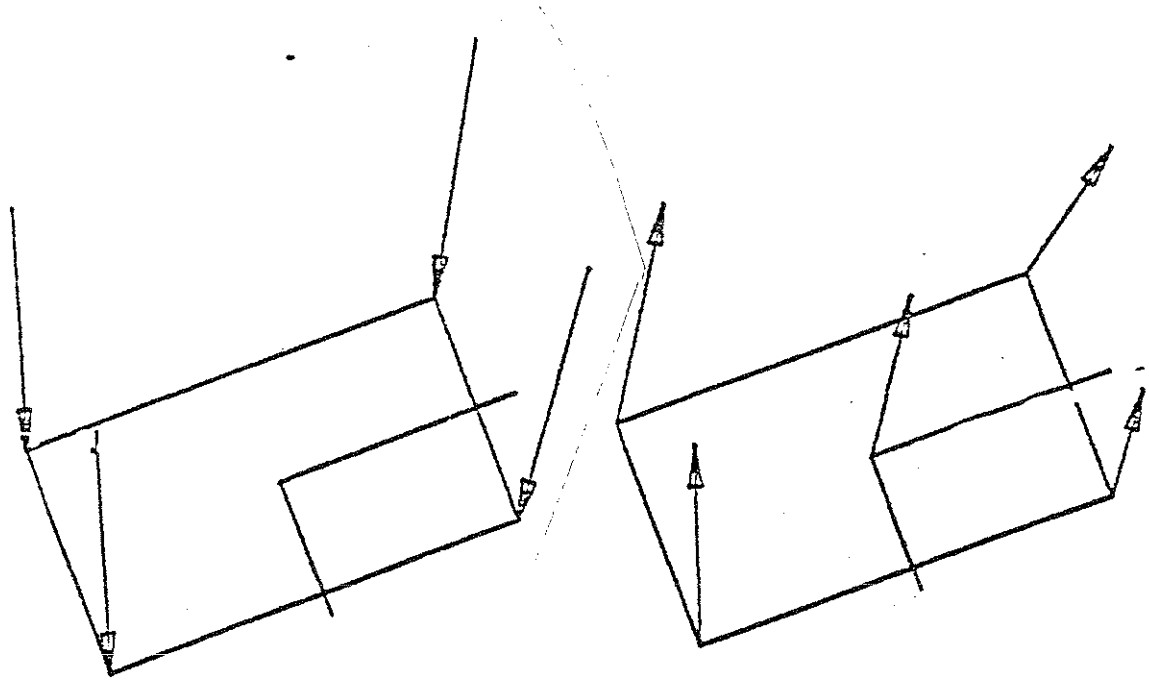
t=0.2

FORCE VELOCITY

RLH

5

: 0.5



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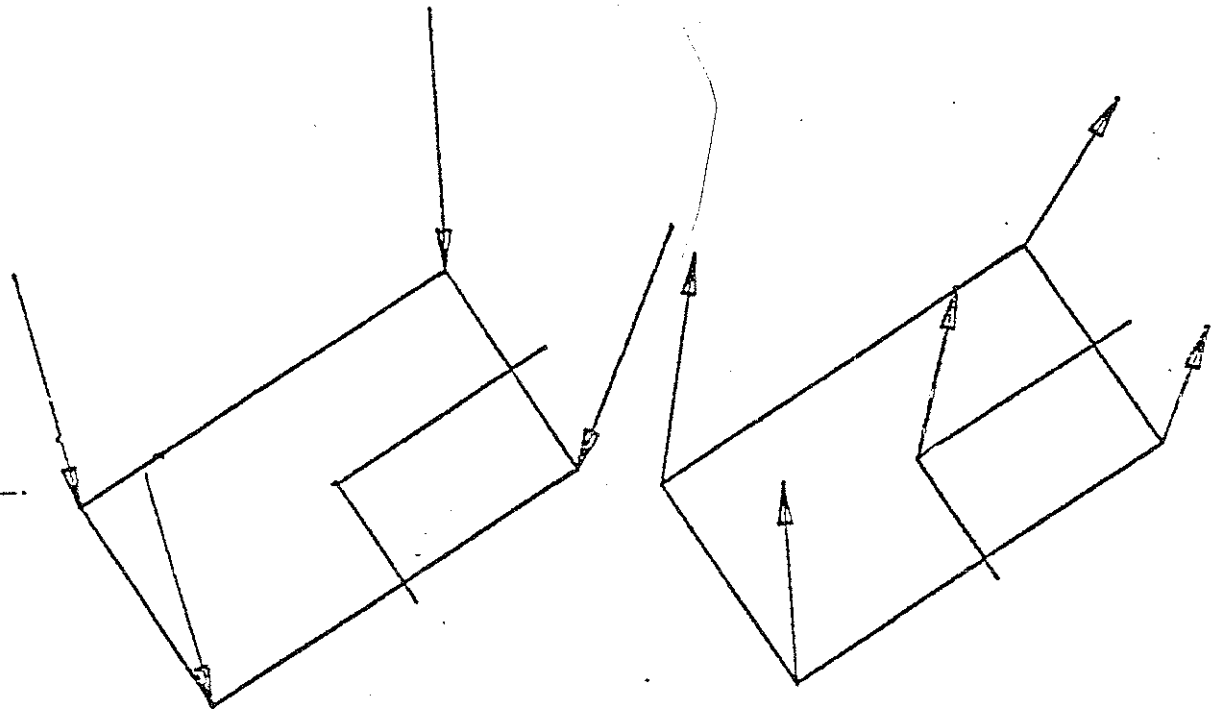

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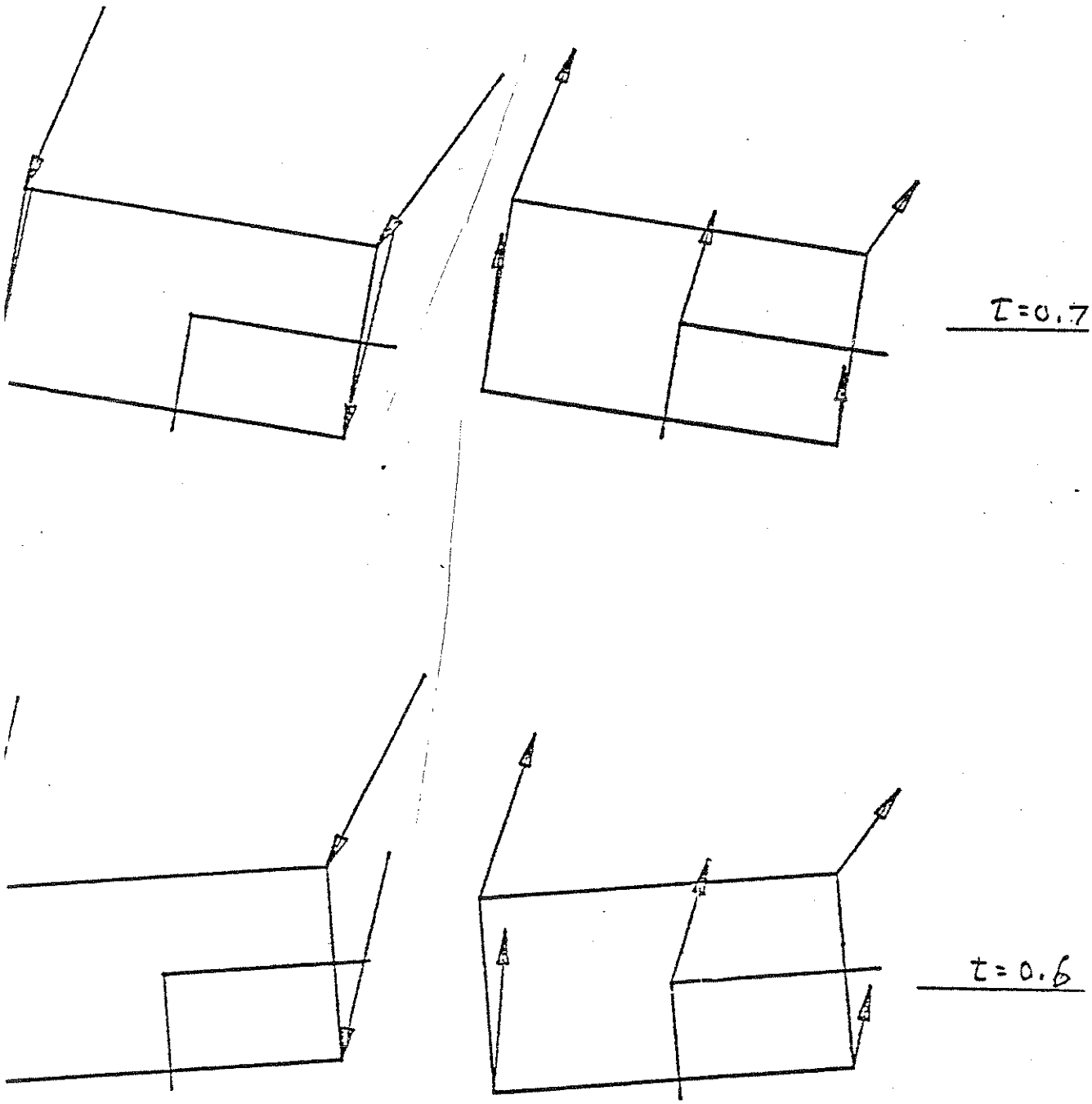
FORCE

VELOCITY

RLH



FORCE

VELOCITY

RLH

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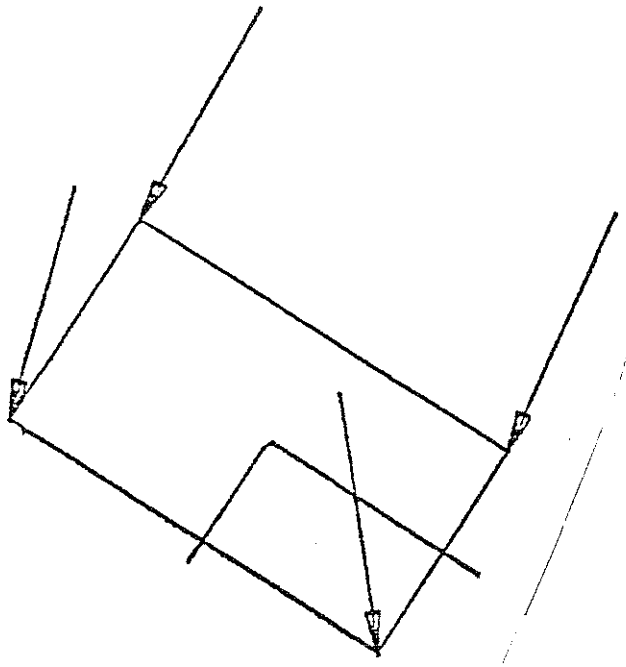

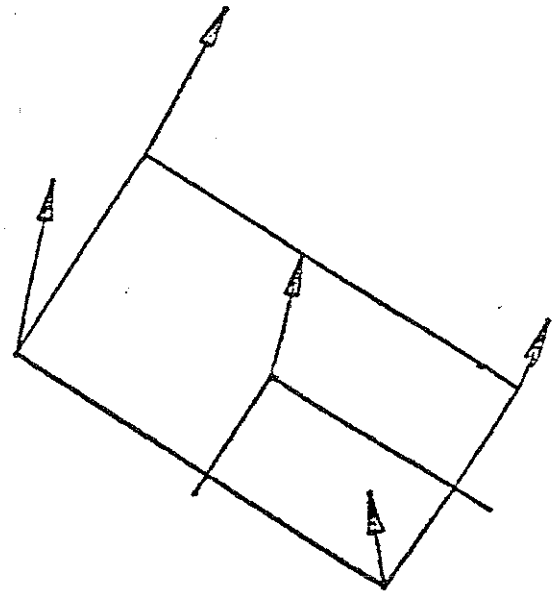

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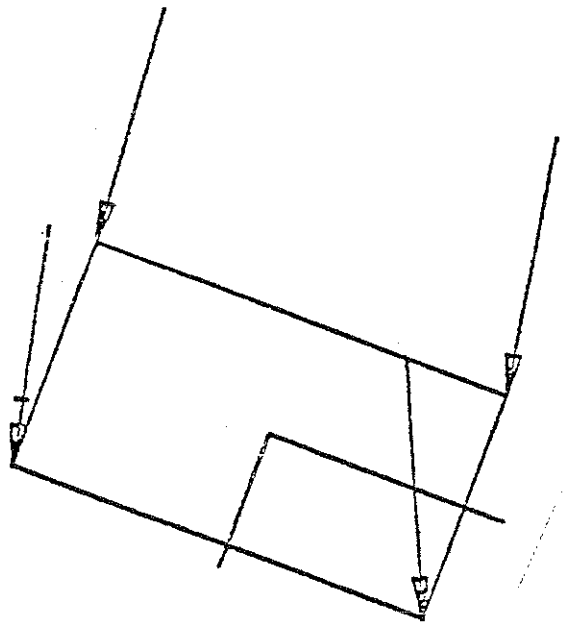
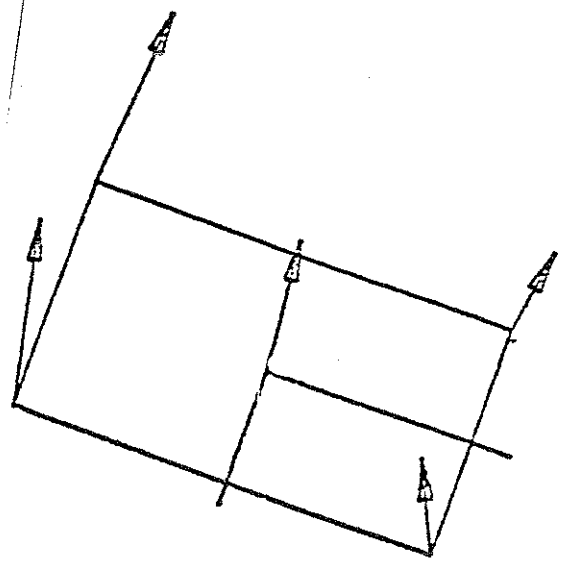




FORCE

VELOCITY

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 4, Vehicle Dynamics

Presented by: Christopher Winkler

Description:

An overview of three-dimensional real-world vehicle dynamics including vehicle handling, braking, and traction as they apply to accident causation and investigation.

Topics:

Definitions
- In-plane and out-of-plane motions
- Motor vehicle components which influence braking and handling

Tire traction and compliance

Handling
- Steady turning, normal and limit
- Directional response to steering
- Influence of wind and road disturbance

Braking
- Brake torque and force distribution
- The steady braking process
- Limit braking and directional stability
- Influence of road surface and tires

Accelerating
- Turning on a reduced friction surface

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 5, Vehicle Force - Deflection Characteristics

Presented by: Kenneth L. Campbell

Description:

An overview of vehicle force-deflection characteristics as they apply to accident reconstruction. These characteristics are a measure of the energy required to crush the vehicle.

Topics:

Energy Considerations
- Energy Balance and Terms
- Estimating Energy Absorbed
- Considerations in Measuring Crush

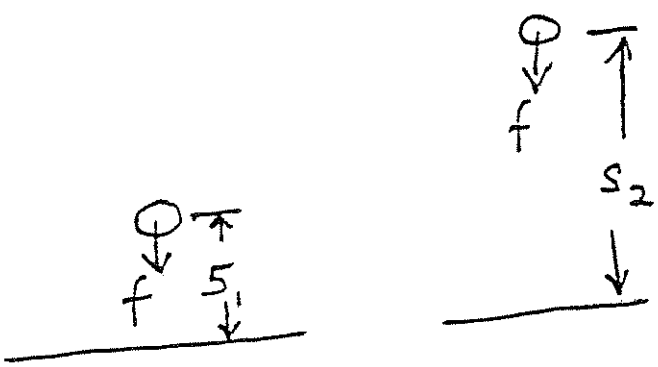
NOTES

I

Energy is the capacity to exert a force through a distance

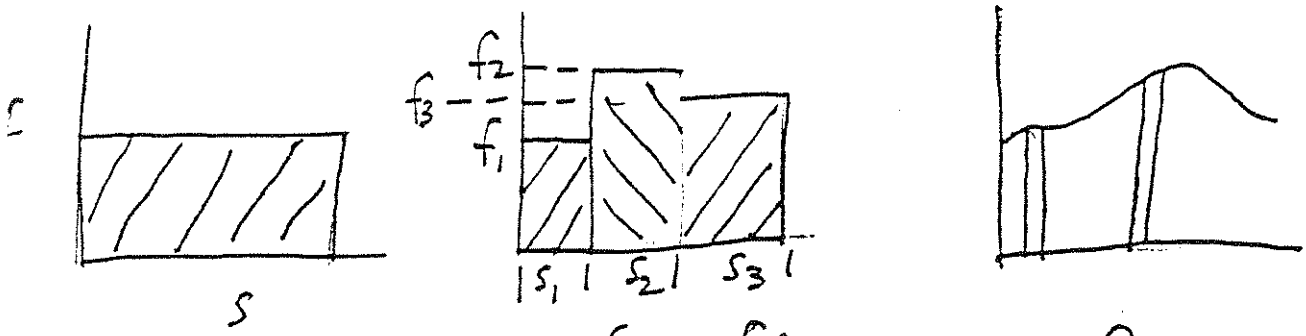
$$\boxed{E = \int f \, ds}$$

if f is constant $E = Fs$



$f = mg$

$E = mg(s_2 - s_1)$



$E = fs$

$E = f_1 s_1 + f_2 s_2 + f_3 s_3$

$E = \int f \, ds$

ENERGY BALANCE

<u>Terms</u>

Kinetic, $E_V = \frac{1}{2} m V^2$

Potential, $E_H = m g h$

Skid Loss, $E_s = m g \mu s (RR)$

Absorbed, E_a

<u>Pre-Impact</u>	<u>Post-Impact</u>
E_{V1}, E_{V2}	E_{a1}, E_{a2}
	E_{s1}, E_{s2}
	E_{H1}, E_{H2}

Note:

$$E_V = E_s$$

$$\frac{1}{2} \cancel{m} V^2 = \cancel{m} g \mu s (RR)$$

$$V = \sqrt{2 g \mu s (RR)}$$

Braking on a Grade

$$E_K = E_S + E_H$$

$$\frac{1}{2} m V^2 = m g s \mu (RR) + m g h$$

$$V = \sqrt{2g\left[s\mu(RR) + h\right]}$$

if grade is small

$$h = s \sin\theta = s\left(\frac{\text{Grade }\%}{100}\right)$$

$$V = \sqrt{2gs\left[\mu(RR) + h\right]}$$

(minus if downhill)

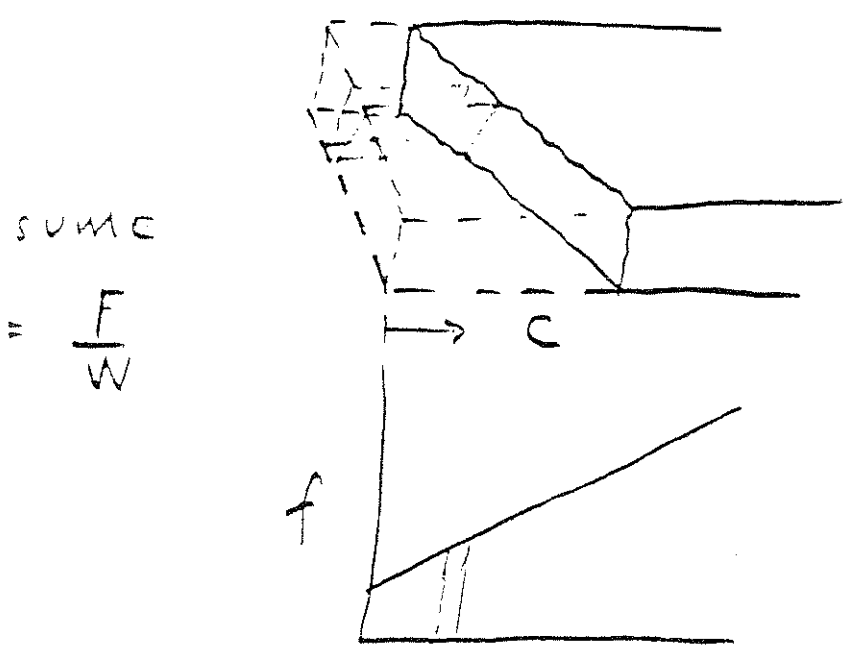
Estimating Energy Absorbed

Damage → [Force Deflection Characteristics] → Energy

Barrier Test Data:

$$V_B = b_0 + b_1 C$$

implies $F = \frac{W}{g}(b_0 b_1 + b_1^2 C)$



$$E_a = \int_W \int_C F \, dC \, dW$$

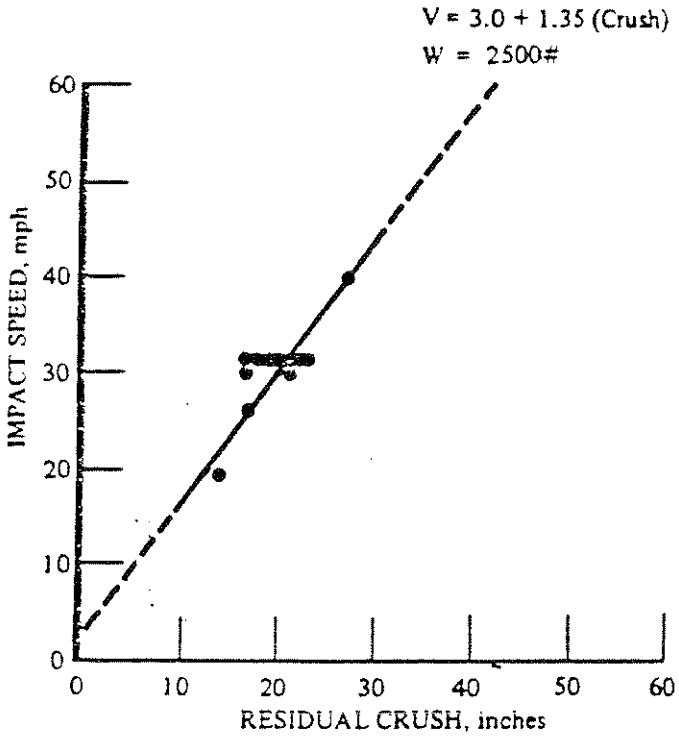


Fig. 2 - Residual crush vs impact speed in full frontal barrier tests for '71-'74 Chevrolet Vegas

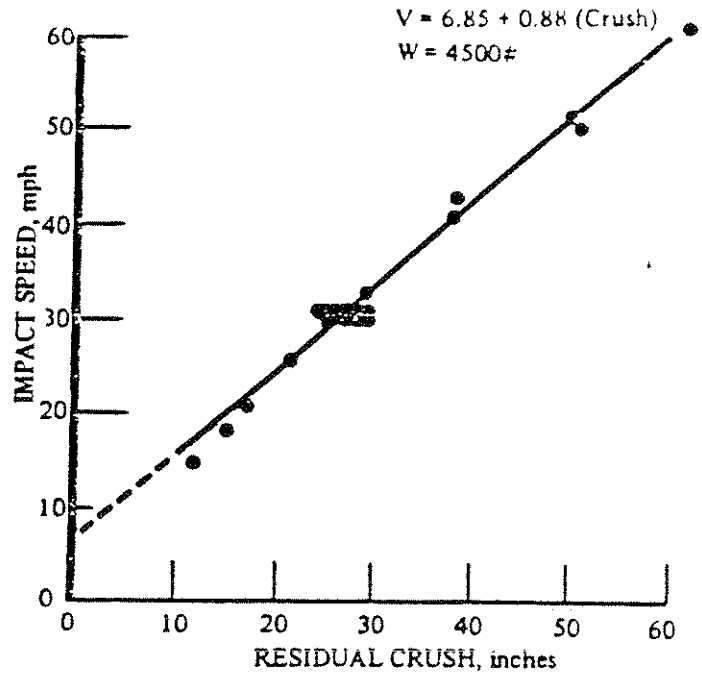


Fig. 1 - Residual crush vs impact speed in full frontal barrier tests for '71-'72 full size GM vehicles

Table 1 - Summary - Frontal Barrier Test Data

$$V(\text{mph}) = b_o + b_1 C(\text{in.})$$

GM Vehicles	b_o mph	b_1 mph/in.	weight lbs.
71-72 Standard Full Size	6.85	0.88	4500
73-74 Standard Full Size	7.5	0.90	4500
73-74 Intermediate	7.5	0.90	4000
71-74 Compact	3.0	1.35	3400
71-74 Subcompact	3.0	1.35	2500

Vehicle Classification	Examples
Standard Full Size	Chevrolet Impala, Pontiac Catalina, Oldsmobile Delta 88, Buick LeSabre
Intermediate	Chevrolet Chevelle and Monte Carlo, Pontiac LeMans and Grand Prix, Oldsmobile Cutlass, Buick Century
Compact	Chevrolet Nova, Pontiac Ventura, Oldsmobile Omega, Buick Apollo
Subcompact	Chevrolet Vega

Note: The following vehicle classifications are not included in the above table:

Sports	(Chevrolet Corvette)
Compact Specialty	(Chevrolet Camaro, Pontiac Firebird)
Luxury Sedan	(Oldsmobile 98, Buick Electra 225, Cadillac Calais)
Personal Luxury	(Oldsmobile Toronado, Buick Riviera, Cadillac Eldorado)

Energy Considerations for Crush Measurements

Direct vs Induced

Underride / Override (frame)

Angle

Narrow Object (frame)

Side (zones)

Rear - station wagons
rear engine

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 6. Skid Marks and Analysis

Presented by: Peter Cooley

Description:

A presentation on the practical aspects of describing and measuring skid and other pavement marks and on the estimating of changes of vehicular velocity based on skid distance and coefficient of friction.

Topics:

Importance of Tiremarks and Roadway Evidence
Overlapping Skidmarks
Curved Skidmarks
Skip Skids
Gaps in Skidmarks
Acceleration Marks
Variable Skidmarks
Tread Pattern Marks and Striations
Point of Impact
Tire Tread Imprints
Scuff Marks
Scratches
Gouges
Ruts
Furrows
Holes and Mounds
Debris
Speed from Skidmarks

NOTES

PHYSICAL EVIDENCE HELPFUL TO RECONSTRUCTION

A. Roadway

1. Surface composition and condition (include ambient)
2. Coefficient of friction
3. Grade, crown/superelevation
4. Pavement width
5. Roadway geometry (curves, tangents, etc.)
6. Intersection geometry (curb lines, angles, turn features, converging angles, etc.)
7. Median description (composition, dimension, etc.)
8. Lateral and longitudinal placement of lines and other surface markings
9. Site distances
10. Pavement edge
    a. Curbs (composition, condition, dimensions)
    b. Shoulders (composition, condition, dimensions, coefficient of friction if applicable)
    c. Drainage gutters (composition, condition, dimensions, coefficient of friction if applicable)

B. Roadside

1. Bridge abutments, overpass abutments, etc. (lateral and longitudinal dimensions/vertical dimensions where applicable)
2. Sidewalks (location, composition, width)
3. Guardrails/Guardposts (type, post spacing, face height, features, etc.)
4. Signs in accident area and relevant ones along approach
5. Private driveways/commercial accesses (dimensions and descriptive features)
6. Utility poles, light standards (luminaires), signal light support standards, etc. (locations, type, diameter, fastening methods, etc.)
7. Delineator posts/mile posts (locations)
8. Ditches (location, width, depth, etc.)
9. Embankments (location, elevation gain/loss)

10. Trees and shrubbery (location, size/diameter of trunk)
11. Waterways (location/depth where pertinent)
12. View obstructions (location, size, etc.)
13. Any other obstacles struck by or which influenced the control/trajectory of accident vehicles
14. Estimate of coefficient of friction of all off-road surfaces traversed by accident vehicles and record the slopes of these surfaces (elevation gains and losses)

C. Pre-Impact or Pre-Key Events

1. Free-rolling tire prints
2. Skidmarks
3. Acceleration scuffs
4. Braking scuffs
5. Critical curve scuffs/yaw marks
6. Flat tire scuffs
7. Liquid spills
8. Gouges or scratches
9. Converging vectors
10. Vehicle rotation (attitude versus vector)

D. Impact or Key Events

1. Point of impact (positions of all vehicles), departure from pavement edge, overturn, etc.
2. Gouges, pavement chops, etc.
3. Debris - liquid spills, vehicle components, underbody dirt, etc.
4. Damage to objects contacted
5. Deflection angle from pre-impact vector

E. Post-Impact or Post-Key Event

1. Other collisions - identify objects struck and document damage to these objects
2. Scuffmarks or skidmarks
3. Gouges
4. Liquid spills
5. Free-rolling tire prints
6. Vector changes during travel
7. Vaults - dimensions

8. Rollover - dimensions
9. Free fall - dimensions
10. Post-impact rotation
11. Final rest positions - all wheels and orientation

F. Vehicle

1. Damage description (include sketches)
2. Crush measurements (include sketches)
3. Physical dimensions (wheelbase, front overhang, rear overhang, width, etc.)
4. Ejection portals
5. Injury producing features
6. Interior contacts from occupants
7. Crash performance (investigator assessment)
8. Vehicle defects (potential, evident, etc.)

G. Human (Occupants)

1. Final rest positions (document, measure with reference)
2. Injury description(s)
3. Restrained (belted)
4. Condition of clothing (tears, rips, removed, etc.)
5. Location of body fluids or matter (identify and locate)

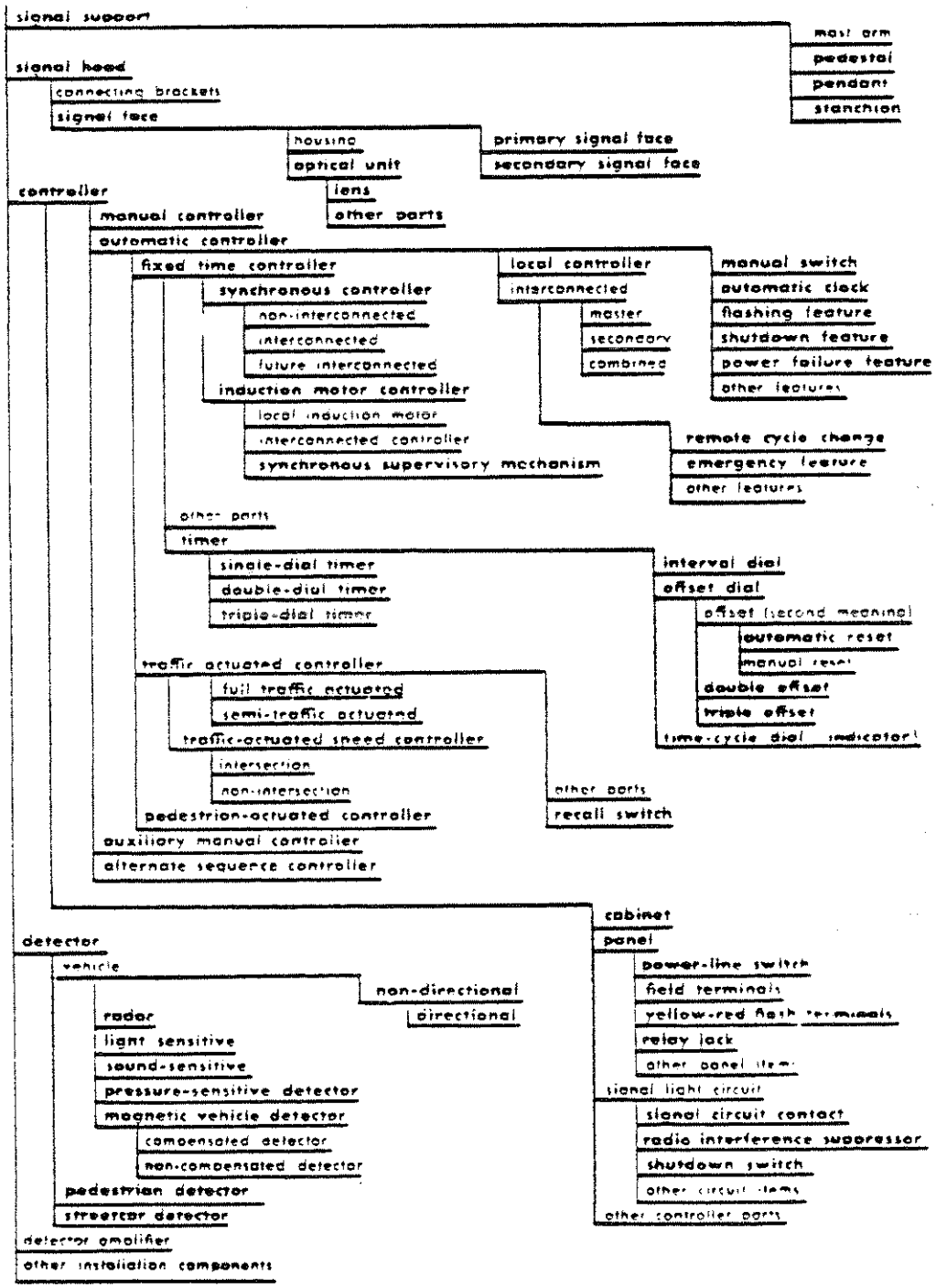
ROADWAY JUNCTIONS & CROSSINGS

- junction of streets or highways (with exchange of traffic)
  - three-way junction
    - T junction
    - Y junction
      - forked junction
  - four-way junction
    - right-angle junction
      - offset junction
    - oblique junction
  - multi-way junction
  - at-grade junction
    - unchannelized junction
      - flared junction
    - channelized junction
      - separated right turns
      - rotary junction
        - traffic circle
        - small rotary
  - interchange
    - directional interchange
      - three-way only
        - trumpet
        - braided Y
        - 3-level Y
      - four-way only
        - partial cloverleaf
        - cloverleaf
        - diamond
    - rotary interchange
      - bridged rotary
      - turbine interchange
    - other types
- crossings without junction (no exchange of traffic)
  - with street or highway
    - street or highway
    - railway
    - pedestrian cross walk
    - cycle path
    - bridle path
  - at grade (no turns)
  - grade separation
    - overpass
    - underpass

TRAFFIC CONTROL DEVICES

- traffic sign
  - regulatory sign
    - stop sign
    - yield sign
    - no left turn
    - speed limit
    - parking restriction and limits
    - one-way street, etc.
    - permanent sign
    - temporary sign
    - painted
      - word
      - symbol
      - combination
    - reflectorization
    - illuminated
    - post mounted
    - overhead mounted
  - guide sign
    - route marker
    - destination sign
    - place identification
    - advance information, etc.
    - warning
      - railroad
        - advance warning
        - crossbuck
      - curve sign
- traffic marking
  - pavement marking
    - longitudinal marking
      - center line (division line)
      - lane line
      - barrier line
      - pavement edge
      - pavement width transition
      - painted
        - word markings
        - line symbol
        - etc.
      - button
      - other markings
        - insert
        - cone, etc.
    - transverse marking
      - clearance line
        - streetcar clearance
      - stop line
      - cross walk line
      - railroad approach
      - directional pavement marking
      - parking marking
        - parking line
        - curb marking
  - object marking
    - reflector marker
      - hazard marker
    - painted marker
  - posts
    - guide post
    - delineator
- traffic signal
  - railroad-highway grade-crossing signal
    - flashing-light signal
    - wig-wag
    - no-turn signal
  - flashing beacon
    - caution signal (yellow)
    - stop signal (red)
  - lane-direction traffic signal
  - traffic-control signal (signal control)
    - manually-controlled traffic signal
      - vehicle
      - special pedestrian signal
    - automatic control
      - with manual control
      - fixed-time signal (fixed-time control)
      - traffic-actuated signal
        - full traffic-actuated signal
        - semi-traffic-actuated signal
        - independent
        - coordinated
          - two phase
          - three phase
          - split phase
        - without speed control
        - traffic-actuated speed control
          - intersection
          - non-intersection
- barricades
  - portable
    - conventional
    - traffic cone
  - permanent
    - fixed
    - movable
      - railroad crossing gate
        - railroad-highway grade-crossing automatic gate
        - manually operated gate
      - drawbridge gate

SIGNAL INSTALLATION COMPONENTS

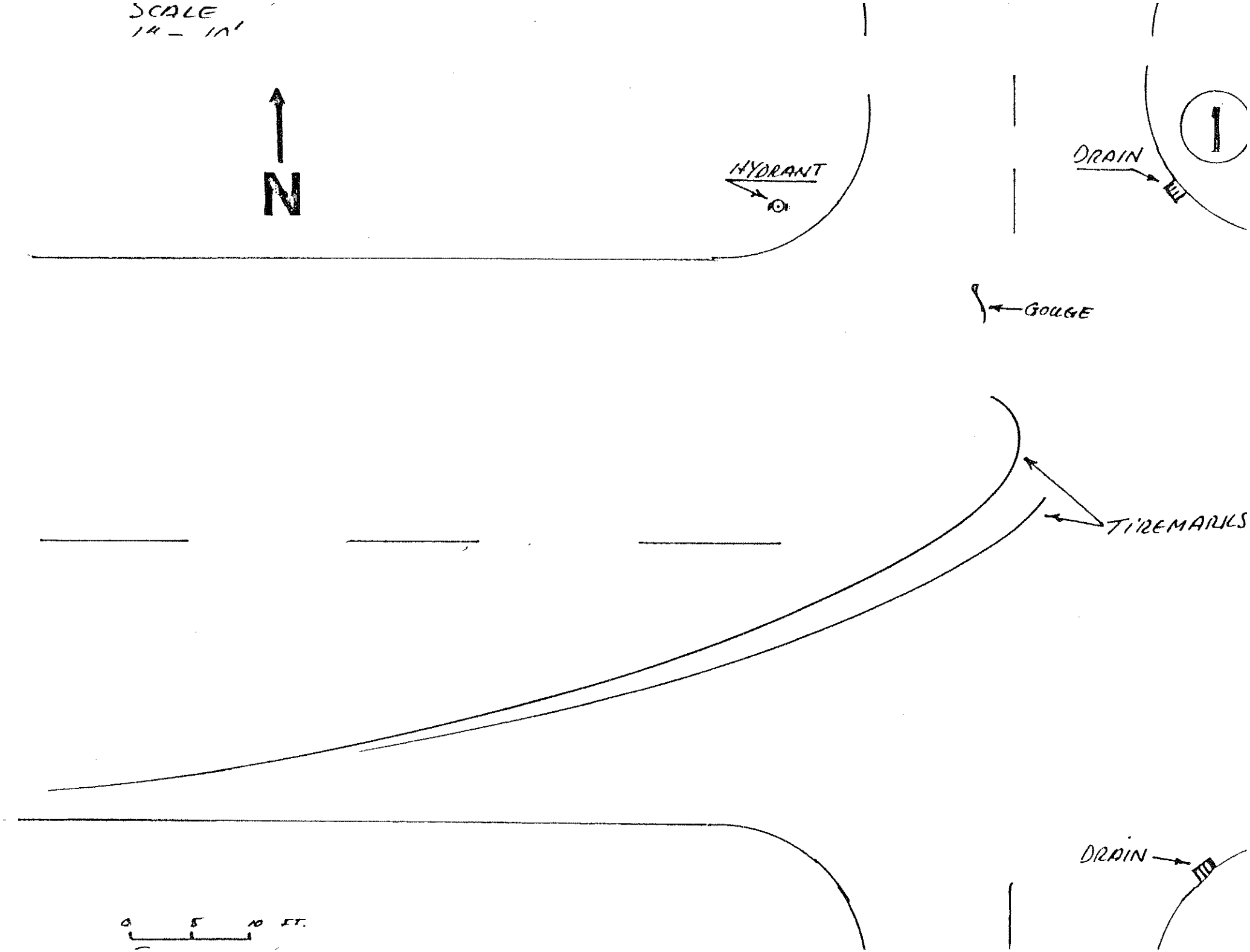


TRAFFIC CONTROL SIGNAL OPERATIONS

- cycle
  - traffic phase
    - vehicular phase
    - pedestrian phase
  - interval (interval sequence)
    - right of way interval
    - clearance interval
      - normal
        - vehicle
        - pedestrian
      - all red period
- signal indication
  - green alone
  - steady yellow alone
  - steady red alone
  - steady red with green arrow
  - walk
  - wait (don't walk)
  - flashing red
  - flashing yellow
- signal system
  - simultaneous system
  - alternate system
  - progressive system
    - simple progressive system
    - flexible progressive system
  - through band
    - in-time operation
    - out-of-step
    - recoordination
  - offset
- traffic-actuated operations
  - initial portion
  - extendible portion
    - unit extension
    - extension limit
  - minimum period
  - carry-over
  - passage period
  - red delay portion
  - demand (actuation)
  - vehicle warning indication

RIGHT OF WAYS

- limits
  - property line
  - building line
  - setback line
- road
  - roadway
    - parts
      - traveled way (composed of traffic lanes)
        - right lane
        - second lane
        - third lane
        - left lane
        - center lane
        - car-track lane
        - directional lane
          - right-turn lane
          - through-traffic lane
          - left-turn lane
        - unrestricted lane
        - curb-loading zone
          - passenger
          - freight
        - speed zone
        - no-passing zone
        - safety zone (workmen's)
        - decelerating zone
        - repair zone
        - work area
      - auxiliary lane (not separated)
        - stopping lane
          - parking lane
          - turn-out lane
        - speed-change lane (added turning lane)
          - acceleration lane
          - deceleration lane
          - median lane
      - curb
    - kinds
      - one-way
      - two-way
      - pavement
      - unpaved
      - laned
      - unlaned
    - parts
      - face
      - top
    - kinds
      - railed
      - vertical
      - sloped
      - lip
      - ~~mountable~~
      - barrier
  - shoulder
    - unguarded
      - unpaved
      - paved
    - guardrail
      - vehicle
      - pedestrian
      - ~~steel-plate beam~~
      - steel-plate
      - wire cable
      - timber
        - conventional
        - rustic guardrail
      - wall
- separator (traffic island in parts of crossings and junctions)
  - lane separator
  - directional separator
    - median
    - outer separator
- roadside
  - traffic areas
    - sidewalk
    - cycle path
    - bridle path
  - non-traffic areas
    - parking area
    - service area
    - park development
    - side strip
    - ditch
    - side slopes
    - other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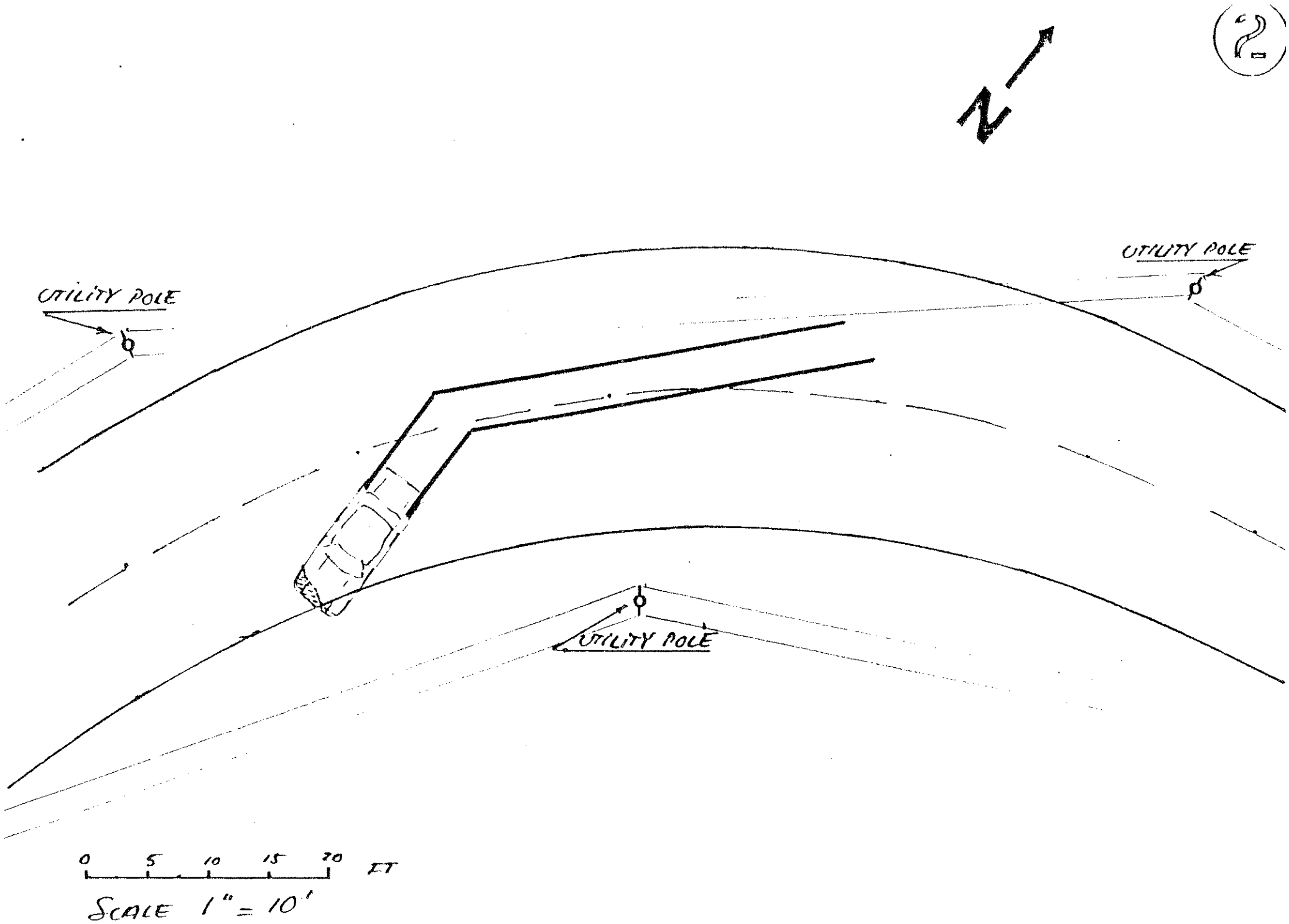
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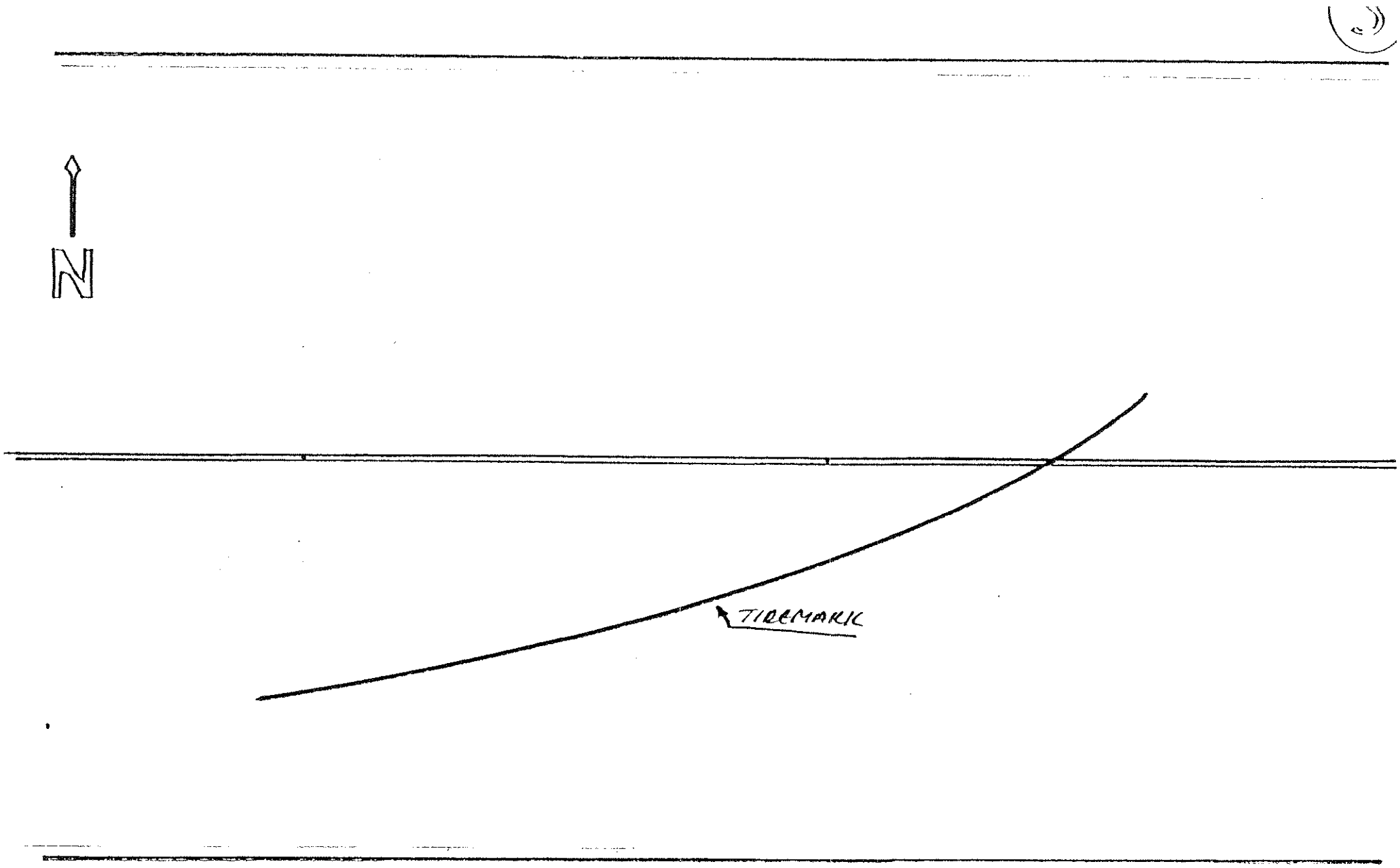

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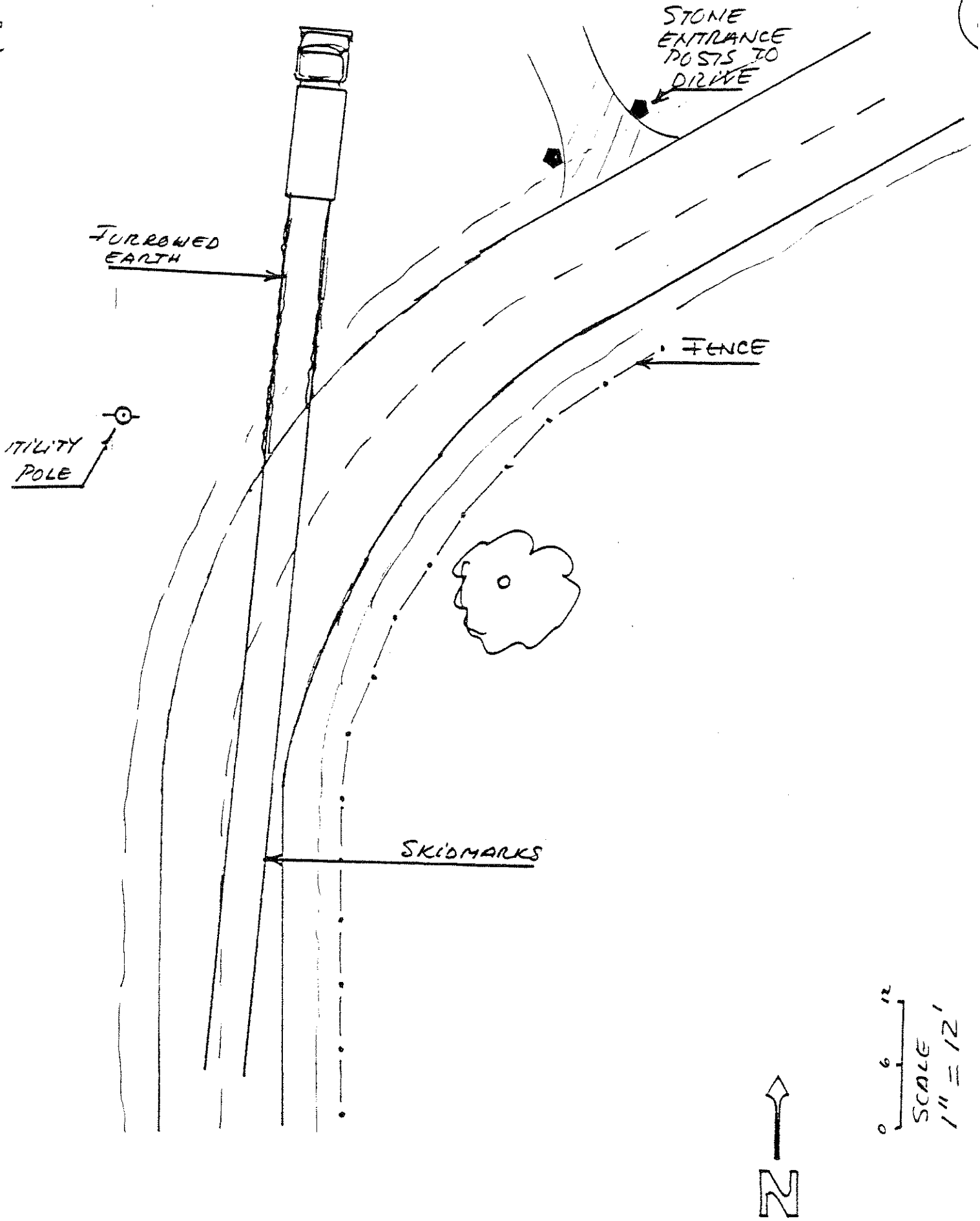
STONE ENTRANCE POSTS TO DRIVE
TURROWED EARTH
FENCE
UTILITY POLE
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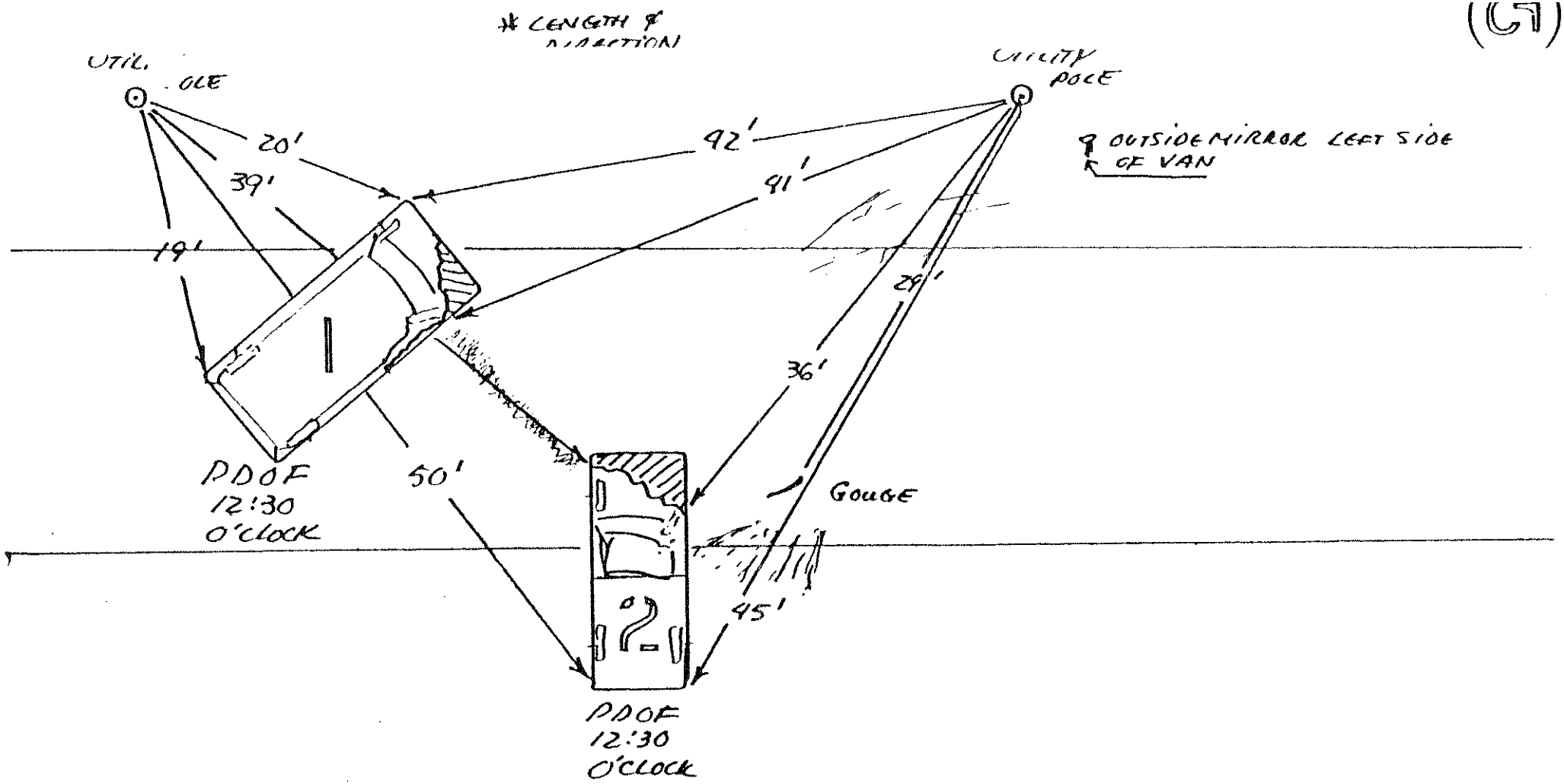


0 5 10 FT.
SCALE 1" = 10'

FROM PAR

MEASUREMENTS FROM:

* UTILITY POLES TO VEHICLE CORNERS
* UTILITY POLE TO GOUGE

FROM POLICE PHOTOS

* PDOF OF BOTH VEHICLES & GENERAL DAMAGE PATTERN
* ROADSIDE DEBRIS LOCATIONS

FROM SCENE EXAM'N

* CONFIRMATION OF POLICE GOUGE LOCATION & MEASUREMENTS
* TIRE SCUFF OF LOCKED (FROM DAMAGE) L.F. WHEEL OF VAN

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 7, Scene Inspection

Presented by: Peter Cooley

Description:

A presentation on the information available at an accident scene with techniques utilized to identify the scene elements necessary for accident reconstruction and to review methods of recording these data.

Topics:

Scene examination overview

Measurement Benchmarks

Measurement Equipment

Vehicle Final Rest Position

Roadway Examination

Road Characteristics

Traffic Controls

Off-road Features

Environmental Factors

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 7L, Scene Inspection Laboratory

Presented by: Peter Cooley

Description:

A laboratory session to accompany the presentation on Scene Inspection. Available will be several accident scenes for the student to observe, measure, and work.

Topics:

Observation of accident scenes

Relevant Roadway Features

Skid marks demonstration

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 8, Vehicle Inspection

Presented by: Peter Cooley

Description:

A presentation to identify the vehicle-related data elements necessary for accident reconstruction and to review the methods utilized in observing, interpreting, and recording these data.

Topics:

I. General Vehicle Factors
II. On-scene Information
III. Vehicle Identification
IV. Tow Operator Practices and Storage
V. Exterior Damage Assessment
VI. Vehicle Exterior Examinations
VII. Vehicle Impact Orientations
VIII. Vehicle Field Forms

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 8L, Vehicle Inspection Laboratory

Presented by: Peter Cooley

Description:

A laboratory session to accompany the presentation on Vehicle Inspection. Available will be several accident-involved vehicles for the student to observe, measure, and record.

Topics:

Observe and measure accident involved vehicles.

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 9, Data Documentation

Presented by: Peter Cooley

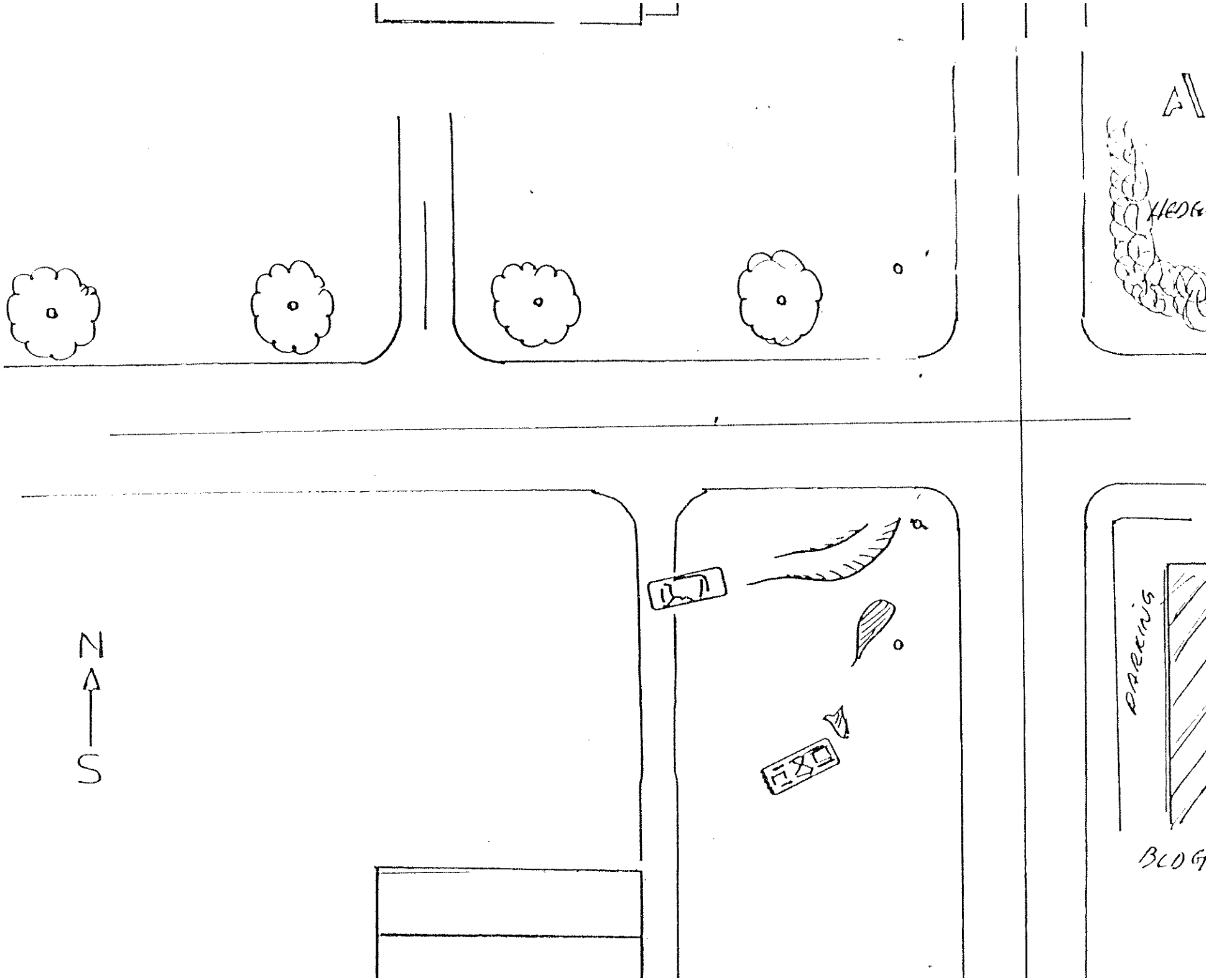
Description:

A presentation to review the techniques available to the investigator for observing and recording the necessary data for accident reconstruction. Techniques to be covered include measurement techniques, marking and mapping techniques, photographic aids, and interviewing techniques.

Topics:

Investigator as an Observer
Field Data Recording Methods
Symbols for Documenting Crash Scene
Field Measurements
Sketching and Drawing To Scale
Photography

NOTES


HEDGE
PARKING
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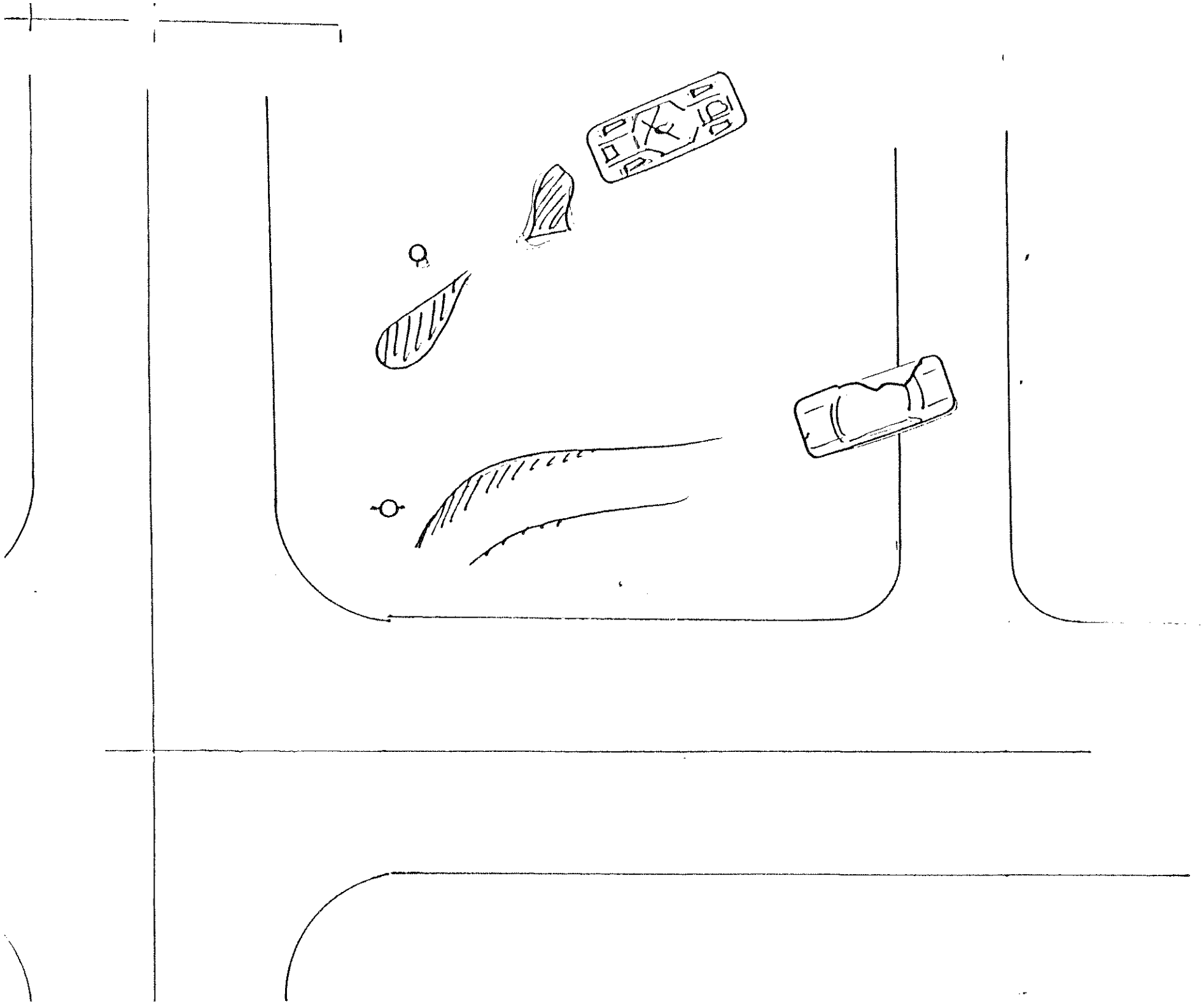
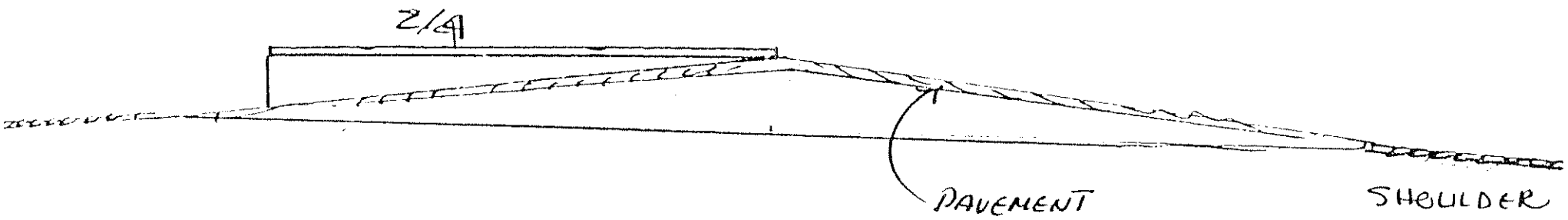
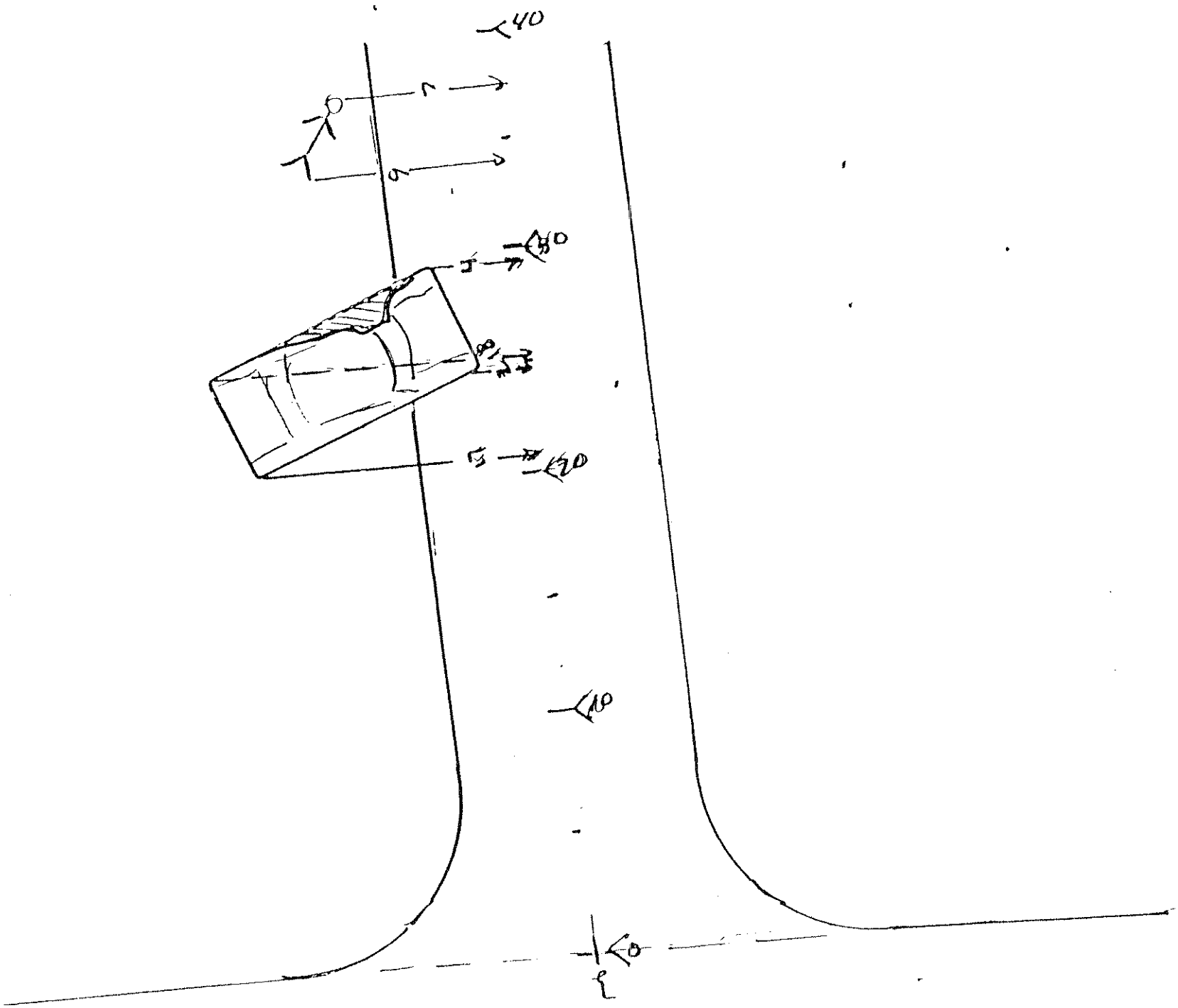




$$\frac{2''}{48''} = 4.2\%$$

SECTION A.A.


40
30
20
10
0

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 10, Classical Reconstruction

Presented by: Robert L. Hess

Description:

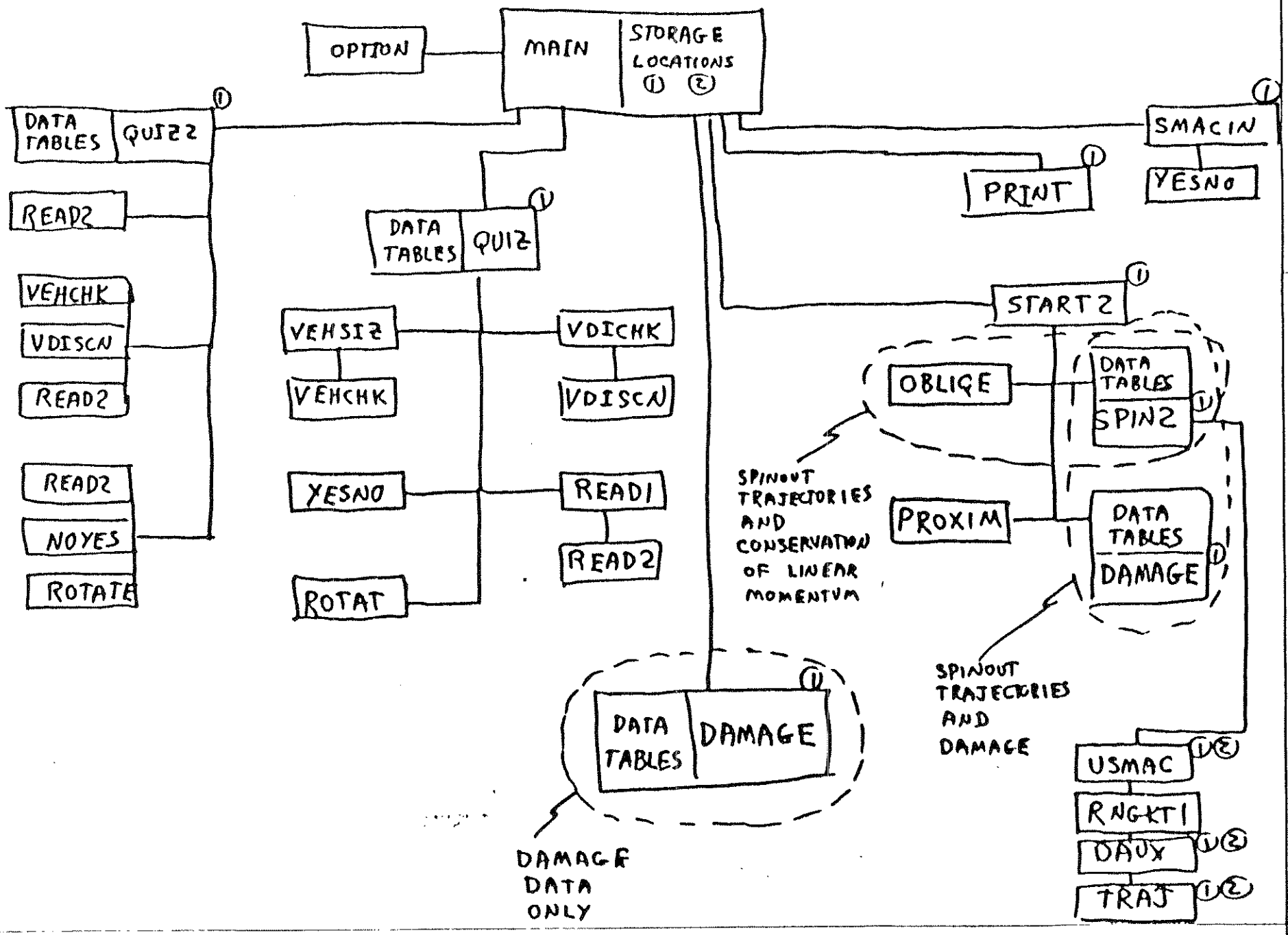
To provide an introduction to calculation used in accident reconstruction, to illustrate the steps necessary to perform a reconstruction, to provide an introduction to computer-assisted reconstruction calculations, and to illustrate the logic of various computer-oriented reconstruction programs and sub-programs.

Topics:

Kinematic Equations of Motion Applied to Vehicle Trajectory
Simplified Marguardt Equations
Step-by-Step Integration
Plots of Linear and Angular Velocity
Trajectory Simulation
Kinematic Equations Applied to Impact

NOTES

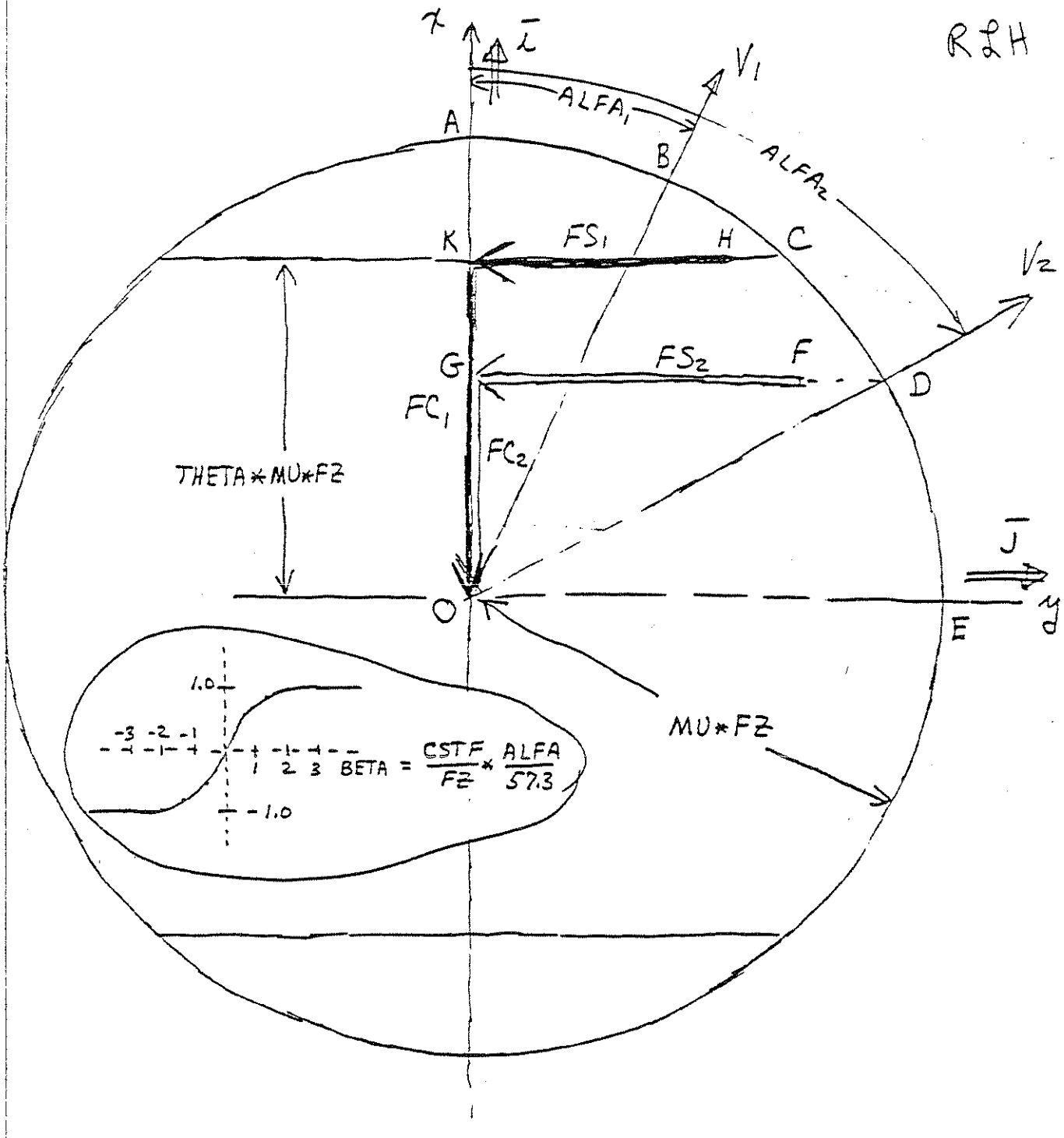
CRASH2
ORGANIZATION CHART



HSRI

Dec 5, 78

RCH



Shear Force on Tire

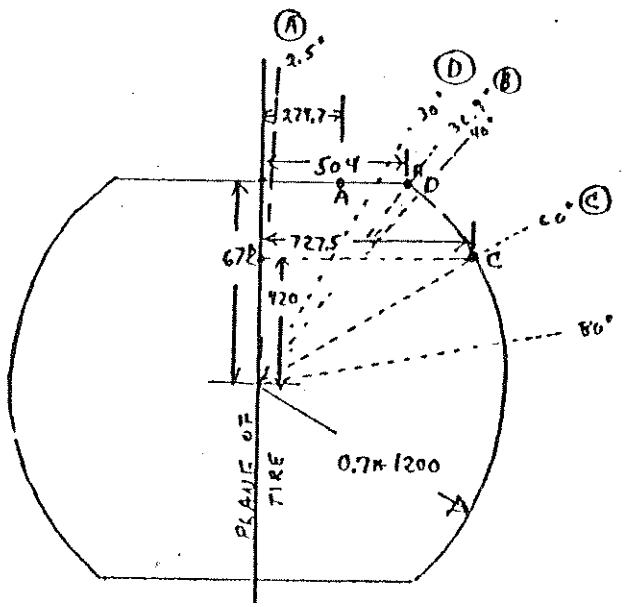
$$\bar{F}_1 = -FC_1\,\bar{i} - FS_1\,\bar{J}$$

or

$$\bar{F}_2 = -FC_2\,\bar{i} - FS_2\,\bar{J}$$

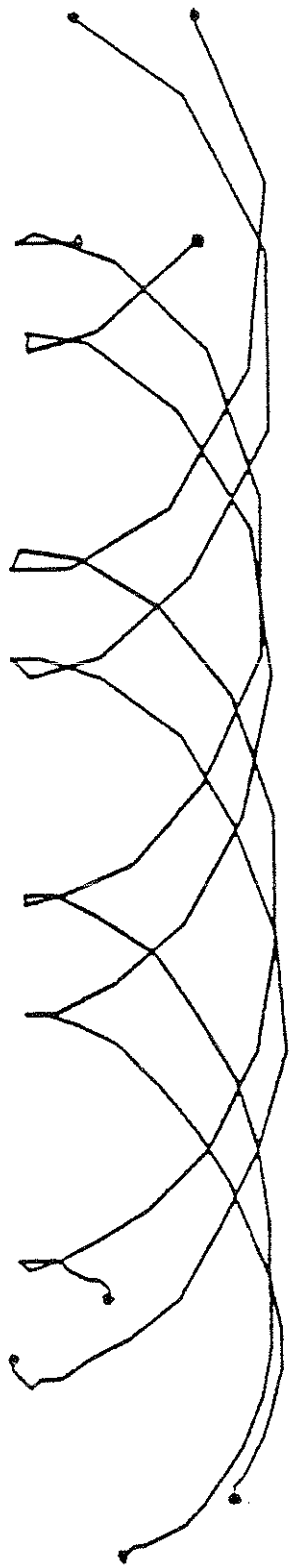
FZ = 1200 CSTF = -8000 $BETA = \left(\frac{CSTF}{FZ} * \frac{SLIP\ ANG}{57.3}\right)$ FCMAX = 0.8 * 0.7 * 1200

Friction	Lockup	Slip Ang	BETA	FC	FC/FCMAX	FS	FSMAX	FS/FSMAX
0.7	0.8	A 2.5	- 0.69	-672.00	1.00	-274.68	504.00	0.545
		5.0	- 1.39	-672.00	1.00	-425.40	504.00	0.844
		7.5	- 2.08	-672.00	1.00	-489.36	504.00	0.971
		10.0	- 2.77	-672.00	1.00	-503.77	504.00	0.999
		12.5	- 3.46	-672.00	1.00	-504.00	504.00	1.000
		20.0	- 5.54	-672.00	1.00	-504.00	504.00	1.000
		D 30.0	- 8.31	-672.00	1.00	-504.00	504.00	1.000
		35.0	- 9.70	-672.00	1.00	-504.00	504.00	1.000
		B 36.9	- 9.97	-672.00	1.00	-504.00	504.00	1.000
		37.0	-10.22	-670.85	0.998	-505.52	505.52	1.000
		40.0	-10.34	-643.48	0.96	-539.94	539.94	1.000
		C 60.0	-11.52	-420.00	0.63	-727.46	727.46	1.000
		80.0	-13.50	-145.87	0.22	-827.24	827.24	1.000
		90.0	-14.96	0.0	0.00	-840.00	840.00	1.000

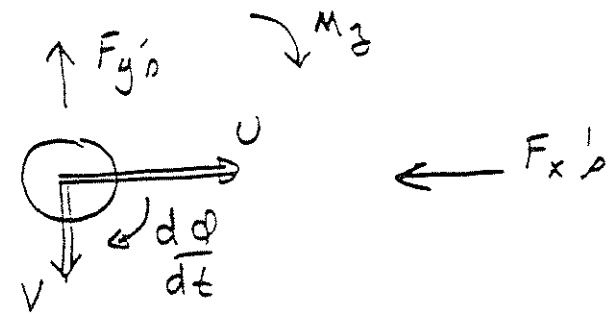


R 96 II

Receipt 510170, Plot 006 SAHP 15:49:53 05-10-79

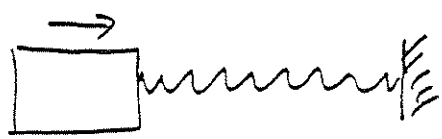


From (25)



$$\boxed{\begin{aligned} \frac{du}{dt} - v\frac{d\phi}{dt} &= \frac{-\Sigma F_x}{m} \\ \frac{dv}{dt} + u\frac{d\phi}{dt} &= \frac{-\Sigma F_y}{m} \\ \frac{d}{dt}\left(\frac{d\phi}{dt}\right) &= \frac{\Sigma M_z}{m\, k_{gy}^2} \end{aligned}}$$

in the case here, i.e.,



$$x = y = \phi = v = \frac{d\phi}{dt} = \Sigma F_y = \Sigma M_z = 0$$

$$\boxed{\frac{du}{dt} = -\frac{k}{m}x}$$

Illustrate a solution of the equation

$$\boxed{dV = -\frac{k}{m}x\,dt}$$

(note change in variable)

```
Real   K , M
K  = 1.0
M  = 1.0
X  = 0.0
V  = 10.0
WRITE ( 6, 100)  T, X , V
DELT = 0.1
 V  =  V - K*X*DELT/M
 X  =  X + V*DELT
WRITE (6,100)  T, X, V
T = T + DELT
if ( T. LE . 1.6)  GO TØ  10
FØRMAT ( 3 (1X, F8.2))
STØP
END
```

	T	X	V
nitial Conditions	0.0	0.0	10.0

$= 10.0 - 1 * 0 * 0.1/1 = 10.0$

$X = 0.0 + 10.0 * 0.1 = 1.0$

$T = 0.0 + 0.1 = 0.1$

0.1	1.0	10.0

$V = 10.0 - 1 * 1.0 * 0.1/1 = 9.9$

$X = 1.0 + 9.9 * 0.1 = 1.99$

$T = 0.1 + 0.1 = 0.2$

0.2	1.99	9.9

$V = 9.9 - 1 * 1.99 * 0.1/1 = 9.70$

$X = 1.99 + 9.70 * 0.1 = 2.96$

$T = 0.2 + 0.1 = 0.3$

0.3	2.96	9.70

$V = 9.70 - 1 * 2.96 * 0.1/1 = 9.404$

$X = 2.96 + 9.404 * 0.1 = 3.900$

$T = 0.3 + 0.1 = 0.4$

0.4	3.900	9.404

$V = 9.404 - 1 * 3.900 * 0.1/1 = 9.014$

$X = 3.900 + 9.014 * 0.1 = 4.801$

$T = 0.4 + 0.1 = 0.5$

0.5	4.801	9.014

T	X	V

$V = 9.014 - 1 * 4.801 * .1/1 = 8.534$

$X = 4.801 + 8.534 * 0.1 = 5.654$

$T = 0.5 + 0.1 = 0.6$

0.6	5.654	8.534

$V = 8.534 - 5.654 * .1 = 7.969$

$X = 5.654 + 7.969 * 0.1 = 6.451$

$T = 0.6 + 0.1$

0.7	6.451	7.969

$V = 7.969 - 6.451 * .1 = 7.324$

$X = 6.451 + 7.324 * 0.1 = 7.183$

$T = 0.7 + 0.1 = 0.8$

0.8	7.183	7.324

$V = 7.324 - 7.183 * .1 = 6.606$

$X = 7.183 + 6.606 * .1 = 7.844$

$T = 0.8 + 0.1 = 0.9$

0.9	7.844	6.606

$V = 6.606 - 7.844 * .1 = 5.822$

$X = 7.844 + 5.822 * .1 = 8.426$

$T = 0.9 + 0.1 = 1.0$

1.0	8.426	5.822

T X V

$V = 5.822 - 8.426 * .1 = 4.979$

$X = 8.426 + 4.979 * .1 = 8.934$

$T = 1.0 + 0.1 = 1.1$

1.1	8.934	4.979

$V = 4.979 - 8.934 * 0.1 = 4.086$

$X = 8.934 + 4.086 * .1 = 9.343$

$T = 1.1 + 0.1 = 1.2$

1.2	9.343	4.086

$V = 4.086 - 9.343 * .1 = 3.152$

$X = 9.343 + 3.152 * .1 = 9.658$

$T = 1.2 + 0.1 = 1.3$

1.3	9.658	3.152

$V = 3.152 - 9.658 * .1 = 2.186$

$X = 9.658 + 2.186 * .1 = 9.877$

$T = 1.3 + 0.1 = 1.4$

1.4	9.877	2.186

T X V

$V = 2.186 - 9.877 * 0.1 = 1.198$

$X = 9.877 + 1.198 * 0.1 = 9.997$

$T = 1.4 + 0.1 = 1.5$

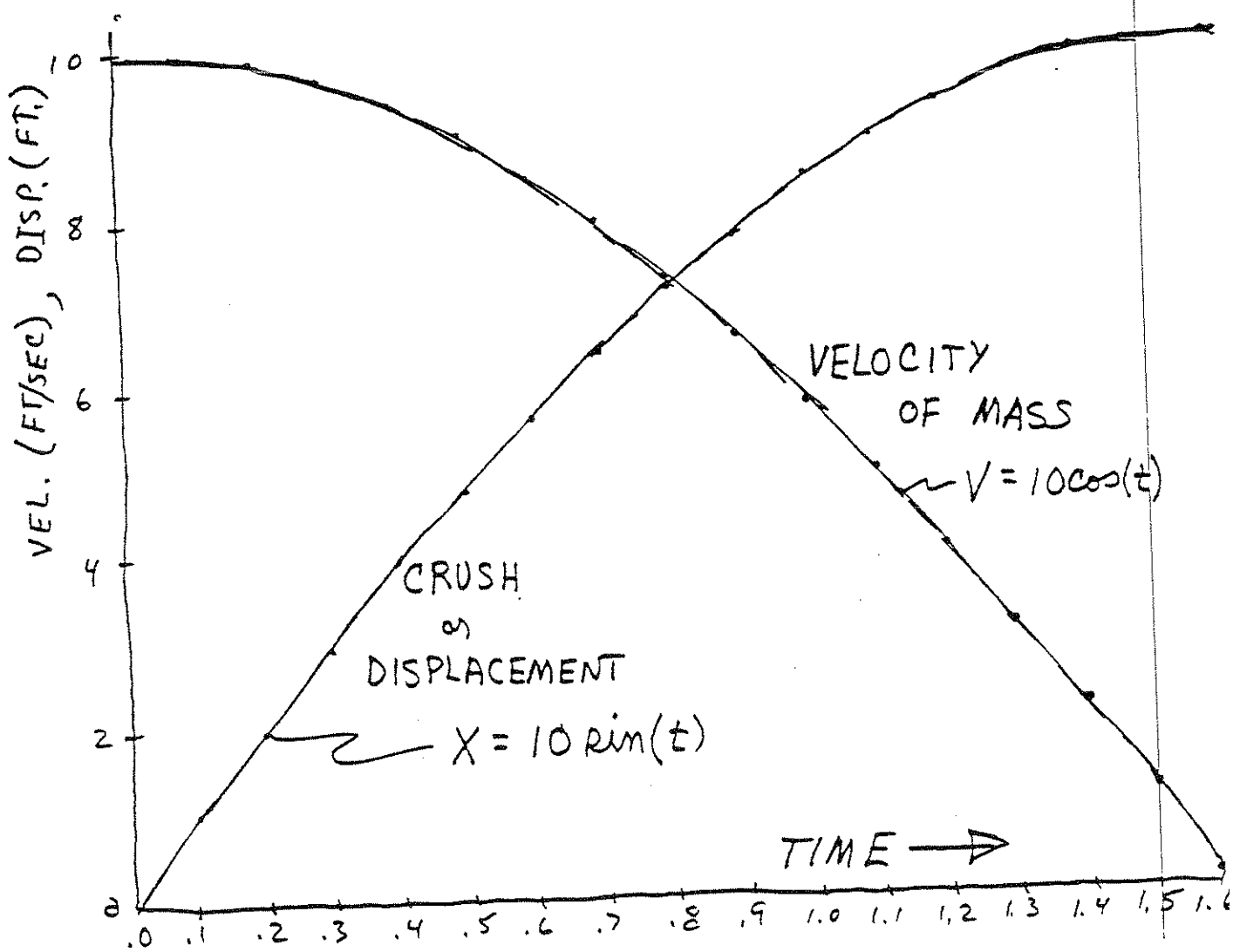
1.5	9.997	1.198

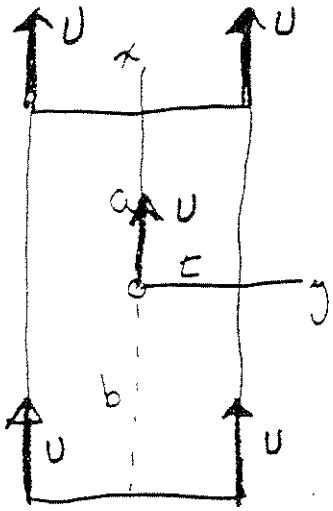
$V = 1.198 - 9.997 * 0.1 = 0.198$

$X = 9.997 + .198 * .1 = 10.017$

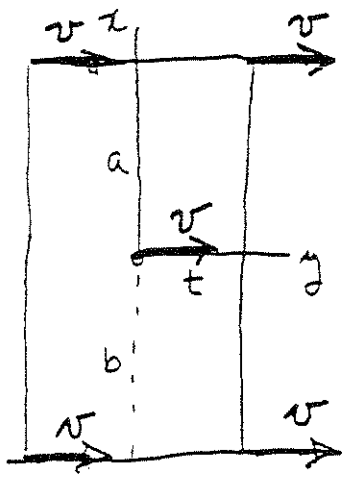
$T = 1.5 + 0.1 = 0.6$

1.6	10.017	0.198

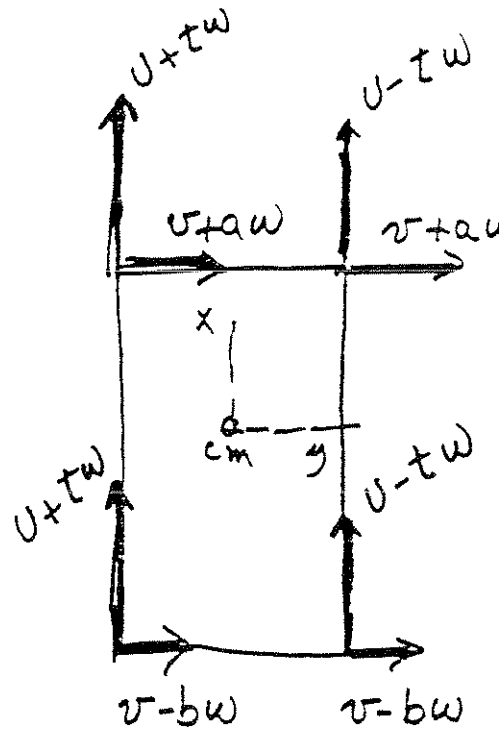




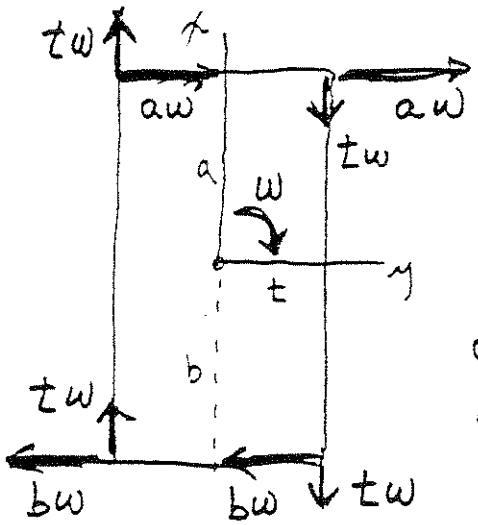
longitudinal velocity of c.m. contribution to wheel velocities



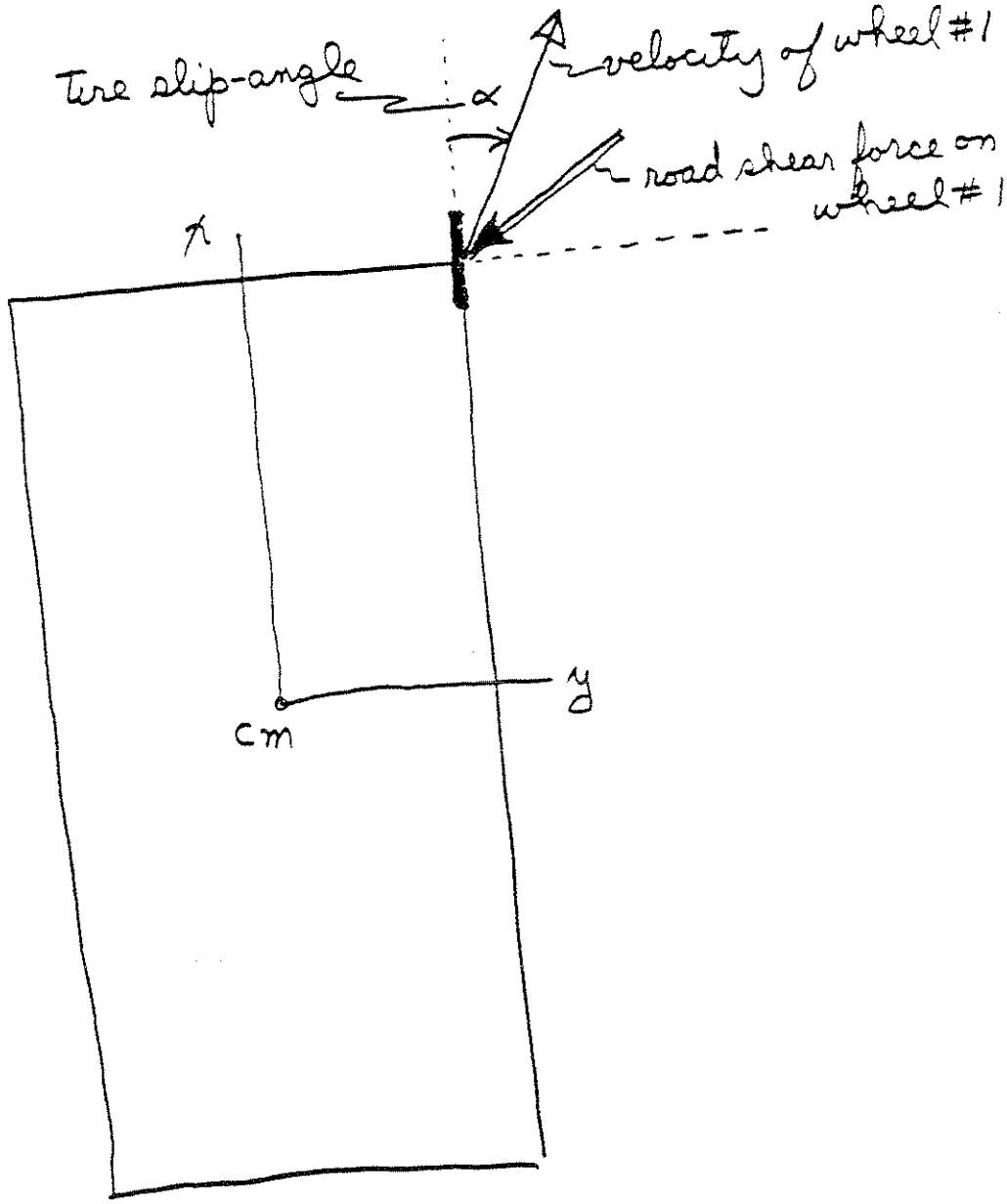
lateral velocity of cm contribution to wheel velocities

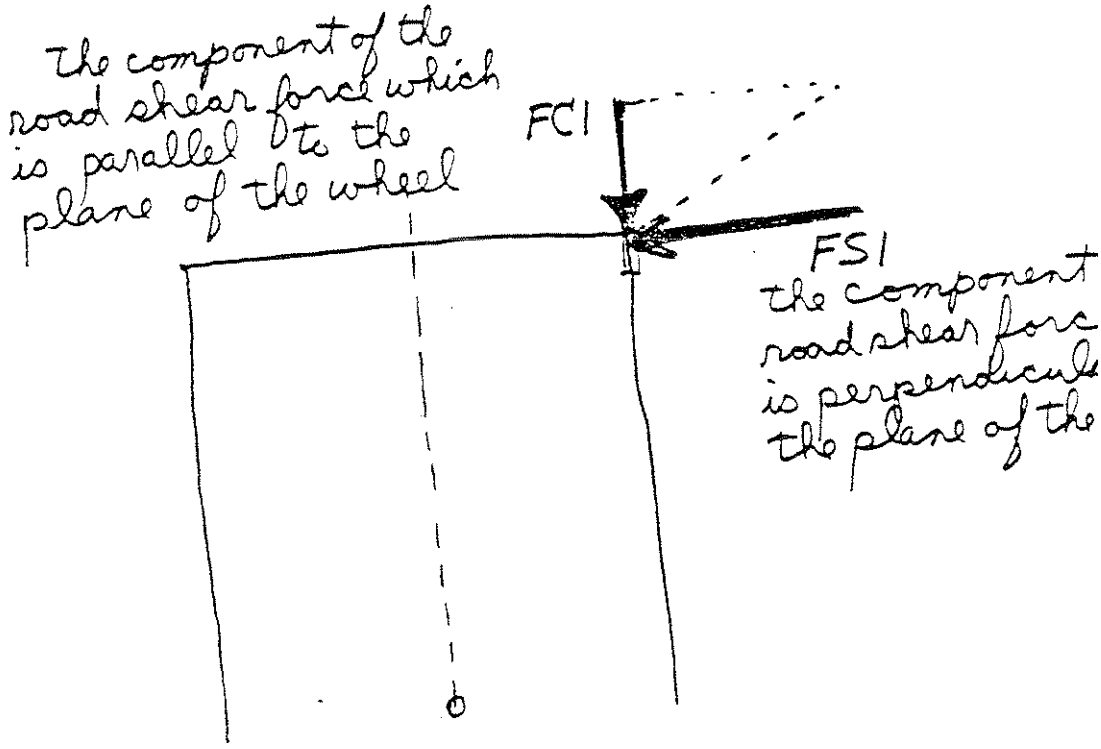


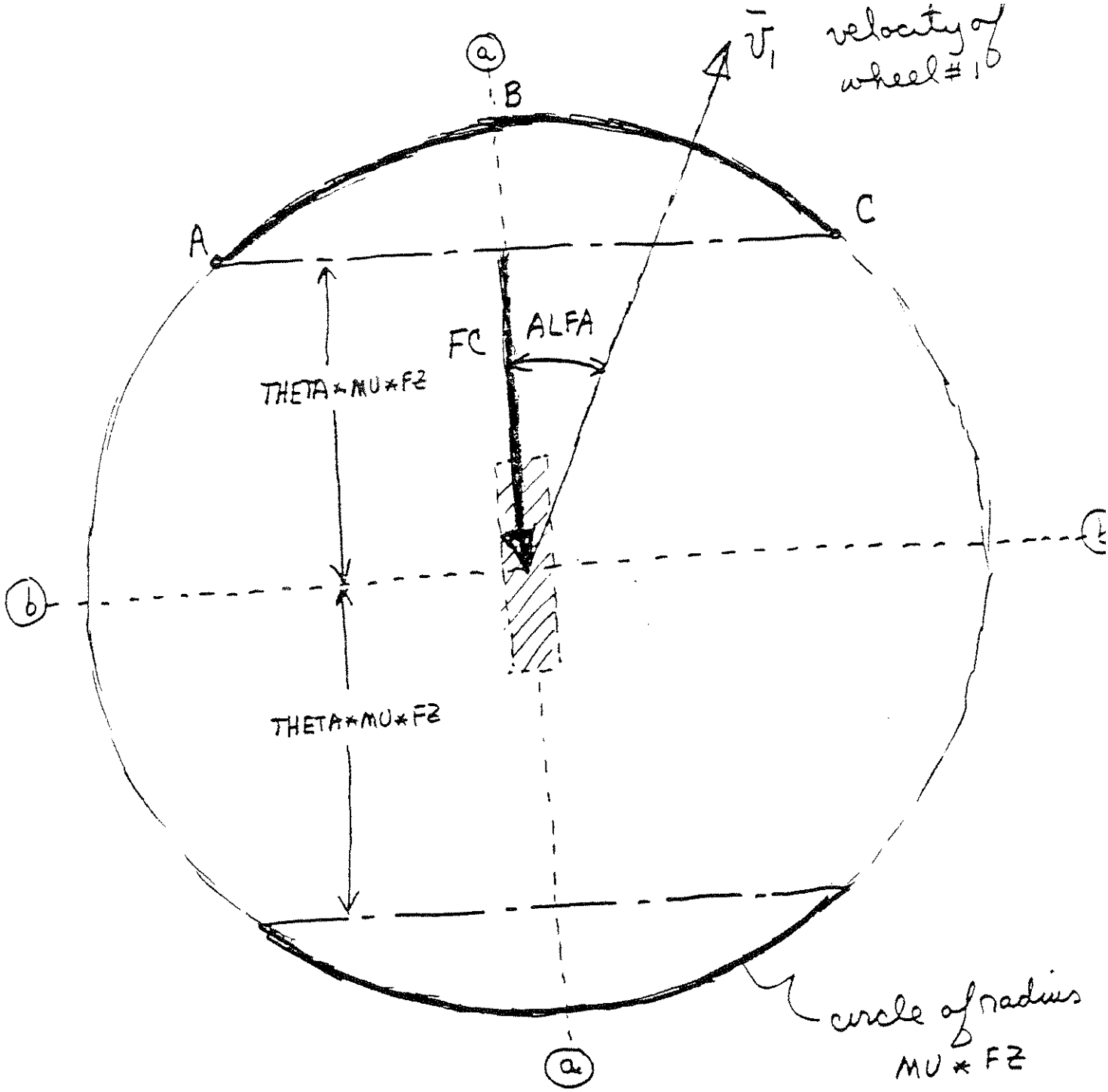
total longitudinal and lateral velocity of each wheel



angular velocity of body contribution to wheel velocities


Tire slip-angle
α
velocity of wheel #1
road shear force on wheel #1
x
y
cm


The component of the road shear force which is parallel to the plane of the wheel
FC1
FS1
the component road shear forc is perpendicul the plane of the

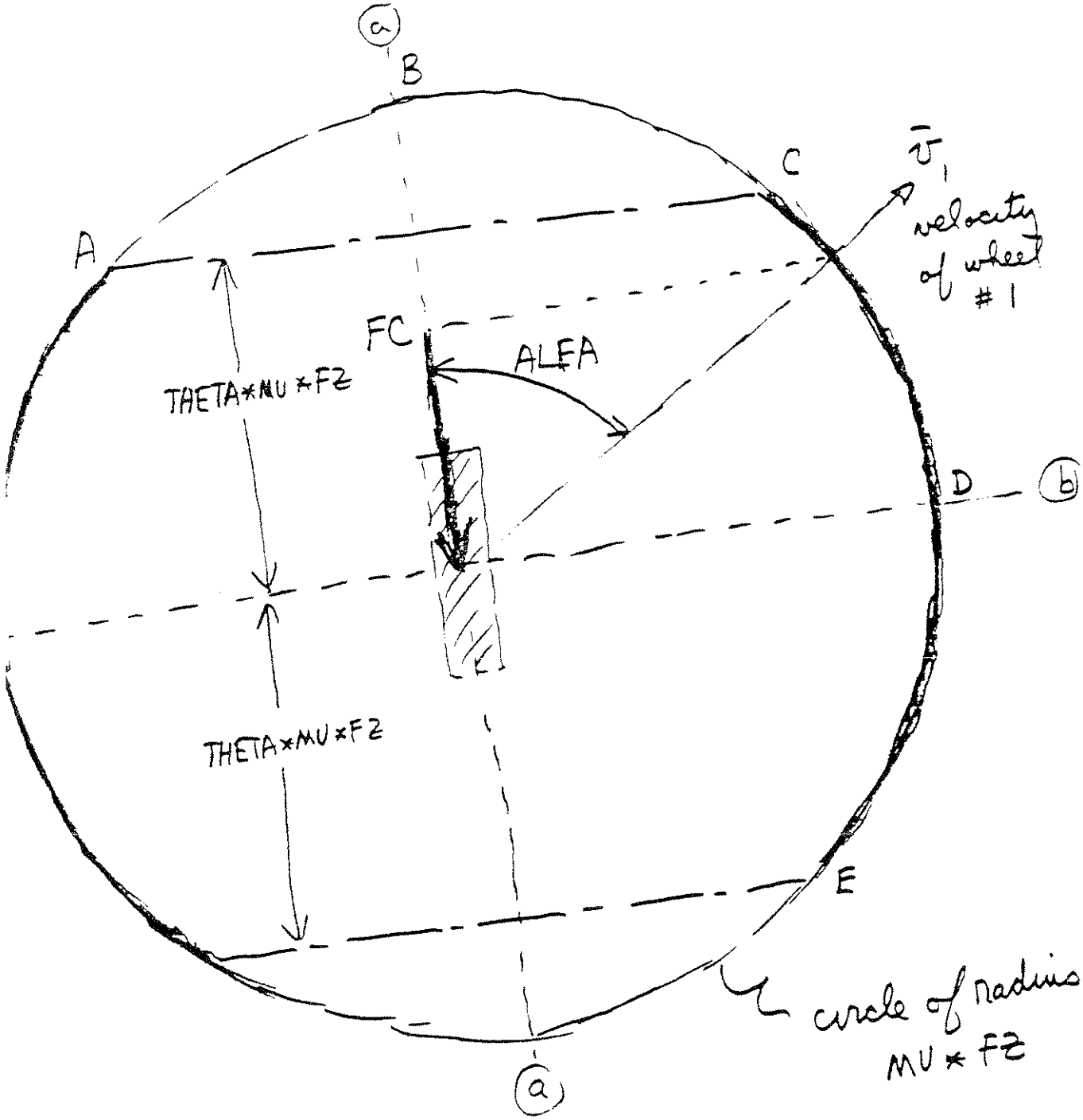


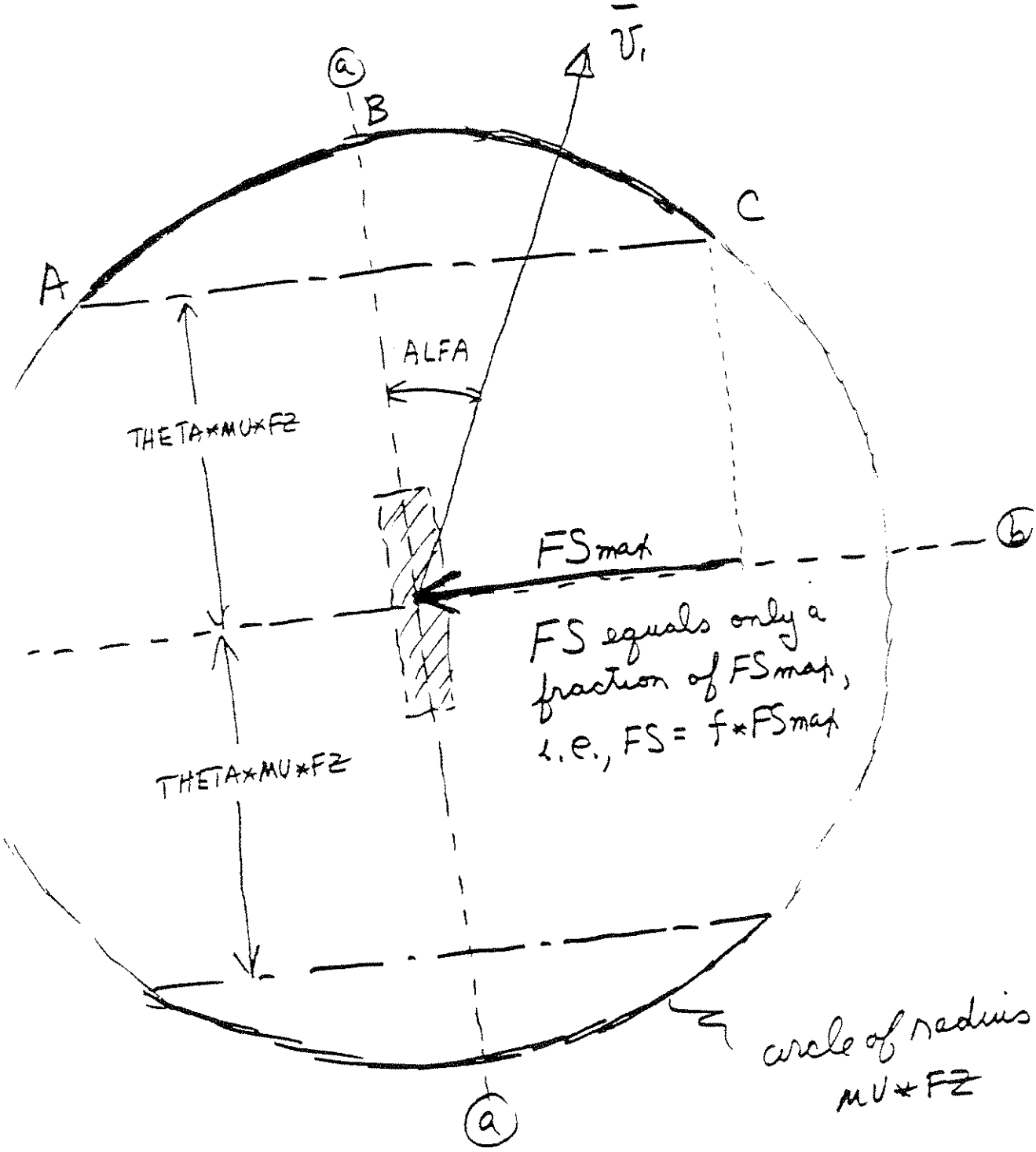
MU = coef. of friction
FZ = vertical load

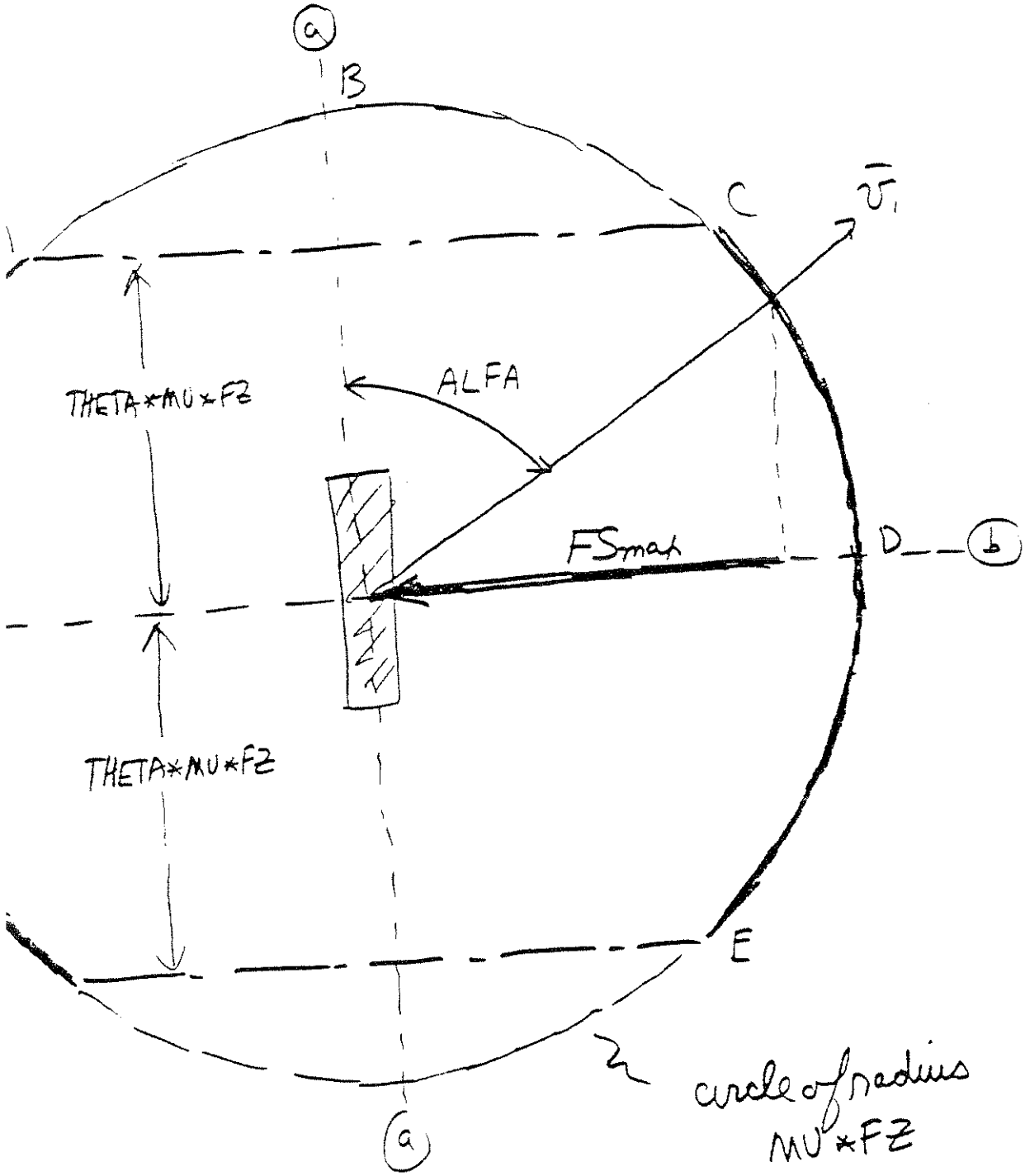
an overhead (plan) view of tire

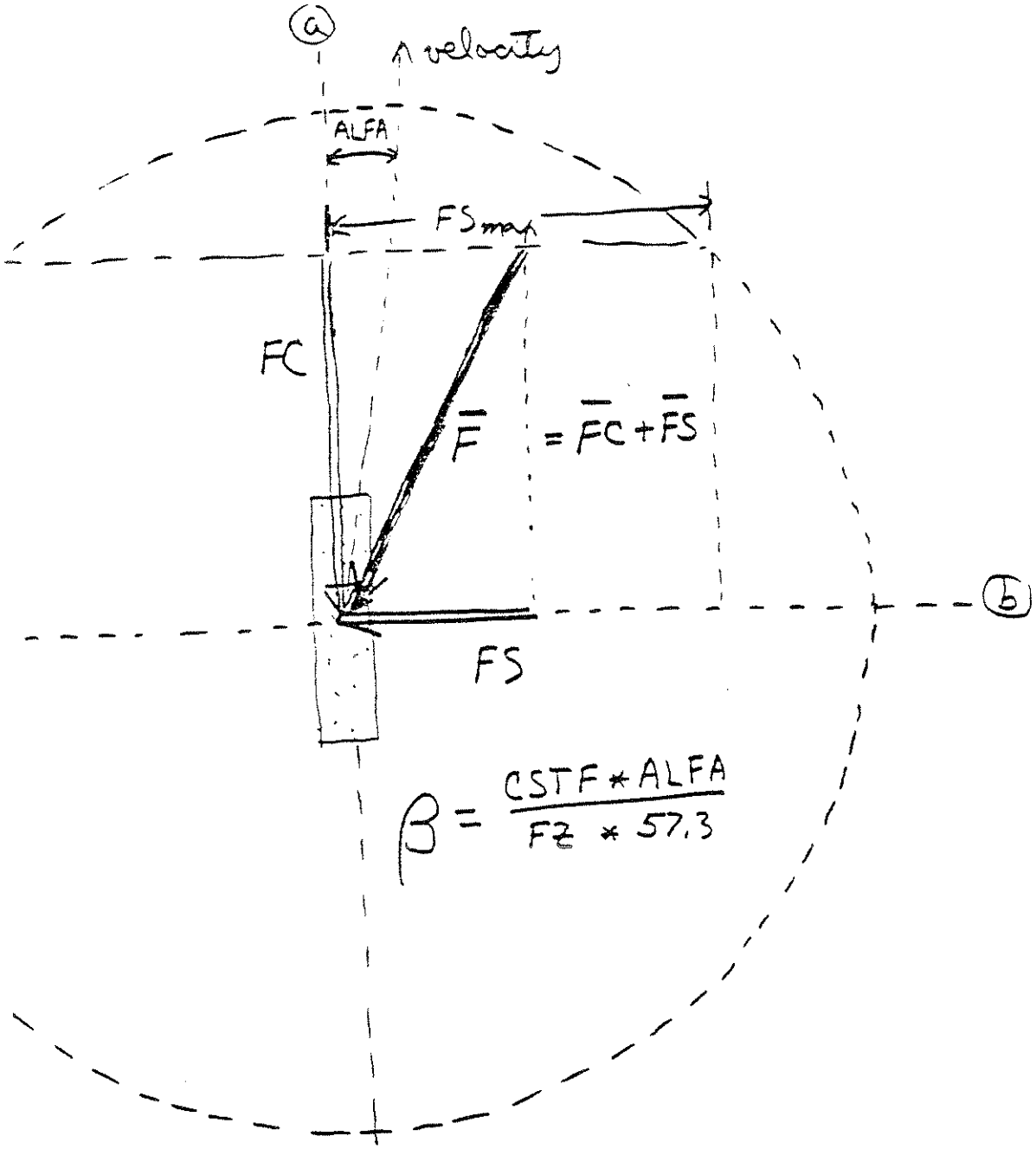
a — a line parallel to wheel

b — b line perpendicular to wheel


a
B
C
$\bar{v}_1$
velocity of wheel #1
A
FC
ALFA
THETA*MU*FZ
D
b
THETA*MU*FZ
E
circle of radius MU*FZ
a


$\bar{v}_1$
a
B
C
A
ALFA
THETA*MU*FZ
FSmax
b
FS equals only a fraction of FSmax, i.e., FS = f*FSmax
THETA*MU*FZ
circle of radius MU*FZ
a


a
B
C
$\bar{v}_1$
ALFA
THETA*MU*FZ
FSmax
D
b
THETA*MU*FZ
E
a
circle of radius
MU*FZ



$$\beta = \frac{CSTF * ALFA}{FZ * 57.3}$$

$$S_{max} = \sqrt{(MU * FZ)^2 - (FC)^2}$$

$$f = \left(\frac{\beta^3}{27.0} - \frac{\beta|\beta|}{3.0} + \beta \right)$$

$$S = f * FS_{max}$$

RLH

MARQUARD , FREE WHEELING

```
RLH005(1,50)
      U=440.
      V=0.
      CHID=4.365
      R=51.3
      G=386.4
      FMU=0.7
      KGYSQ=3324.
      DELTAT=0.005
      PRNT=0.
      T=0.
      VTOT=SQRT(U*U+V*V)
      WRITE(6,101) T,CHID,VTOT
   10 VFY=V+R*CHID
      VRY=V-R*CHID
      IF(VFY.GT.10.) VFF=0.5*G*FMU
      IF(VFY.LT.-10.) VFF=-0.5*G*FMU
      IF(VFY.GE.-10..AND.VFY.LE.10.) VFF=0.05*G*FMU*VFY
      IF(VRY.GT.10.) VFR=0.5*G*FMU
      IF(VRY.LT.-10.) VFR=-0.5*G*FMU
      IF(VRY.GE.-10..AND.VRY.LE.10.) VFR=0.05*G*FMU*VRY
      UD=V*CHID
      VD=-U*CHID-(VFF+VFR)
      CHIDD=(VFR-VFF)*R/KGYSQ
      U=U+UD*DELTAT
      V=V+VD*DELTAT
      CHID=CHID+CHIDD*DELTAT
      VTOT=SQRT(U*U+V*V)
      DELTAT=0.01
      T=T+DELTAT
      PRNT=PRNT+DELTAT
      IF(PRNT-.098) 12,11,11
   11 PRNT=0.
      WRITE(6,101) T,CHID,VTOT
   12 IF(T-4.) 10,20,20
  101 FORMAT(1X,3E10.3)
   20 STOP
      END
END OF FILE
```

initial velocity conditions

— friction

— rad. of gyration sq'd.

initializing Δt, printcontrol, Time

— Total velocity

velocity of front and rear tire

DIFFERENTIAL EQUATIONS OF MOTION

accel (longitudinal) = 0

accel (lateral) = − (VFF+VFR)

accel (yawing) = $\frac{(VFR-VFF)*R}{KGYSQ}$

— update velocities at each time step

— total velocity

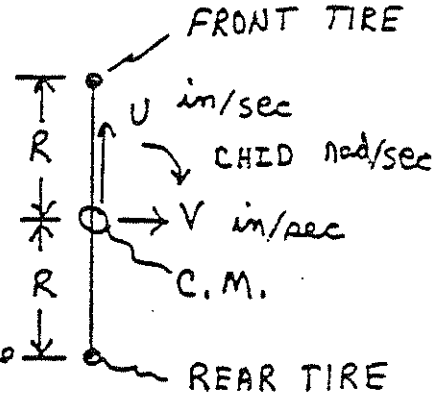
— new time step established on 1st pass

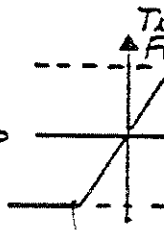
— update time

output and recycle control

prints each 0.1 seconds

runs for 4.0 seconds





RLH

```
:R *FTN PAR=I=RLH005(1,500)
 :EXECUTION BEGINS
 :NO ERRORS IN MAIN
:EXECUTION TERMINATED
:R -LOAD 6=RLH005(200)
:EXECUTION BEGINS
:EXECUTION TERMINATED
:L RLH005(200)
```

Compiling of RLH(005) Creates machine instructions in file - ie

Run program and write output into the file RLH005 starting at line #200

	TIME	CHID	VTOT
200	0.0	0.436E+01	0.440E+03
201	0.100E+00	0.397E+01	0.444E+03
202	0.200E+00	0.387E+01	0.435E+03
203	0.300E+00	0.387E+01	0.416E+03
204	0.400E+00	0.387E+01	0.393E+03
205	0.500E+00	0.387E+01	0.369E+03
206	0.600E+00	0.387E+01	0.349E+03
207	0.700E+00	0.369E+01	0.341E+03
208	0.800E+00	0.328E+01	0.343E+03
209	0.900E+00	0.286E+01	0.345E+03
210	0.100E+01	0.257E+01	0.343E+03
211	0.110E+01	0.257E+01	0.329E+03
212	0.120E+01	0.257E+01	0.311E+03
213	0.130E+01	0.257E+01	0.290E+03
214	0.140E+01	0.257E+01	0.266E+03
215	0.150E+01	0.257E+01	0.240E+03
216	0.160E+01	0.257E+01	0.214E+03
217	0.170E+01	0.257E+01	0.190E+03
218	0.180E+01	0.244E+01	0.175E+03
219	0.190E+01	0.203E+01	0.176E+03
220	0.200E+01	0.161E+01	0.176E+03
221	0.210E+01	0.119E+01	0.176E+03
222	0.220E+01	0.776E+00	0.176E+03
223	0.230E+01	0.359E+00	0.176E+03
224	0.240E+01	0.452E-01	0.176E+03
225	0.250E+01	0.406E-02	0.176E+03
226	0.260E+01	0.365E-03	0.176E+03
227	0.270E+01	0.328E-04	0.176E+03
228	0.280E+01	0.294E-05	0.176E+03
229	0.290E+01	0.265E-06	0.176E+03
230	0.300E+01	0.238E-07	0.176E+03
231	0.310E+01	0.213E-08	0.176E+03
232	0.320E+01	0.192E-09	0.176E+03
233	0.330E+01	0.172E-10	0.176E+03
234	0.340E+01	0.155E-11	0.176E+03
235	0.350E+01	0.139E-12	0.176E+03
236	0.360E+01	0.125E-13	0.176E+03
237	0.370E+01	0.112E-14	0.176E+03
238	0.380E+01	0.101E-15	0.176E+03
239	0.390E+01	0.905E-17	0.176E+03
240	0.400E+01	0.813E-18	0.176E+03

END OF FILE

RIH

A PROGRAM TO READ A DATA FILE AND PLOT ON THE TERMINAL A TIME HISTORY OF A VARIABLE.

```
07
    /COMPILE
          CHARACTER G*10
          DIMENSION G(3)
(Note#1)  SCALE=0.125
          DATA G //(1X,F5.3,',',1X,E10.3,T',' '/
          WRITE(6,10)
       10 FORMAT(' ',T16,'---+',19('-'),'+',19('-'),'+',19('-'),'+')
        5 READ(5,15,END=20) A,B
          NB=ABS(B)*SCALE+18
          WRITE(G(3),12) NB,'*'/)'
          WRITE(6,G) A,B
          GO TO 5
(Note#2)15 FORMAT(1X,E10.3,10X,E10.3)
       12 FORMAT(I3,A5)
       20 CONTINUE
          WRITE(6,10)
          RETURN
          END
    /DATA
    /STOP
FILE
```

Notes

#1 Scale * max. variable should be less than 60

#2 Format must be adjusted to fit data file being read

I-3

RLH

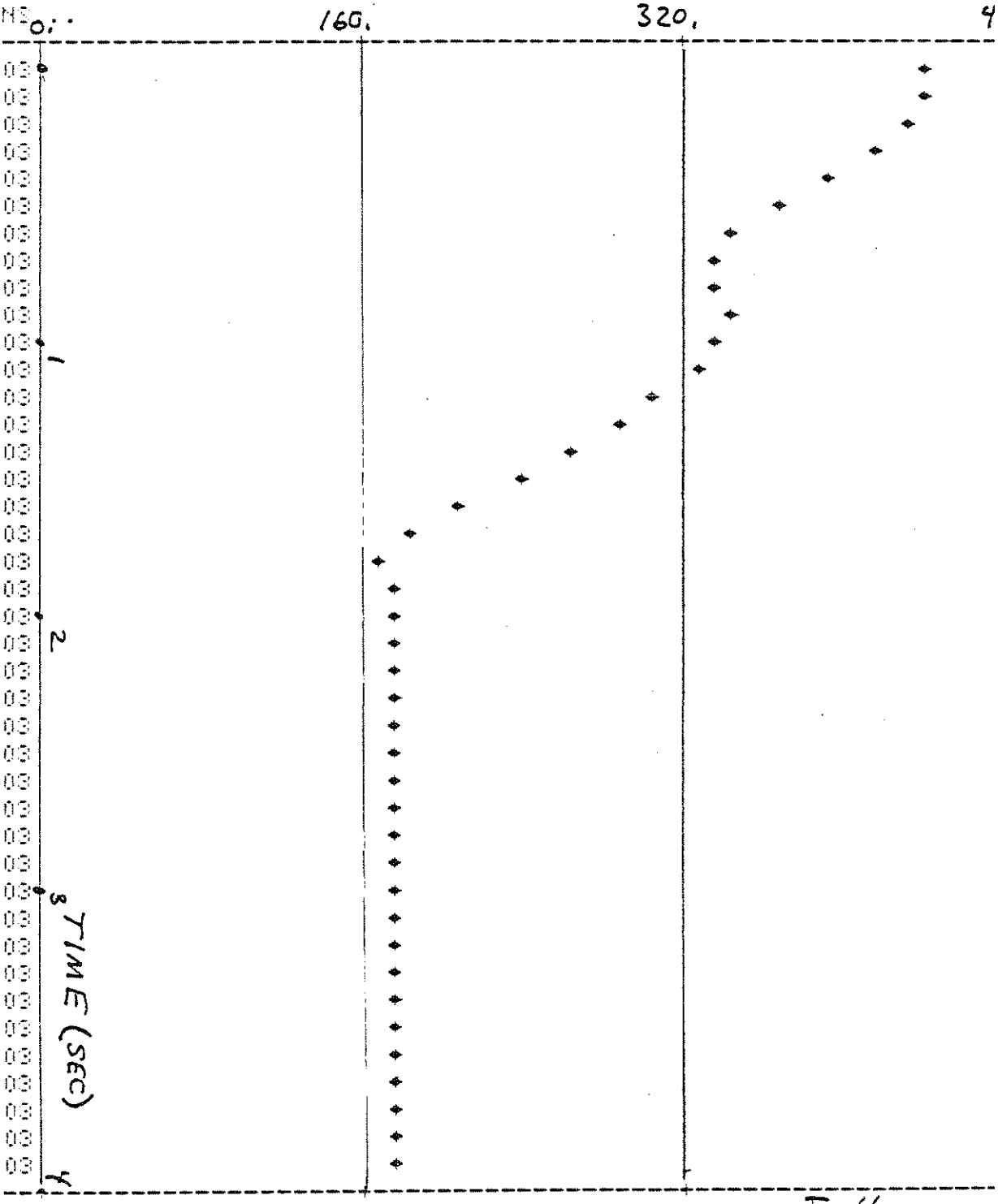
Running a Terminal plot
(Scale = 1/8)

```
*WATFIV SCARDS=PLH007 5=PLH005(200)
EXECUTION BEGINS
*COMPILE
```

LINEAR VELOCITY (IN/SEC)

```
'80.EXECUTION BEGINS
```

TIME	V TOT
0.000	0.440E 03
0.100	0.444E 03
0.200	0.435E 03
0.300	0.416E 03
0.400	0.393E 03
0.500	0.369E 03
0.600	0.349E 03
0.700	0.341E 03
0.800	0.343E 03
0.900	0.345E 03
1.000	0.343E 03
1.100	0.329E 03
1.200	0.311E 03
1.300	0.290E 03
1.400	0.266E 03
1.500	0.240E 03
1.600	0.214E 03
1.700	0.190E 03
300	0.175E 03
1.900	0.176E 03
2.000	0.176E 03
2.100	0.176E 03
2.200	0.176E 03
2.300	0.176E 03
2.400	0.176E 03
2.500	0.176E 03
2.600	0.176E 03
2.700	0.176E 03
2.800	0.176E 03
2.900	0.176E 03
3.000	0.176E 03
3.100	0.176E 03
3.200	0.176E 03
3.300	0.176E 03
3.400	0.176E 03
3.500	0.176E 03
3.600	0.176E 03
3.700	0.176E 03
3.800	0.176E 03
3.900	0.176E 03
4.000	0.176E 03



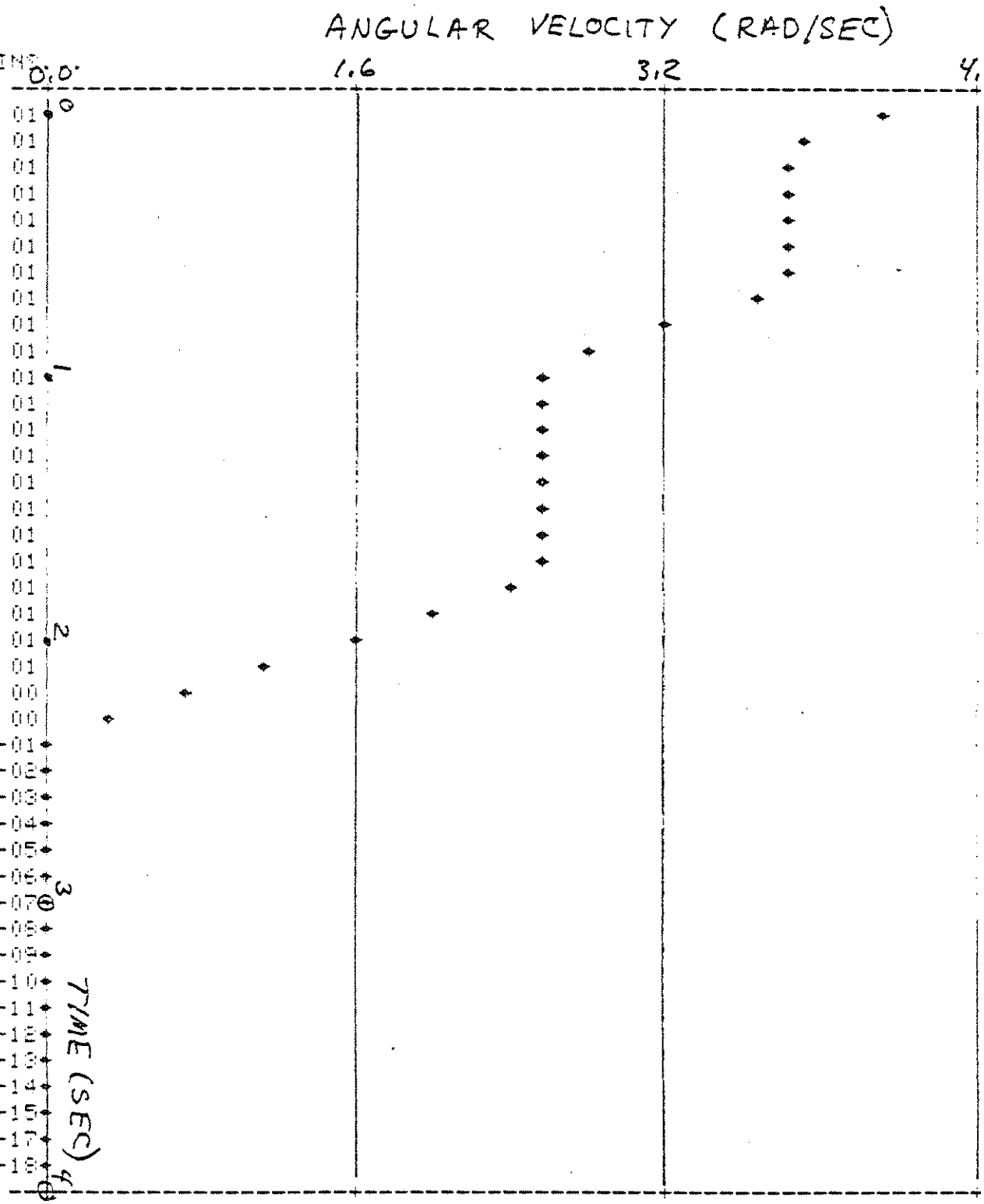

```
ECUTION TERMINATED
```

```
 16 :0.125:12.5:
   16              SCALE=12.5
 60 :10X,::
   60        15  FORMAT(1X,E10.3,E10.3)
ITS
```

Edit to Scale = 100/8
Edit to new format
RLH

```
 *WATFIV SCARDS=RLH007 5=RLH005(200)
EXECUTION BEGINS
COMPILE
EXECUTION BEGINS
```

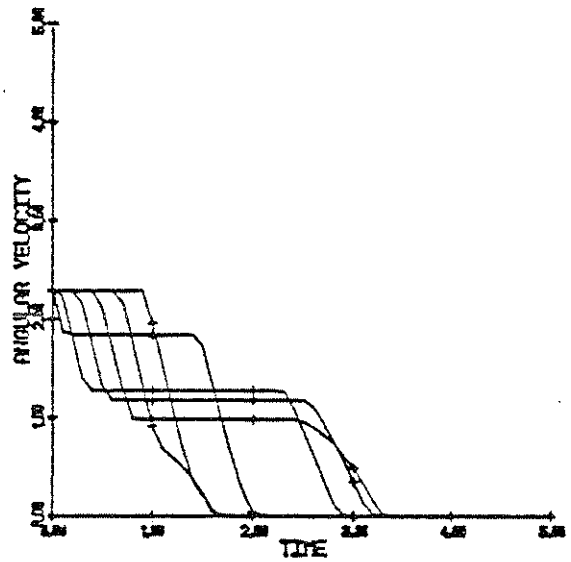
TIME	CHID
.000	0.436E 01
.100	0.397E 01
.200	0.387E 01
.300	0.387E 01
.400	0.387E 01
.500	0.387E 01
.600	0.387E 01
.700	0.369E 01
.800	0.328E 01
.900	0.286E 01
.000	0.257E 01
.100	0.257E 01
.200	0.257E 01
.300	0.257E 01
.400	0.257E 01
.500	0.257E 01
.600	0.257E 01
.700	0.257E 01
.800	0.244E 01
.900	0.203E 01
.000	0.161E 01
.100	0.119E 01
.200	0.776E 00
.300	0.359E 00
.400	0.452E-01
.500	0.406E-02
.600	0.365E-03
.700	0.328E-04
.800	0.294E-05
.900	0.265E-06
.000	0.238E-07
.100	0.213E-08
.200	0.192E-09
.300	0.172E-10
.400	0.155E-11
.500	0.139E-12
.600	0.125E-13
.700	0.112E-14
.800	0.101E-15
.900	0.905E-17
.000	0.813E-18

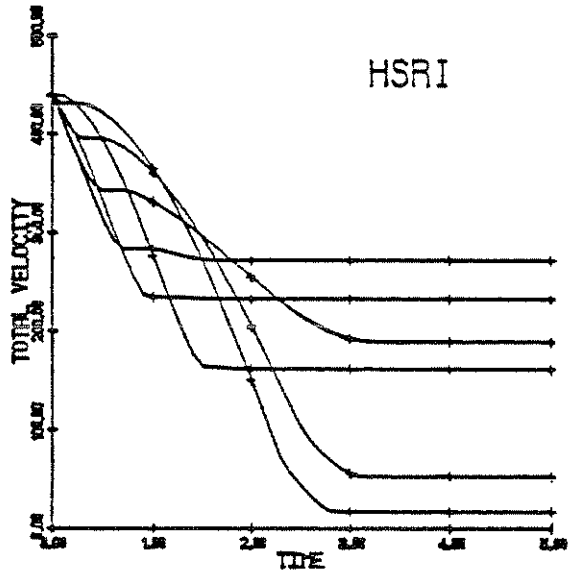


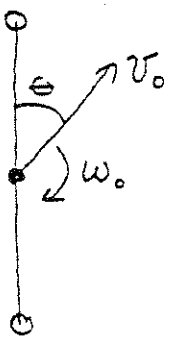

```
EXECUTION TERMINATED
```

RLH

Receipt 507960, Plot 013 SAHP 08:23:08 05-07-79



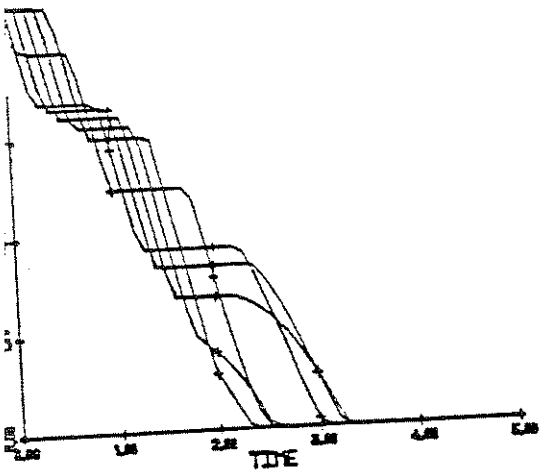


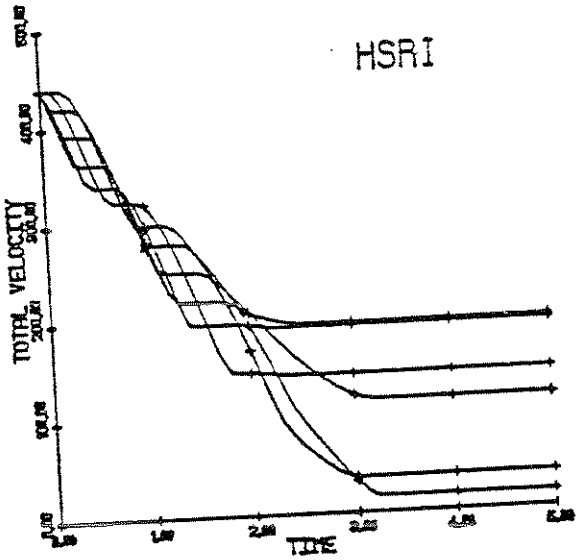


Marquard's free-wheeling symmetric two-wheeled vehicle model.

ω_0, v_0 same on 6 runs with θ varied:
θ = 0°, 30°, 60°, 90°, 120°, 150°.

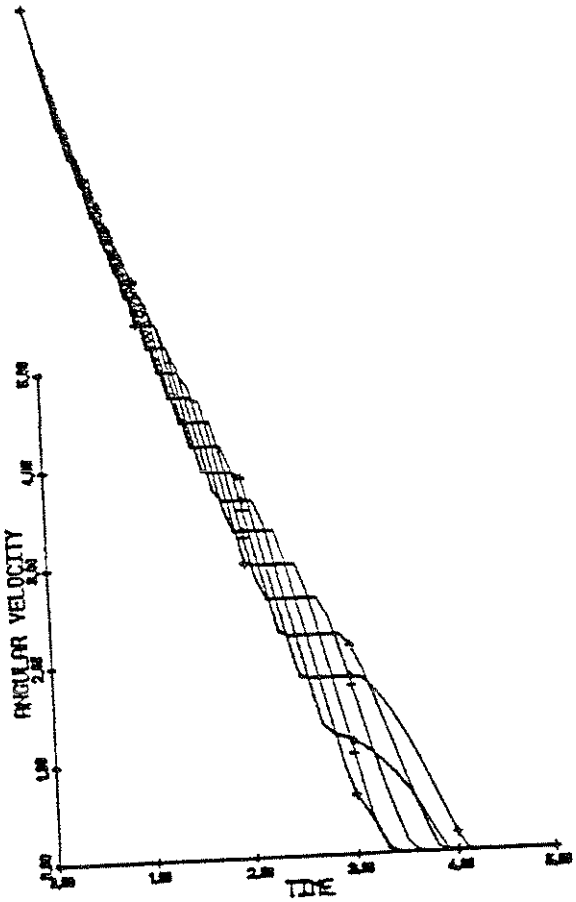
KZM

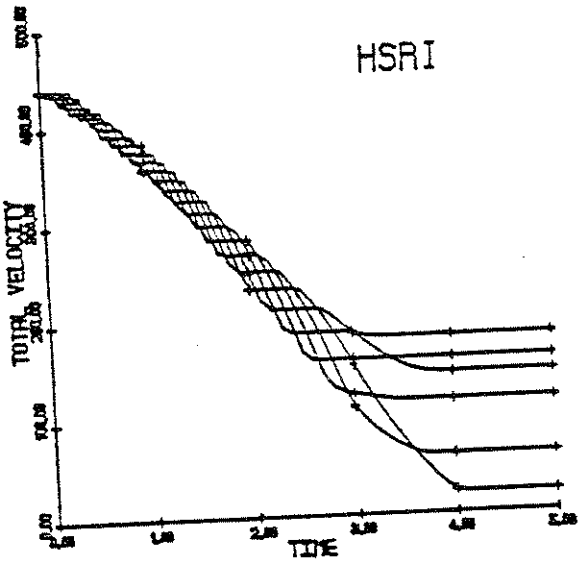


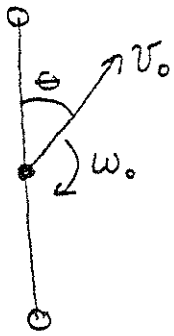


$\nearrow v_0$ Marquard's free-wheeling symmetric two-wheeled vehicle model.

ω_0 ω_0, v_0 same on 6 runs with θ varied:
$\theta = 0°, 30°, 60°, 90°, 120°, 150°$.







Marguard's free-wheeling symmetric two-wheeled vehicle model.

ω_0, v_0 same on 6 runs with θ varied:
θ = 0°, 30°, 60°, 90°, 120°, 150°.

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 11, Applications of Classical Accident Reconstruction

Presented by: Robert L. Hess

Description:

A discussion of the application of classical, hand calculations to accident reconstruction and of the application of CRASH to crash configurations, of the sensitivity of CRASH results to certain input parameters and of operational problems encountered in using CRASH.

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 11E, Classical Reconstruction Exercises

Presented by: Kenneth L. Campbell

Description:

A structured exercise laboratory session for the group solving of basic reconstruction problems.

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 11L, CRASH Lab.

Presented by: Robert L. Hess

Description:

A laboratory session devoted to individual or small group solution of reconstruction problems using CRASH and other problem solving techniques. Input data will come from either instructor generated problems or from student-generated data based on previous laboratory sessions.

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 12, CRASH Lab Discussion

Presented by: Robert L. Hess

Description:

A discussion of the CRASH Laboratory Session

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 13, Collision Severity Measures

Presented by: Kenneth L. Campbell

Description:

The need, definition, and uses of information on collision severity are described in order to instill a sense of "purpose" in the investigator.

Topics:

Need for Collision Severity
Definition of Collision Severity
A Model
Measures of Collision Severity
Applications
Examples

NOTES

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 14, Final Examination

Presented by: Thomas L. McDole

Description:

A final course examination

Advanced Accident Reconstruction

Lesson Outline

Lesson No. 15, Summary and Closure

Presented by: Thomas L. McDole
Kenneth L. Campbell

Description:

Final class meeting, closing remarks, course evaluation, etc. as necessary to wrap-up the course.

NOTES